

# Effect of Supply Unbalance on Harmonic Emissions of Uncontrolled Six-Pulse Rectifiers

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**Abstract.** Standards dealing with power quality disturbances and emission limits for equipment and installations are constantly evolving as additional knowledge is gained. Often, each type of disturbance, such as harmonics, is considered individually and without regard for the possible effects of other types of disturbances, such as unbalance. The interdependencies between the disturbance phenomena are often disregarded. The work reported in this paper is focused on quantifying and correlating the effect of supply system unbalance on the harmonic emissions of the most basic harmonic-producing converter, the uncontrolled six-pulse rectifier. Six-pulse rectifiers with different loading conditions are supplied by sources with varying levels of unbalance to determine the effect of the unbalance on harmonic emissions via simulation. Different unbalanced supply conditions, all within credible levels, are considered and the effects on the harmonic emissions of the six-pulse rectifier are evaluated. The results of these simulations show that as source unbalance increases, THD and triplen harmonics also increase in a nearly-linear pattern. This type of relationship may be appropriate for consideration in the ongoing efforts in standardization related to both harmonics and unbalance so that more coordinated decisions can be made.

**Key words.** AC-DC power converters, power conversion harmonics, power quality, rectifiers, total harmonic distortion.

## 1. Introduction

The quality of power is an integral part of overall power systems. Higher quality power helps efficiency, saves money, and minimizes wear and tear on equipment. A major aspect of power quality is associated with voltage and current harmonics. Harmonics reduce voltage quality, increase losses, and contribute to innumerable other undesirable conditions in the power system. As a result, there are standards in place for acceptable harmonic levels and limits exist so that their impact is minimized. [1],[2].

An AC-DC rectifier is the most basic converter which produces harmonics that are injected into the power system. The expected harmonic emissions from a six-pulse uncontrolled rectifier are well known assuming a balanced three-phase supply, but an unbalanced supply can impact the emissions in a significant way. In addition, the combination of voltage unbalance and harmonic distortion can potentially lead to greater cumulative levels of disturbance effects, such as unacceptable levels of motor vibrations [3]. Continuing the evaluation of the interdependencies of these two disturbance types is the main purpose of the work reported in this paper.

Considering the effects of unbalance on harmonic emissions is not a new field of research. Previous efforts have provided advanced techniques for analyzing emissions under unbalance and also made efforts to develop correlative relationships between the two disturbance types [4],[5]. For simplicity, the work in this paper uses basic simulation techniques to evaluate emissions to limit the focus of this work on clarifying and further quantifying the relationship. These and other prior works generally support that a relationship exists, but its form has not been sufficiently evaluated and simplified to the point that it is useful for standardization.

To evaluate the effects an unbalanced source could have on the harmonic emissions from an uncontrolled six-pulse rectifier, a series of simulations has been performed. This series consists of a balanced and fourteen different unbalanced three-phase source variations applied to the rectifier. Both series resistive-inductive (RL) load and parallel resistive-capacitive (RC) load conditions are considered. Of the fourteen unbalanced sources, seven have an unbalance of 2% while the other seven have an unbalance of 4%. The results of these simulations are compared to determine what relationship, if any, may exist between amount of unbalance and amount of harmonic emission and how this relationship may be quantified.

## 2. Background

### A. Harmonics

Harmonics are integer multiples of a signal's fundamental frequency. Harmonic levels are typically quantified two ways: as individual magnitudes and collectively via indices such as total harmonic distortion (THD). Individual harmonics are commonly considered as a percentage of the power frequency fundamental term's magnitude and THD is evaluated as shown in (1). In (1),  $h$  is the harmonic number,  $I_1$  is the rms current at the fundamental (power) frequency, and  $I_h$  is magnitude of the  $h^{\text{th}}$  harmonic current. Previous work, summarized in a following subsection, has shown that an increase in source unbalance can increase individual harmonic emissions, particularly for the triplen harmonics, as well as THD [4]-[6].

$$THD = \frac{\sqrt{\sum_{h=2}^{\infty} (I_h)^2}}{I_1} * 100\% \quad (1)$$

The basic circuit used for the simulation-based analysis in this work is shown in Fig. 1. Under balanced supply conditions, the harmonics produced are the non-triplen odd harmonics [7]. These harmonics are described by (2) and (3), where  $h$  is the harmonic number and  $k$  is an integer.

