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Optimization of Offshore Wind Farms Configuration Minimizing the Wake Effect

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Abstract. Currently, there has been a great development of the wind energy market, which is accompanied by an increase in the number of wind farms at sea, the offshore wind farms.
Therefore, it is crucial to ensure that efficiency in energy production is maximum and that the levelized cost of energy (LCOE) is minimal.
In this paper, a mixed-integer linear programming model (MILP) is proposed to find the best wind farm layout taking into account the wake effect in order to maximize energy production. The design of an offshore wind farm located at the North Sea is considered as a case study, contemplating three situations regarding the number of wind turbines to be installed and to determine the best positioning of them in order to maximize energy production, taking into account the wake effect and the lowest LCOE.

Keywords. Wake effect; offshore wind farm; optimization; levelized cost of energy.

1. Introduction

The big investment in renewable energies started in the 70s, with the need to guarantee diversity, security of energy supply, and the obligation to protect the environment. Nowadays, among several types of renewable energy, wind energy is seen as one of the most promising. In the last two decades, there has been a significant development in the exploitation of wind energy, increasing the number of onshore and offshore wind farms (OWF). However, the most outstanding progress that has been made was installing turbines at sea, OWF. Offshore wind farms have huge advantages over onshore. Among them, can stand out the higher efficiency, less visual impact, and the absence of obstacles that allow the wind to reach a greater constant speed. However, the costs are pretty high due to the maintenance and installation of wind turbines (WTs) at sea. The European Union recently presented a study (An EU Strategy to harness the potential of offshore renewable energy for a climate neutral future) [1] exposing the enormous potential that remains to be tapped in Europe's seas. The goal is to multiply offshore wind energy by 20 until it reaches 450GW, thus meeting energy decarbonization targets and achieving carbon neutrality by 2050 [1]. According to [2], expected by 2050, 15% of capacity is located in the South Seas and the remaining 85% in North Seas.

Consequently, it is crucial to optimise the efficiency of wind farms. One of the relevant aspects to consider is the layout of the turbines, given the impact that the wake effect has on energy production.
So far, several studies have been carried out to find the best solution and the most efficient models to optimise the WTs location. In [3], the authors address some issues to be considered when building OWF, such as the distance between WTs. If they are too close, the wake effect will affect energy production. On the other hand, the grid cost will be pretty high if they are too far apart.
In [4], the authors propose a Mixed-Integer Linear Programming to design the layout of the wind farm, in which the turbines are distributed to obtain the maximum output of the wind energy while minimising the energy losses. In addition, other authors have addressed the issue of optimising the location of WTs using models based on genetic algorithms (GA), such as Mosetti et al. [5].
In this paper, to study the impact of the wake effect, the Jensen model was used. The Jensen model is a simplified way to calculate the velocity considering the wake effect, initially presented by Jensen in [6]. The model was later improved in 1986 by Katic et al. [7], widely adopted in the design of wind farms.
In this work, the main aim is to obtain the optimal layout of the WTs, minimising the wake effect and therefore maximising the energy production. Furthermore, it is also intended to calculate the Levelized cost of energy (LCOE), which identifies the most economically favourable solutions.

The paper is organised into five sections. In Section 2, the importance of wind energy is analysed, and it is shown how to obtain energy from wind. Section 3 presents the linear programming optimisation model used for the optimal placement of WTs. In Section 4, the case studies are addressed, and the obtained results are presented and discussed. Finally, Section 5 highlights some conclusions.

2. Wind Resource

To evaluate the production of electricity is essential to analyse the potential of the wind resource, taking into account technical and physical aspects that affect the energy production of a wind farm. In this section, some aspects will be discussed, among which the Weibull wind speed distribution, Prandtl's logarithmic law, the power contained in the wind, thrust coefficient and Jensen's model for the wake effect are highlighted.

To obtain wind energy potential is essential to know the wind distribution throughout the year. Several probabilistic distributions portray the wind regime, but the Weibull distribution is usually considered the most appropriate [8]-[10].

