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GA-based Optimization of a Surface-mounted Permanent Magnet Synchronous Motor

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Abstract. The paper deals with a simple procedure to design surface mounted permanent magnet synchronous machines, based on genetic algorithms scripted in Python library *deap*. The paper includes the formulas involved in the motor design, the operational constraints, the fitness functions (torque constant and motor weight), as well as the results obtained after running the algorithm in a case study.

Key words. fitness function, genetic algorithms, surface mounted permanent magnet, motor constant, Pareto front.

1. Introduction

Permanent magnet synchronous machines (PMSMs) have brought a great interest in the last decades due to their high torque density, high efficiency and fast dynamic performance. Two main classes of PMSM are used today: surface-mounted (SPMSM) and interior magnet arrangements. The SPMSM are quite popular for their capability of reaching a wide constant power speed range with rather simple geometry. This paper will deal with this kind of PMSM (Fig.1).

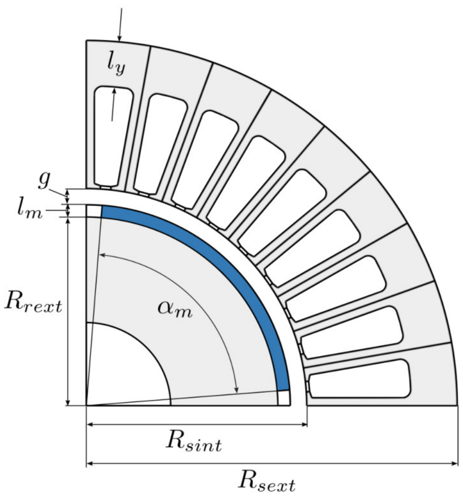

Fig. 1. Surface mounted permanent magnet machine.

Numerous studies have focused on the optimization of SPMSMs. They can be classified into two types of optimization problems: those that design some performance indicator of the whole machine (torque, weight, efficiency, etc.) [1-5] and those that optimize a part of the machine (pole shape, core shape, tooth tips, etc.). We will focus on the whole machine optimization by reviewing previous research.

In [1] a complex analytical model is introduced, composed of 17 free parameters. The objective functions are the torque density (ratio of torque per volume) and the ratio of torque per conduction losses. The torque value is commanded, not actual, so it is needed to introduce a constraint on the actual torque. In [2] a simple model, with just 6 free parameters, is applied. The objective function is a combination of efficiency and the inverse of the motor weight. In [3] another simple model, with just 5 free parameters, is used, being the objective functions the torque and the efficiency combined in a single-objective optimization problem. The analytical model is not included in the paper.

In [4,5] an analytical model is applied to the optimization problem. It is a simple but accurate model validated both experimentally and by FEM in [4,5]. A new parametric design method for SPM motors with distributed windings is proposed. The novelty is the introduction of two parameters that clarify the analysis, the rotor-stator radius ratio (x) and the magnet-airgap length ratio (b). In addition, the PMs can have a straight shape (PMs have the same short length regardless of the point in the PM, Fig.1) or rounded shape in order to reduce the torque ripple.

In this paper we rely on the formulation from [4,5] without considering rounded shape. The fitness functions of the optimization problem are the torque constant (ratio between the torque T and the current i_q) and the motor mass. These fitness functions are optimized assuming that the torque the machine is developing is 66 Nm (commanded torque), which allows to compare different solutions with the same torque.

2. Reference machine

We start with a machine whose initial parameter values are shown in Table I [4].

The thermal loading is defined as

$$k_j = \frac{P_{Cu}}{\pi D L} \,.$$

The copper filling factor k_{Cu} is the ratio between the section of all the conductors in a slot and the slot section.

Table I. - Parameters of the reference machine.

Parameter	unit	Value
Number of pole pairs (p)		3
Number of stator slots (q)		36
Stator outer radius (R_{sext})	mm	87.5
Stack length (L)	mm	110
Copper losses (P_{Cu})	W	550
Thermal loading (k_j)	kW/m^2	9.1
Airgap length (g)	mm	1
Copper filling factor (k_{Cu})		0.432
Steel grade		M600-50A
Steel loading (B_{fe})	T	1.5
PM grade		NdFeB 32 MGOe
Remanence (B_r)	T	1.16
Current (I)	A	27.66
Number of turns/phase (N_s)		120
Ph-to-ph resistance	Ω	0.835
Torque	Nm	65.98
x		0.6
b		4.5
Pole span angle (α_m)	rad	171·π/180
winding factor (k_w)		0.95
magnet relative permeability (μ_r)		1.04
Cu resistivity (ρ_{Cu})	Ωm	1.68e-8
Stator/rotor density (ρ_s)	kg/m^3	7020
PM density (ρ_p)	kg/m^3	7500
Conductor density (ρ_c)	kg/m^3	8890