$$h = 6k \pm 1 \quad (2)$$

$$h = 5, 7, 11, 13, 17, 19 \dots \quad (3)$$

Under unbalanced source conditions, significant triplen harmonics, especially the 3<sup>rd</sup>, can be produced [8],[9]. The triplen harmonics are described in (4) and (5).

$$h = 3(2k - 1) \quad (4)$$

$$h = 3, 9, 15, 21 \dots \quad (5)$$

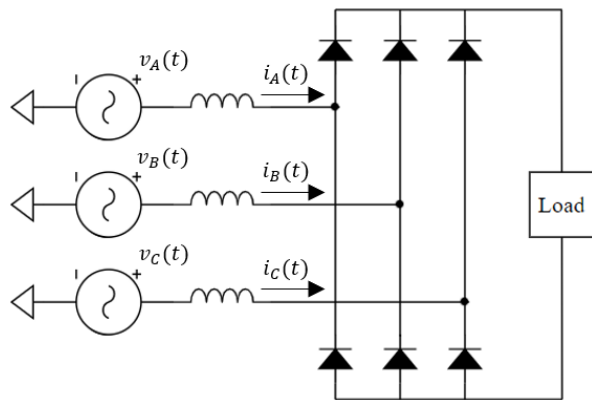


Fig. 1. Three-phase bridge rectifier circuit.

### B. Voltage Unbalance

A balanced three-phase voltage source consists of voltages that are equal in magnitude and exactly 120° apart per phase. Any deviation results in an unbalanced source. Unbalance is normally measured by evaluating positive and negative sequence values based on the symmetrical component transformation as shown in (6) and (7). In (7), the phase quantities are  $\bar{V}_a$ ,  $\bar{V}_b$ , and  $\bar{V}_c$ ;  $\bar{V}_0$  is the zero-sequence voltage,  $\bar{V}_1$  is the positive-sequence voltage, and  $\bar{V}_2$  is the negative-sequence voltage.

$$\alpha = 1 \angle 120^\circ \quad (6)$$

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} \quad (7)$$

Once the transform is applied, the percent unbalance for a positive phase sequence system is calculated using (8). For negative phase sequence systems,  $V_1$  and  $V_2$  would be swapped.

$$\% \text{ Unbalance} = \left( \frac{V_2}{V_1} \right) * 100\% \quad (8)$$

### C. Prior Work

Previous work focused on how load levels on an uncontrolled rectifier impacted the harmonic emissions when balanced and unbalanced supply sources were applied. In this previous work, eight different supply voltage scenarios, each with an unbalance of 2%, were applied to five different rectifier circuits, each with a different loading configuration. The eight supply voltage scenarios consisted of one balanced source, three sources

with the magnitudes unbalanced, two sources with the phase angles unbalanced, and two sources with both of the magnitudes and phase angles unbalanced. In every (unbalanced) scenario, the unbalance was fixed at 2%. The five loading configurations consisted of three parallel RC loads and two series RL loads, each having a different power level. In every case, applying an unbalanced source to these rectifier circuits resulted in the production of triplen harmonics and an increase in THD. A particularly important result from this prior work was that rectifier circuits with a higher THD when supplied from a balanced source, such as lightly-loaded rectifiers supplying parallel RC loads, were more sensitive to source unbalance. In these cases, and consistent with other works, there were noticeably greater increases in THD and triplen harmonics for those circuits. [4],[5],[10].

While both individual and total distortion levels were shown to be dependent on unbalance level and the effects are strongly dependent on loading, there is no general quantification of the effects of increasing (or decreasing) the unbalance level on the harmonic emissions. The work reported in this paper addresses this lack of knowledge with the eventual objective of establishing a relationship between the two disturbance phenomena that is suitable for use in standardization. Applying research works to standardization often requires a compromise between accuracy and simplicity and the specific work of this paper, combined with other prior related works, is one step toward that goal.

## 3. Simulated Scenarios

Balanced and unbalanced voltage sources will be applied to two different three-phase bridge rectifier circuits. One circuit will contain a parallel RC load, while another will contain a series RL load. All rectifier circuits will have a per-phase source impedance of 0.2 mH, while the diodes will have an RC snubber circuit of  $R=5 \text{ k}\Omega$  in series with  $C=0.05 \text{ }\mu\text{F}$ . The RC load consists of a 470  $\mu\text{F}$  capacitor in parallel with a 4  $\Omega$  resistor. The RL load consists of a 5 mH inductor in series with a 5  $\Omega$  resistor.