The friction between the earth's surface and the wind will result in a decrease in wind speed. Thus, the wind speed increases proportionally with height due to the friction with the earth's surface. Therefore, adjusting the recorded wind speed is necessary according to the vertical height of interest. In [11], the importance of having a relationship about the instantaneous wind variation with height and the wind probability distribution parameters is addressed. Prandtl's logarithmic law is a good interpretation of the wind speed variation with height, reproduced in Equation (1).

$$\bar{u}(z) = \bar{u}_r \frac{\ln \frac{z}{z_0}}{\ln \frac{z_r}{z_0}} \qquad (1)$$

where $\bar{u}(z)$ is the average wind speed at height z, z_0 is the surface roughness, $\bar{u}_r$ is mean speed recorded at the reference height z_r [11]. The roughness z_0, is defined as a function of the height of the layers of the earth's surface.

A. Power in the Wind

The power available in the wind results from the kinetic energy associated with an air column moving at a constant and uniform speed u (m/s), and can be calculated using Equation (2)

$$P_{wind} = \frac{1}{2}(\rho A u)u^2 = \frac{1}{2}\rho A u^3 \qquad (2)$$

where P_{wind} is the power available in the wind (W), A is the swept area by the rotor blades (m^2), u is the wind speed (m/s), and ρ is the air density usually considered constant during the year, the standard value being equal to 1,225kg/m^3 [12].

From the analysis of Equation (2), it can be concluded that wind speed strongly impacts on the available power. Furthermore, the wind speed and direction have a fundamental importance in choosing the best positioning of the turbines. However, wind power cannot be fully converted into mechanical power in the turbine, because by Betz's law, it is at most 59,3% [13] of the kinetic energy into mechanical energy to be used in the turbine.

With the knowledge of the wind profile and the turbine power curve, it is possible to determine the annual energy produced, for a single wind direction only, by the conversion system using Equation (3)

$$AEP = T \int_{v_0}^{v_\infty} P_e(v) f_w(v) dv \qquad (3)$$

where AEP is the annual energy produced (kWh), T is the number of yearly hours, f_w is the Weibull probability density function, and $P_e(v)$ is WT power (kW) for v wind speed [12].

One of the fundamental indicators for characterising the wind resource at a given location is the wind's knowledge, indicating the wind's frequency and speed in different directions. This factor is important because it enables to know which are the dominant wind directions, helping to design the wind farm in order to minimise wake effects and maximise energy production [14]. With the data extracted from the wind rose, each wind direction is associated with a Weibull distribution, providing an estimation of the wind distribution during the year. Equation (4) shows the inclusion of the wind directions in Equation (3), where σ represents the wind direction in degrees [12].

$$AEP = T \int_{\sigma_0}^{\sigma_{360}} \int_{v_0}^{v_\infty} P_e(v) f_w(v) dv d\sigma \qquad (4)$$

To calculate the energy production for a wind farm, it is important to analyse the wind speed deficits caused by upstream WTs on downstream turbines. The wake effect and resulting wind deficits from upstream WTs are directly related to the thrust coefficient, C_t, of the upstream wind turbine. This coefficient, also known as the drag coefficient, shows a significant decrease with increasing speed, exemplified in the curve in Figure 1. The power curve is crucial for determining the power output of a single turbine, the thrust coefficient curve is essential for determining the wake effects and power output for a series of WTs [12].