x in Table I is defined as

$$x = \frac{R_{rext} + l_m}{R_{sext}} \,,$$

where R_{rext} is the rotor outer radius and l_m the magnet radial length (Fig. 1). On the other hand, b is defined as

$$b = \frac{l_m}{g} \,.$$

3. Problem formulation

The electromagnetic torque in a SPMSM is given by

$$T = \frac{3}{2} p \lambda_m i_q \,, \tag{1}$$

being λ_m the magnet flux linkage considering the fundamental component of the airgap flux density and neglecting higher order harmonics:

$$\lambda_m = (2 R_{sint} L N_s k_w B_{g1})/p, \tag{2}$$

with R_{sint} the stator inner radius:

$$R_{sint} = R_{rext} + l_m + g = x R_{sext} + g. \tag{3}$$

B_{g1} is the fundamental component amplitude of the analytical flux density distribution over one pole pair [4]:

$$B_{g1} = \frac{4}{\pi} B_{g,avg} sin \, \frac{\alpha_m}{2} \tag{4}$$

$$B_{g,avg} = \frac{\frac{l_m}{g}}{\frac{l_m}{g} + k_c \mu_r} B_r = \frac{b}{b + k_c \mu_r} B_r \,, \tag{5}$$

with k_c the Carter coefficient (see appendix).

On the other hand, i_q is given as in [4]:

$$i_q = \frac{1}{6 N_s} \sqrt{k_j \left(\frac{k_{Cu}}{\rho_{Cu}} \frac{L}{L + L_{end}} 4 \pi R_{sext} A_{slots} \right)} \,, \tag{6}$$

where L_{end} is the end-turn length:

$$L_{end} = \frac{\pi (2 R_{sint} + 5 l_t)}{p \cdot Nspp} \,, \tag{7}$$

being l_t defined in Fig.2 and l_y in Fig. 1:

$$l_t = R_{sext} - l_y - R_{sint}, \tag{8}$$

$$l_y = \frac{2 \pi R_{sext}}{4 p B_{fe}} x B_{g,avg} \,, \tag{9}$$

and N_{spp} is the number of slots per pole and phase:

$$N_{spp} = \frac{q}{6p} \,. \tag{10}$$

On the other hand, A_{slots} is the total area of the q slots:

$$A_{slots} = q A_{slot} \,, \tag{11}$$

where A_{slot} is the section of a single slot (Fig. 2):

$$A_{slot} = \frac{w_1 + w_2}{2} H_2 \,, \tag{12}$$

being w_1, w_2 and H_2 defined as (Fig. 2):

$$w_1 = 2 \pi (R_{sint} + H_0 + H_1) \frac{1}{q} - w_t \,, \tag{13}$$

$$w_2 = 2\pi \frac{R_{sint}+l_t}{q} - w_t \ , \quad (14)$$

$$H_2 = l_t - H_0 - H_1 \ , \quad (15)$$

$$H_0 = 0.05 l_t \ , \quad (16)$$

$$H_1 = 0.02 l_t \quad (17)$$

In the previous expressions the parameter w_t is the width of a tooth (Fig.2). Assuming that all the airgap flux concentrates on the teeth results:

$$w_t = \frac{2\pi R_{sext}}{q} x \frac{B_{g,avg}}{B_{fe}} \ . \quad (18)$$

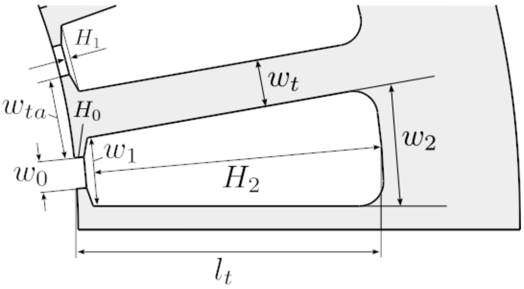

Fig.2. Geometrical magnitudes of the slot.

Regarding the fitness functions, we have considered in our study two of them: torque constant (T/i_q) and motor mass m. Taking into account (1) and (2) it is verified that

$$\frac{T}{i_q} = \frac{3}{2} p \lambda_m = 3 R_{sint} L N_s k_w B_{g1} \ . \quad (19)$$

Therefore, optimizing the torque constant is equivalent to optimizing (19).