All rectifier circuits were constructed and simulated in MATLAB/Simulink. The resulting A-phase currents from a balanced three-phase 120 V source applied to the system containing each load are shown in Figs. 2 & 3. The power levels were 19.6 kW for the RC load and 15 kW for the RL load.

### A. Voltage Source Cases

Both rectifier circuits will have fifteen different three-phase sources applied to them. All of the sources are three-phase positive sequence voltage sources with a 120 V (line-to-neutral) base. Case 1 is a balanced source and will serve as the reference case for the unbalanced cases. Cases 2-8 are various forms of 2% unbalance, while cases 9-15 are (the same) various forms except increased to 4% unbalance. Cases 2, 3, 9, and 10 have unbalanced magnitudes, while they still have balanced phase shifts. Cases 4, 5, 11, and 12 have unbalanced phase shifts and balanced magnitudes. Cases 6, 7, 13, and 14 have both unbalanced magnitudes and phase shifts. Cases 8 and 15 have only one phase magnitude significantly lowered, simulating that phase being heavily loaded compared to the other two phases. The

TABLE I  
Balanced and Unbalanced Voltage Source Cases

| Case 1   | Cases 2 & 9                                                                                                          | Cases 3 & 10                                                                                                         | Cases 4 & 11                                                              | Cases 5 & 12                                                              | Cases 6 & 13                                                                                                                                                                          | Cases 7 & 14                                                                                                                                                                          | Cases 8 & 15                                                                                                         |
|----------|----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Balanced | $ V_A  = 120 \text{ V}_{\text{rms}}$<br>$ V_B  > 120 \text{ V}_{\text{rms}}$<br>$ V_C  < 120 \text{ V}_{\text{rms}}$ | $ V_A  = 120 \text{ V}_{\text{rms}}$<br>$ V_B  < 120 \text{ V}_{\text{rms}}$<br>$ V_C  > 120 \text{ V}_{\text{rms}}$ | $\theta_A = 0^\circ$<br>$\theta_B > -120^\circ$<br>$\theta_C > 120^\circ$ | $\theta_A = 0^\circ$<br>$\theta_B < -120^\circ$<br>$\theta_C < 120^\circ$ | $ V_A  = 120 \text{ V}_{\text{rms}}, \theta_A = 0^\circ$<br>$ V_B  < 120 \text{ V}_{\text{rms}}, \theta_B > -120^\circ$<br>$ V_C  > 120 \text{ V}_{\text{rms}}, \theta_B > 120^\circ$ | $ V_A  = 120 \text{ V}_{\text{rms}}, \theta_A = 0^\circ$<br>$ V_B  > 120 \text{ V}_{\text{rms}}, \theta_B < -120^\circ$<br>$ V_C  < 120 \text{ V}_{\text{rms}}, \theta_B < 120^\circ$ | $ V_A  = 120 \text{ V}_{\text{rms}}$<br>$ V_B  < 120 \text{ V}_{\text{rms}}$<br>$ V_C  = 120 \text{ V}_{\text{rms}}$ |

breakdown for each unbalance condition is shown in Table I, where  $|V|$  represents the voltage magnitude and  $\theta$  represents the phase angle. Because the relatively low levels of unbalance would be difficult to detect in waveforms, example supply voltage plots are not shown.

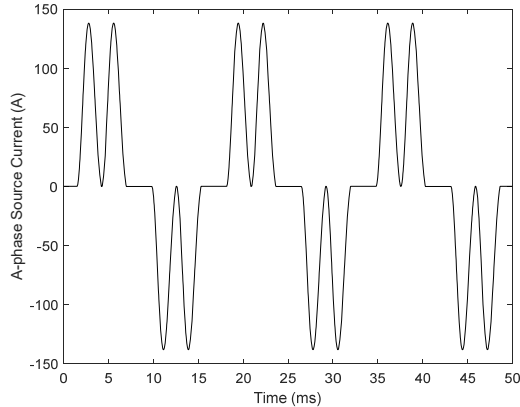


Fig. 2. Resulting source current from balanced source; 4 Ω, 470 μF load.

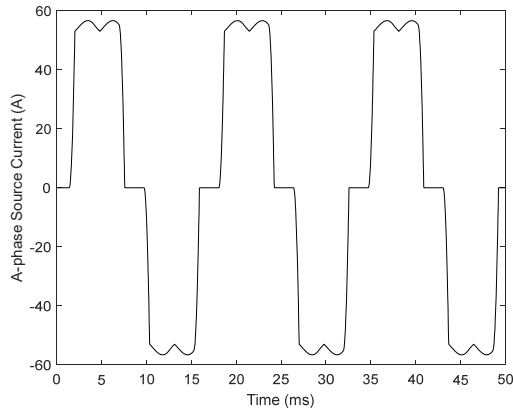


Fig. 3. Resulting source current from balanced source; 5 Ω, 5 mH load.