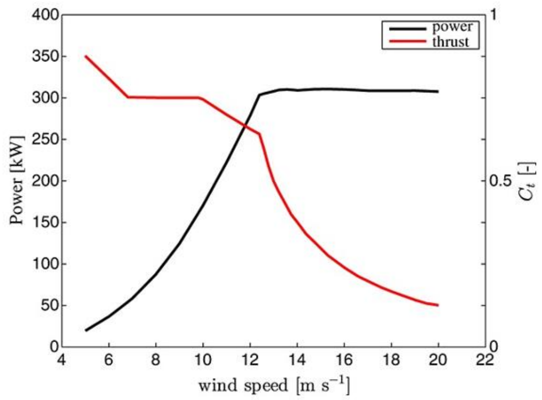

Fig.1 - Power curve and coefficient thrust, *Ct* [15]

B. Jensen model for the wake effect

A moving air mass has kinetic energy, which depends on the air mass and the wind speed. Part of the kinetic energy is converted into mechanical energy by WTs when the air passes through the blades. The wake effect reflects the interference that the wind passing through one turbine exerts on another, reducing the air mass flow and wind speed, reducing wind energy production [16], [17].
The Jensen model, illustrated in Figure 2, is used to model the wake effect in wind farms.

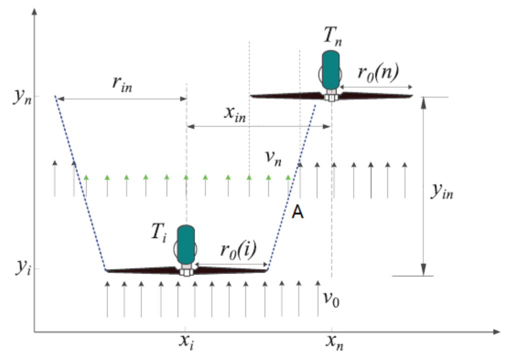

Fig.2 - Principle of the Jensen wake effect model (top view) [17]

Figure 2 represents a single wake effect where T_i is located at coordinates (x_i, y_i) and T_n at coordinates (x_n, y_n), representing the upstream and downstream turbines, respectively. The wake effect is axisymmetric. It depends on the distance between T_i and T_n with respect to the wind direction, as shown in the dashed line A with the wake radius, r_{in}. The wind speed at T_n is given in Equation (5).

$$v_n = v_0\left[1-\sqrt{\sum_{i=0}^{N}\left[\frac{2a}{\left(1+\frac{\alpha y_{in}}{r_{in}}\right)^2}\right]^2}\right] \tag{5}$$

where

$$r_{in} = \alpha y_{in} + r_0(i) \tag{6}$$

$r_0(i)$ is the radius of the upstream turbine, y_{in} is the distance between the turbines, measured in the wind direction, and α is a dimensionless parameter and determines how fast the wake expands, given by Equation (7) [18].

$$\alpha = \frac{1}{2\ln\left(\frac{z}{z_0}\right)} \tag{7}$$

where z is axis height and z_0 is the length of the surface roughness. The value of roughness in water is usually 0.0002, although it may increase with sea conditions.
The fractional decrease in wind speed between the free-flow wind speed, v_0, and the turbine is shown by the axial induction factor, a, as represented in Equation (8).

$$a = \frac{\left[1-\sqrt{1-C_t}\right]}{2} \tag{8}$$

where C_t is thrust coefficient. Thus, the wind speed reduction at T_n for multiple turbines is represented by Equation (9) [18].

$$v_n = v_0\left[1-\sqrt{\sum_{i=0}^{N}\left[\frac{1-\sqrt{1-C_t}}{T}\right]^2}\right] \tag{9}$$

where

$$T = \left[1+\frac{y_{in}}{2\ln\left(\frac{z}{z_0}\right)\left(\alpha . y_{in} + r_0(i)\right)}\right]^2 \tag{10}$$

3. Optimization Model

This section presents a Mixed Integer Linear Programming model to determine the optimal location of WTs in an offshore wind farm.
In this study, the objective is to optimise the locations of WTs to maximise energy production, taking into account the wake effect for a given set of possible turbine locations and a limit on the number of turbines to be installed [20].
The following sets and parameters are considered:

- $N=\{1,\dots,n\}$ is the set of all possible locations for the turbines.
- E_i is the energy generated by the turbine installed at site i, without considering the wake effect, with $i \in N$, calculated from the Equation (4)
- I_{ij} é the interference (loss of produced energy) at site i when a turbine is installed at site j, with $i,j \in N$ (it is considered $I_{ii}=0$). These values are calculated using the Jensen model, equations (5)-(10).
- U maximum number of turbines to be installed (the limit is often related to the available capital).