As for the motor mass, it is given by

$$m = m_s + m_r + m_c + m_p \ , \quad (20)$$

being m_s the stator mass, m_r the rotor mass, m_c the conductor mass and m_p the permanent magnet mass. Each of these masses will be given by the product of volume and density:

$$m_s = v_s \cdot \rho_s \qquad m_r = v_r \cdot \rho_s$$

$$m_c = v_c \cdot \rho_c \qquad m_p = v_p \cdot \rho_p \quad (21)$$

The volume of each part is given by

$$v_s = \pi R_{sext}^2 L - \pi R_{sint}^2 L - A_{slots} L - \frac{w_1 + w_0}{2} H_1 q L$$

$$- w_0 H_0 q L \quad (22)$$

$$v_r = \pi R_{rext}^2 L \quad (23)$$

$$v_c = q A_{slot} k_{Cu} (L + L_{end}) \quad (24)$$

$$v_p = 2 p l_m \frac{\alpha_m}{p} R_{sint} L \quad (25)$$

4. Proposed evolutionary approach

The theory of evolution proposed by Darwin, which posits that individuals better adapted to their environment are more likely to survive and reproduce over successive generations, serves as the foundation for genetic algorithms [6]. At the core of genetic algorithms lies the iterative refinement of a population of candidate solutions, referred to as individuals. Each individual is represented in a chromosome-like structure, where each gene corresponds to a variable in the solution. A fitness score is assigned to each individual, reflecting how effectively it aligns with the objectives of the problem within the search space. Based on this fitness score, certain individuals are probabilistically chosen to act as parents for generating new offspring. Typically, fitter individuals have a higher likelihood of being selected for reproduction.

New offspring are created using two primary operators: crossover and mutation. Crossover involves combining genetic material from two parent individuals, while mutation introduces random alterations to the genetic information of a single individual. By carefully adjusting the probabilities of these operators, genetic algorithms can achieve an effective balance between exploring new areas of the search space and exploiting known high-performance regions. To ensure a good performance of the algorithm, proper calibration of both probabilities ensures that the algorithm effectively navigates the search landscape, identifying high-quality solutions efficiently.

In this study, we implement a $\mu+\lambda$ genetic algorithm [7]. In each generation, an offspring population of size λ is generated from a parent population of size μ. The offspring are produced through stochastic applications of the crossover and mutation operators, meaning that new individuals in λ may result from either crossing over or mutating the parent individuals in μ. Once the combined population of size $\mu + \lambda$ is formed, a selection process is applied to choose the μ individuals that will form the next generation. This selection is performed via a tournament mechanism, where the best individual is chosen from a randomly sampled subset of the combined population. Consequently, offspring must compete with their parent generation to secure a place in the subsequent iteration.

4.1 Individual representation and generation

In this work, each individual represents a possible motor configuration. It is proposed that the coding of individuals is based on a sequential enumeration of the motor parameters defined in Table II. Thus, for instance, the reference machine described in Table I can be codified in a chromosome-like structure as follows:

$$\Delta = [3, 36, 87.5, 110, 9.1, 120, 0.6, 4.5, 171 \cdot \pi/180],$$

where Δ represents an individual of the problem.

Table II.- Variables of the problem.

Parameter	Unit	Feas. interval
Number of pole pairs (p)		2, 3
Number of stator slots (q)		24, 36
Stator outer radius (R_{sext})	mm	75 ÷ 100
Stack length (L)	mm	95 ÷ 130
Number of turns/phase (N_s)		90 ÷ 132
x		0.5 ÷ 0.7
b		3 ÷ 6
Pole span angle (α_m)	rad	160·π/180 ÷ π

In sight of the proposed formulation, it is clear that individuals can be generated purely from the random selection of the different motor parameters. In this context, each gene of the individual is obtained as a random number. The real numbers are generated following a uniform distribution bounded between the extreme values described in Table II. The discrete values are obtained with a random selection between all the possible configurations.

4.2 Fitness function

The fitness function weights the performance of the individuals given a specific objective. In this case, the main objective of the motor design will be to minimize the current required to provide the reference machine torque (65.98 Nm). Thus, when evaluating an individual, the set of equations defined in Section 3 will be solved considering the parameters listed in Table II (those defined by the individual Δ) as known. Once all the equations have been solved, the fitness function returns the value of i_q, which must be minimized by the algorithm.

4.3 Genetic operators

To ensure a good performance of the proposed methodology, tailored algorithms are proposed for mutation and crossover operations. On the one hand, the mutation operator consists in randomly changing one of the parameters of the motor. This is done by analogy to the individual generation, that is, changing randomly the value of the gene within the permissible band defined in Table II. On the other hand, a two-point crossover operator is used. This operator randomly selects two points (two positions) to exchange between the genetic information of the individuals. Thus, the two individuals are modified in place and both retain their original length.