## 4. Results

The results of the RC load and RL load are evaluated separately. For each case, the resulting harmonics up to the 20<sup>th</sup>, as well as the THD, were recorded for each phase current. The average and maximum values were taken of each respective harmonic for the 2% and 4% unbalance results. To assess the impact on non-triplen odd harmonics, the averages and maximums of the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonics are considered. To assess the impact on triplen harmonics, the averages and maximums of the 3<sup>rd</sup>, 9<sup>th</sup>, and 15<sup>th</sup> harmonics are considered

### A. RC Load

The different unbalanced sources applied to the RC load produced a higher THD in at least one phase for each case. The average THD produced by the 2% unbalanced sources as well as the 4% unbalanced sources were both within 0.05% of the THD produced by the balanced source. The

maximum THD produced by the 2% unbalanced sources was 76.01%, 1.27% higher than the THD produced by the balanced source. The maximum THD produced by the 4% unbalanced sources was 77.31%, 2.57% higher than the THD produced by the balanced source and 1.30% higher than the THD produced by the 2% unbalanced sources. These results are displayed in Table II. Cases 5 and 12 produced the highest level of THD for the 2% and 4% unbalance, respectively. The comparison of the balanced source, case 5, and case 12 is shown in Fig. 4.

TABLE II  
THD Results for RC Load

| Source and Result    | THD    | Increase from Balanced |
|----------------------|--------|------------------------|
| Balanced Average     | 74.74% | --                     |
| 2% Unbalance Average | 74.75% | 0.01%                  |
| 2% Unbalance Maximum | 76.01% | 1.27%                  |
| 4% Unbalance Average | 74.78% | 0.04%                  |
| 4% Unbalance Maximum | 77.31% | 2.57%                  |

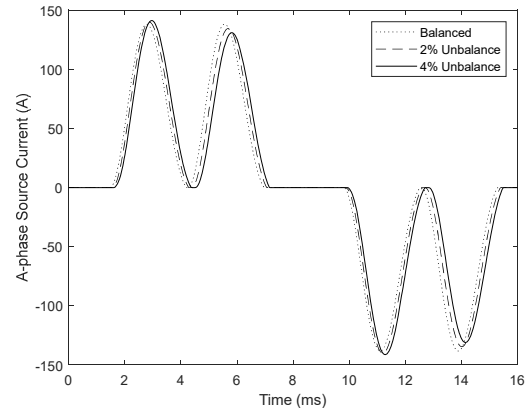


Fig. 4. Comparison of unbalanced sources to the balanced source applied to the 4 Ω, 470 μF load. Cases 5 and 12.

The impact that the unbalanced sources applied to the RC load had on the non-triplen odd harmonics is similar to the impact on THD. There was a small difference in the average values of the non-triplen harmonics, slightly decreasing as the source unbalance increased. The maximum values of the non-triplen harmonics increased as the source unbalance increased. The different unbalanced sources applied produced a higher value of each non-triplen harmonic in at least one phase for each case compared to what was produced by the balanced source. Direct comparisons of the non-triplen harmonics are shown in Figs. 5 & 6. While the effects of the various unbalance sources on non-triplen harmonics are relatively small, there

is a generally-linear association between increasing unbalance levels and increasing (non-triplen) harmonic emissions.

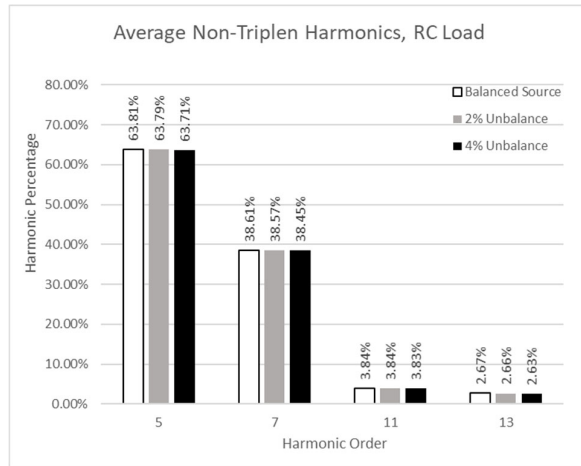


Fig. 5. Comparison of average values of non-triplen odd harmonics for the RC load.