The decision variables are as follows:

- for $i \in N$, the binary variable x_i which takes the value 1 if a turbine is installed at site i and takes the value 0 otherwise;
- for $i \in N$, the variable w_i represents the total interference caused at site i.

If a turbine is installed at site i, the total interference, w_i, caused by all the other turbines is given by the sum of the interference caused by each one, i.e.,

$$w_i = \begin{cases} \sum_{j \in N} I_{ij} x_j & , x_i = 1 \\ 0 & , x_i = 0 \end{cases} \tag{17}$$

The total interference, w_i, corresponds to the reduction in energy production by the wake effect at site i. Therefore, if a turbine is installed at site i the energy produced is given by E_i - $E_i w_i$.

The objective function in the optimization model is given by

$$maximize \sum_{i \in N} (E_i x_i - w_i E_i) \quad (18)$$

To ensure that it is valid (17), the following constraint is considered

$$\sum_{j \in N} I_{ij} x_i \leq w_i + M(1 - x_i) \quad (19)$$

where M is sufficiently large number. Effectively, if $x_i=0$, by the inequality (19), $\sum_{j \in N} I_{ij} x_i \leq w_i + M$, and therefore there is no lower bound for the value of w_i. However, since the coefficient w_i in the objective function is negative, and the model is of maximization, then w_i will assume the smallest possible value, which is zero because of the constraints. Furthermore, if $x_i=1$, by the inequality (19), $\sum_{j \in N} I_{ij} x_i \leq w_i$ and, since the coefficient w_i is negative in the objective function in the maximization model, w_i assumes the smallest possible value, which is $\sum_{j \in N} I_{ij} x_i$ and in this way, it is verified (17).

The optimization model for determining the location of WTs can be written as follows:

$$maximize \sum_{i \in N} (E_i x_i - w_i E_i) \quad (20)$$

Subject to

$$\sum_{j \in N} x_i \leq U \quad (21)$$

$$\sum_{j \in N} I_{ij} x_i \leq w_i + M(1 - x_i) \quad (22)$$

$$x_i \in \{0,1\}, i \in N \quad (23)$$

$$w_i \geq 0, i \in N \quad (24)$$

The objective function (20) corresponds to the maximization of the energy produced, considering the losses by the wake effect. The constraint (21) limits the number of turbines to install. The restriction (22) relates the variables x and w ensuring that (17) is valid. Finally, the constraints (23) and (24) are sign constraints on the variables.

4. Results and Discussion

In this section, a case study will be presented that has the main objective to obtain the optimal location of the WTs in an OWF to maximize the energy production, and minimize the wake effect and LCOE. The results are presented and discussed, as well as the effectiveness of the developed optimization model. The model was solved using FICO Xpress Optimization software (Xpress installed January 2021 with Solver Xpress-Optimizer 37.01.02 and Xpress-Mosel 5.4.1) [21].

The turbines used were Vestas v164-8.0, a Danish turbine widely used in OWF, with the specifications shown in Table III [22].

Table III - Wind turbine specifications

Power	8 MW
Cut-in wind speed	4 m/s
Cut-out wind speed	25 m/s
Rotor diameter	164 m
Area swept by the blades	21124 m^2
Shaft height	140 m

For the placement of the turbines, the chosen location was in the North Sea, the type of turbine arrangement for the offshore wind farm was 4x4, as it is possible to see in Figure 3.