5. Simulation results

In this section, two blocks of simulation results are presented. First, the performance of the proposed methodology for the motor design is tested with a single objective in consideration. In this context, it is attempted to design the motor by minimizing the current required to obtain the torque of the reference machine. On the other hand, the design of the machine is carried out taking into consideration two conflicting objectives such as the minimization of the current to obtain the reference torque as well as minimizing the weight of the system.

The design of the motor has been addressed by means of a genetic algorithm, using the selection, crossover and mutation rules proposed in Section 4. Table III contains the main configuration parameters of the genetic algorithm implementation. It can be noticed that the proposed genetic algorithm is tested under different configuration parameters in terms of crossover and mutation probabilities in order to evaluate the best values for the hyperparameters used by the evolutionary approach. A set of 30 simulations have been carried out for each combination of mutation and crossover probabilities considered.

To show the convergence of the proposed genetic algorithm, Fig. 3 depicts the evolution of the fitness of the individuals throughout the considered number of generations. It is shown that 100 generations sufficed to guarantee convergence.

Table III. - Simulation parameters for both the single- and multi-objective cases.

	Single-objective	Multi-objective
μ	1000	1000
λ	1000	1000
Generations	100	100
Selection	Tournament (size=3)	NSGA-II
p_{cx}	[0.5, 0.6, 0.7, 0.8]	0.5
p_{mut}	[0.5, 0.4, 0.3, 0.2]	0.5

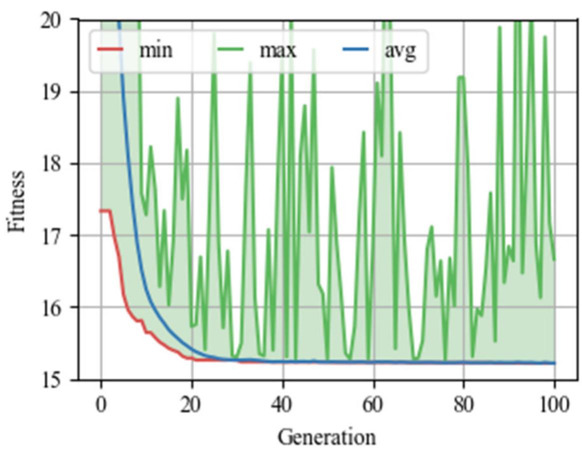

Fig.3. Evolution of the proposed genetic algorithm with hyperparameters $p_{cx} = p_{mut} = 0.5$.

5.1 Single-objective simulations

This section considers the design of the machine with the intention of minimizing the current required to generate a given torque. In this case, the torque has been set at 66 Nm, since it is the one that defines the reference machine (see Table I). The best solutions obtained using the genetic algorithm are listed in Table IV.

Table IV. - Results of the tests proposed to tune (p_{cx}, p_{mut}) for the single-objective case.

p_{cx}	0.5	0.6	0.7	0.8
p_{mut}	0.5	0.4	0.3	0.2
min	15.2038	15.2038	15.2038	15.2039
avg	15.2100	15.2114	15.2131	15.2198
σ	0.0033	0.0054	0.0047	0.0098

Observing Table IV, it can be seen that the proposed approach led to very similar solutions, reaching the best with p_{cx} = 0.5 and p_{mut} = 0.5. In particular, the best individual achieves a fitness of 15.2038 A and it is defined by the parameters presented in Table V.

Observe the marked improvement in the current value from 27.66 A (Table I) to almost half (15.20 A). As seen from Table V, all of the parameters are situated in their highest feasible value, leading to a weighty machine (close to 30 kg). Regarding the rest of parameters, the main ones are shown in Table VI, together with the same parameters of the reference machine. We can see that the area of a slot is much lower with the single-objective machine than with the reference solution due to the relationship between slot area and current in (6).

Table V. - Single-objective best solution.

Parameter	unit	Value
Number of pole pairs (p)		3
Number of stator slots (q)		36
Stator outer radius (R_{sext})	mm	100
Stack length (L)	mm	130
Number of turns/phase (N_s)		132
x		0.7
b		6
Pole span angle (α_m)	rad	π

Table VI.- Rest of the parameters corresponding to the single-objective best solution and the reference machine.