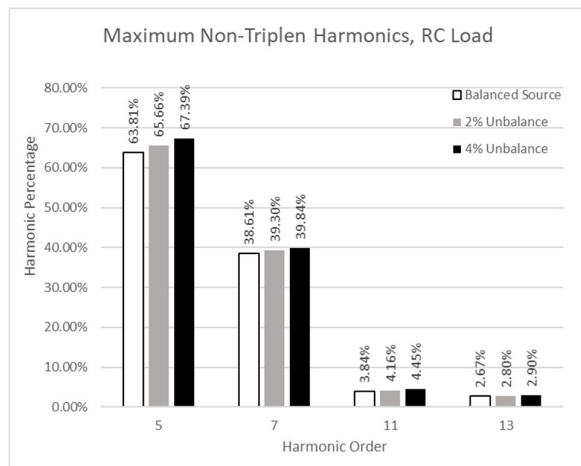


Fig. 6. Comparison of maximum values of non-triplen odd harmonics for the RC load.

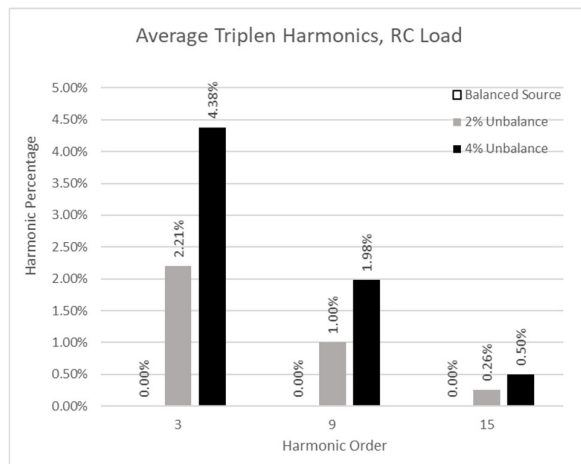


Fig. 7. Comparison of average values of triplen harmonics for the RC load.

The balanced source applied to the RC load did not produce any triplen harmonics, while the unbalanced sources did produce triplen harmonics. The average value

of the 3<sup>rd</sup> harmonic with 2% unbalance was 2.21% and with 4% unbalance the value was 4.38%, a nearly linear increase. The 9<sup>th</sup> and 15<sup>th</sup> harmonics, though smaller than the 3<sup>rd</sup>, also had near linear increases. The maximum values of the triplen harmonics also reflected the near linear patterns in the average values. The average value of the 3<sup>rd</sup> harmonic with 2% unbalance was 3.19% and with 4% unbalance the value was 6.40%. The triplen harmonic results are shown in Figs. 7 & 8.

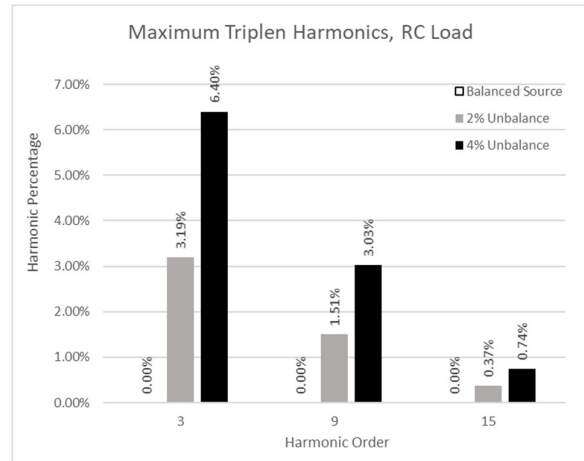


Fig. 8. Comparison of maximum values of triplen harmonics for the RC load.

## B. RL Load

The results for an RL load were similar to those of the RC load in that the different unbalanced sources produced a higher THD in at least one phase for each case. The average THD produced by the 2% unbalanced sources as well as the 4% unbalanced sources were both within 0.08% of the THD produced by the balanced source. The maximum THD produced by the 2% unbalanced sources was 27.03%, 0.98% higher than the THD produced by the balanced source. The maximum THD produced by the 4% unbalanced sources was 28.04%, 1.99% higher than the THD produced by the balanced source and 1.01% higher than the THD produced by the 2% unbalanced sources. These results are displayed in Table III. Cases 8 and 15 produced the highest level of THD for the 2% and 4% unbalance, respectively. The comparison of the balanced source, case 8, and case 15 is shown in Fig. 9.