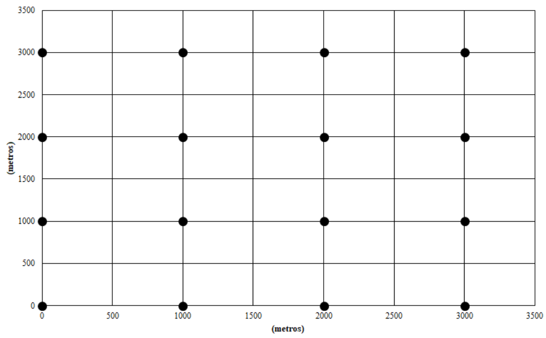

Fig.3 - Possible location of the turbines

The wind resource used, presented in Table IV [23] with the average annual speeds, enables to obtain the Wind Rose illustrated in Figure 4, where the average speeds in each wind direction are shown. It can be concluded that the predominance of the wind is north (0º) although it presents a balanced distribution in the various directions.

Fig.4 - Wind Rose - Average Speed

Next, in Figure 5, the relative frequency of occurrence in each wind direction is presented. In the situation of 330º relative frequency of occurrence is 14.6 %.

The average speeds presented are at the height of 150 m. To adjust the wind speed to the turbines' axis height, 140 m, Equation (1) is used.

Fig.5 - Wind Rose - Frequency

Then, the frequency of occurrence for the various wind speeds and directions is calculated, using the Weibull distribution. In Figure 6 an example of the occurrence of the wind speed for 0º is presented.

Applying Equation (3), the expected value for the total annual production in each wind speed considered is calculated. Subsequently, Equation (4) is used to calculate the expected value for the total annual production at each wind speed and direction.

Table IV - Average speed and annual frequency of occurrence

Direction (º)	Average speed (m/s)	Frequency (%)
0	12,11	12,1
30	11,90	8,5
60	10,38	6,4
90	8,14	6,7
120	9,77	6,3
150	8,34	5,9
180	7,93	5,5
210	10,18	7,8
240	8,14	8,3
270	8,24	6,5
300	9,05	11,4
330	11,59	14,6

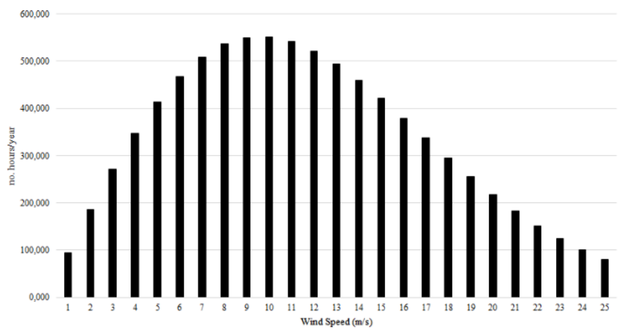

Fig.6 - Number of hours when wind speed in the 0º direction occurs

For example, at 0º, the highest production, 4.11 GWh, corresponds to the speed of 12 m/s, as shown in Figure 7. Adding up all the energy values, the total annual energy production for this turbine will be 44.5 GWh.

In the study case 16 possible locations for turbines are considered, with four rows in which each row has four locations, as shown in Figure 3. The horizontal and vertical distance between two neighbouring locations is 1000 meters.

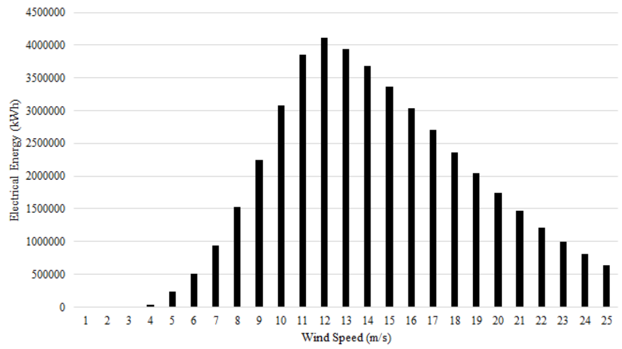

Fig.7 - Electric energy produced in a year by a turbine driving a 0º direction

The Jensen model for the wake effect is used to calculate the input wind speed on each turbine v_n, according to what is presented in Figure 2. The energy production in each turbine is different because not all turbines receive the same amount of wind. Using the thrust coefficients and applying the Equations (5)-(10), the wind speed, v_n, is obtained. Thus, if all turbines are installed, the amount of energy produced by each will decrease, depending on the wind direction.