Parameter (unit)	Reference machine	Single-objective best solution
l_t (mm)	16.887	5.015
l_y (mm)	17.113	23.985
H_2 (mm)	15.705	4.664
A_{slot} (mm^2)	81.825	22.688

5.2 Multi-objective simulations

For the multi-objective case, the well-known NSGA-II algorithm is used [8]. In the NSGA-II, a $\mu + \lambda$ approach is employed to create an extended population through selection and genetic operators. Then, the extended population is sorted by Pareto dominance, placing at the first level those solutions that are dominant. The next step consists of selecting the μ best individuals according to the Pareto-based ranking. In the case that all individuals of the last level cannot pass to the next generation (μ size is the limit), a distance metric is employed to preserve diversity in the Pareto front.

In this context, the design of the machine will be carried out with two objectives in mind. On the one hand, it is desired to minimize the current required to generate a given torque. On the other hand, it is important to minimize the weight of the motor. Note that the objectives are opposite, i.e., the lower the current the higher the weight and vice versa. The resulting Pareto front is shown in Fig. 4. Note that both ends of the line correspond to the optimal solution when evaluating a single-objective optimization design. All intermediate values achieve a balance between both objectives.

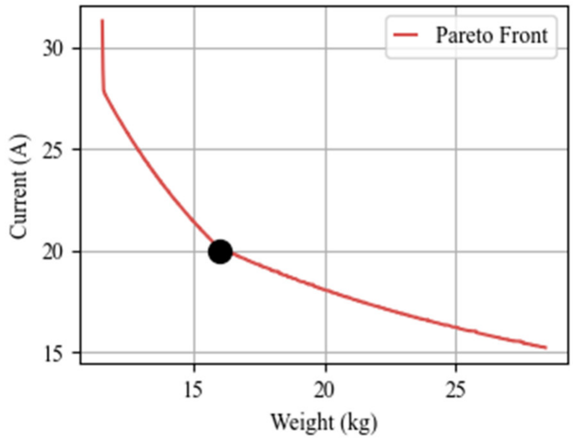

Fig. 4. Pareto front of the proposed multi-objective problem.

A compromise solution is presented in figure 4 by a point situated at a current of around 20 A and a weight of around 16 kg. The parameter values corresponding to this point are in Table VII. It should be noted that the stator outer radius is at its lowest value (around 75 mm), leading to a long motor with a short diameter.

Some other parameters of the intermediate point are shown in Table VIII, where it is seen that the slot area is even smaller than with the single-objective best solution. This leads us to think that area behaves in the same way with respect to mass and current, so that a reduction in area implies a reduction in current and mass.

Table VII. - Pareto front intermediate solution.

Parameter	Unit	Value
Number of pole pairs (p)		3
Number of stator slots (q)		36
Stator outer radius (R_{sext})	Mm	75.16
Stack length (L)	Mm	130
Number of turns/phase (N_s)		132
x		0.7
b		5.96
Pole span angle (α_m)	Rad	$177 \cdot \pi/180$

Table VIII.- Rest of the parameters corresponding to the Pareto front intermediate solution.

Parameter	Unit	Value
l_t (mm)	mm	3.552
l_y (mm)	mm	18.030
H_2 (mm)	mm	3.3033
A_{slot} (mm^2)	mm^2	12.137

6. Conclusion

In this paper a simple procedure to optimize the design of surface mounted permanent magnet synchronous motors has been presented. The optimization method has been the genetic algorithm. The fitness functions have been the torque constant (or equivalently the current for a constant torque) and the weight. It is observed that the optimization improves dramatically the fitness function, as can be seen from the Pareto front, where the current spans from 15 to around 30 A and the weight from around 11 to around 30 kg. In a future version the model will be completed to take into account the heat transfer from inside the slots due to the copper losses.

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Appendix. Carter's coefficient

In slotted electrical machines, the airgap is not uniform due to the slots. Instead of passing straight through the airgap, part of the magnetic flux spreads laterally around the slot openings, causing an increase in the effective airgap and hence in the permeance and the leakage reactance:

$$k_c = \frac{g_{effective}}{g}$$

being g the airgap length and $g_{effective}$ the airgap length of a slotless machine with the same permeance than the real slotted machine. The expression of the Carter's coefficient is the following [5]:

$$k_c = \frac{1}{1 - k_{so}\gamma'}$$

where k_{so} has the following expressions:

$$k_{so} = \frac{w_0}{w_0 + w_{ta}}$$

being w_0 and w_{ta} defined in Fig. 2. On the other hand, γ' is given by

$$\gamma' = \frac{2}{\pi}\left(\tan^{-1}\frac{w_0}{2g} - \frac{2g}{w_0}\ln\sqrt{1 + \left(\frac{w_0}{2g}\right)^2}\right)$$