Figure 8 shows the energy produced in each row, from Row 1 to Row 4. For example, the impact of the wake effect in Row 2 results in a 7.84 % reduction in energy production, while in Row 3, the reduction is 13.20%, and in Row 4, it is 2.07%.

In the application of the optimization model, regarding the number of turbines to be installed, three situations were considered: U=8 turbines, U=10 turbines and U=12 turbines.

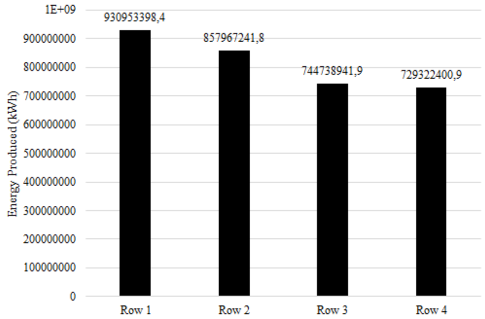

Fig.8 - Energy production in each row considering the wake effect

Not considering the wake effect, selecting the eight turbines, the value of annual energy produced is 7450 GWh, where the average production per turbine is 931 GWh. Setting ten turbines, the value of yearly energy produced is 9270 GWh, and the average annual production per turbine is 927 GWh. Furthermore, selecting 12 turbines, the value of yearly energy produced is 11100 GWh, where the average production per turbine is 925 GWh. Figures 9, 10 and 11 show the optimal solutions for the distribution of turbines considering at most 8, 10 and 12 turbines, respectively, contemplating the wake effect.

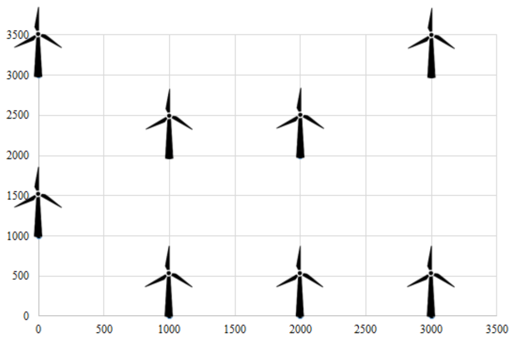

Fig.9 - Optimal location of WTs when U=8

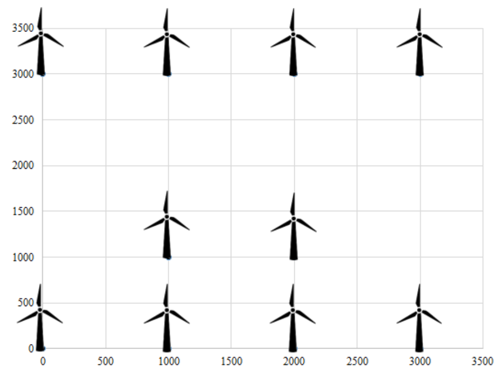

Fig.10 - Optimal location of WTs when U=10

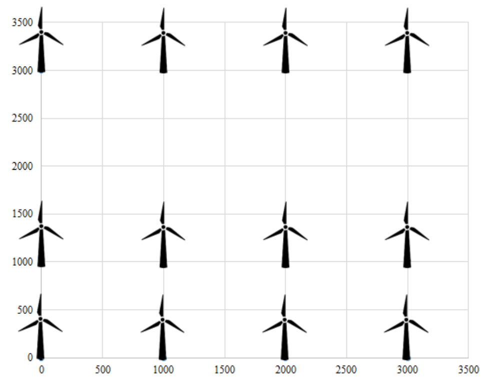

Fig.11 - Optimal location of WTs when U=12

It can be seen that, with eight turbines, considering the wake effect, there is a decrease of approximately 937 GWh of energy produced, which corresponds to a reduction of 12.58 %. With ten turbines, there is a decrease of approximately 1140 GWh of energy produced, which corresponds to a reduction of 12.30 %. With 12 turbines, there is a decrease of approximately 1470 GWh of energy produced, which corresponds to a reduction of 13.29 %.
Table V shows a comparison of the results of the three situations considered.

Table V – Comparing results

	8 turbines		10 turbines		12 turbines	
	with wake effect	without wake effect	with wake effect	without wake effect	with wake effect	without wake effect
Annual energy production (GWh)	6510	7450	8130	9250	9620	11100
Total installed power (MW)	64	64	80	80	96	96
Average annual production per turbine (GWh)	814	931	813	927	802	925

In this work, the levelized cost of energy, LCOE, was calculated, which is an important indicator that allows the producer to evaluate the average cost of energy produced over the project's useful life and can be calculated using Equation (25). Moreover, that indicator is widely used in projects involving renewable energy sources. Therefore, the minimization of LCOE is an important objective to consider [24], [25].

$$LCOE = \frac{\sum_{t=1}^{n} \frac{I_t + M_t + F_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{E_t}{(1+r)^t}} \tag{25}$$

where

$$E_t = E_0\left(1 - \frac{DR}{100}\right) \tag{26}$$

where I_t is the investment of the offshore wind farm in year t, approximately 4555 €/kW, [26]; M_t is the operation and maintenance costs of the offshore wind farm, approximately 15 % of the investment I_t, [26]; F_t is the fuel costs, in this case, it is zero since we are talking about renewable energy; E_t is the amount of energy produced in one year; E_0 is the energy produced in the first year of installation; r is the discount rate, 10 %; n is the expected lifetime of the wind farm in years, considered 25 years; DR is the degradation factor, for WTs, an annual production decrease of 1.6% [25].
In Figure 12, the $LCOE$ values are presented for each proposed situation, applying the equation (25). For ten turbines case, the levelized energy cost results in 0.02091 €/kWh.

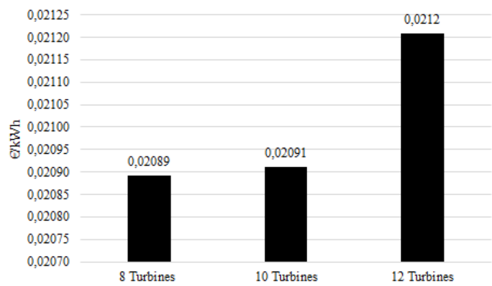

Fig.12 - LCOE for each of the situations

5. Conclusions

In this work, the mathematical model for the wind characterization was addressed, and a Mixed Integer Linear Programming model was proposed to obtain the optimal distribution of a given number of turbines to maximize energy production.
The study was carried out considering all wind directions and their relative frequency of occurrence.

A conclusion drawn is that the relative position of the turbines influences energy production, i.e., the wake effect significantly influences energy production in an offshore wind farm.

From the optimal solutions for 8, 10 or 12 turbines, we can conclude that in the first case, 8 turbines, there is a decrease of approximately 12.58 in energy production, taking into account the wake effect and has LCOE of 0,02089 €/kWh. In the second case, 10 turbines, we can observe a decrease of approximately 12.30% in energy production and has LCOE of 0.02091 €/kWh. Finally, in the third case, 12 turbines, it can be observed that there is a decrease of approximately 13.29% in energy production and LCOE rounds 0.0212 €/kWh.

The results obtained show that the MILP-based optimization model is able to achieve, with low processing times, exact optimal solutions allowing to significantly increase the efficiency of OWF.

The offshore wind farm with 8 turbines has a lower levelized cost of energy and an average output per turbine of 814 GW, which is the best value of the three cases.

Acknowledgement

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