TABLE III  
THD Results for RL Load

| Source and Result    | THD    | Increase from Balanced |
|----------------------|--------|------------------------|
| Balanced Average     | 26.05% | --                     |
| 2% Unbalance Average | 26.07% | 0.02%                  |
| 2% Unbalance Maximum | 27.03% | 0.98%                  |
| 4% Unbalance Average | 26.12% | 0.07%                  |
| 4% Unbalance Maximum | 28.04% | 1.99%                  |

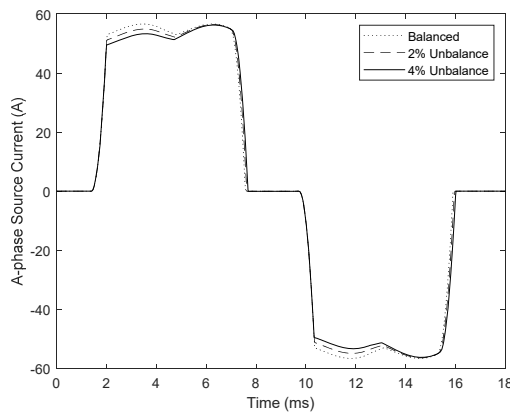


Fig. 9. Comparison of unbalanced sources to the balanced source applied to the 5  $\Omega$ , 5 mH load. Cases 8 and 15.

The impact that the unbalanced sources applied to the RL load had on the non-tripen odd harmonics is similar to the impact on THD. There was a small difference in the average values of the non-tripen harmonics, slightly decreasing as the source unbalance increased. The maximum values of the non-tripen harmonics increased as the source unbalance increased. The different unbalanced sources applied produced a higher value of each non-tripen harmonic in at least one phase for each case compared to what was produced by the balanced source. Comparisons of the non-tripen harmonics are shown in Figs. 10 & 11 and the nearly-linear correlation is easily seen.

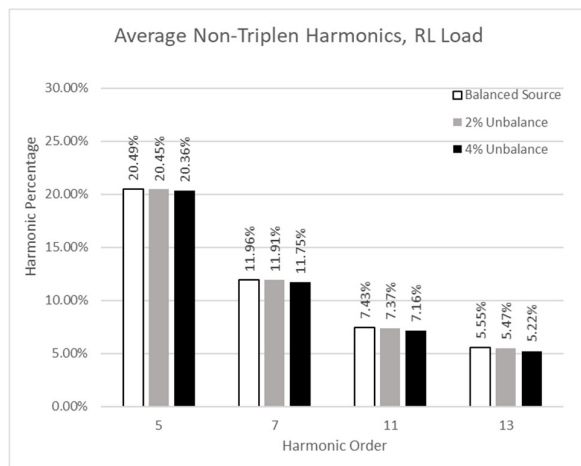


Fig. 10. Comparison of average values of non-tripen odd harmonics for the RL load.

The balanced source applied to the RL load did not produce any triplen harmonics, while the unbalanced sources did produce triplen harmonics. The average value of the 3<sup>rd</sup> harmonic with 2% unbalance was 1.35% and with 4% unbalance the value was 2.69%, a nearly linear increase. The 9<sup>th</sup> and 15<sup>th</sup> harmonics, though smaller than the 3<sup>rd</sup>, also had near linear increases. The maximum values of the triplen harmonics also reflected the near linear patterns in the average values. The average value of the 3<sup>rd</sup> harmonic with 2% unbalance was 1.85% and with 4% unbalance the value was 3.76%. The triplen harmonic results are shown in Figs. 12 & 13.

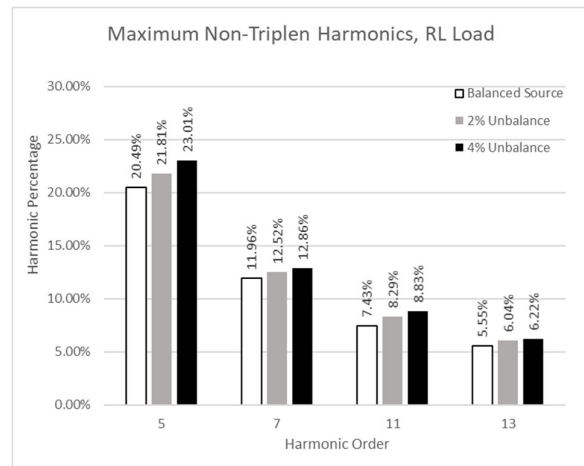


Fig. 11. Comparison of maximum values of non-tripen odd harmonics for the RL load.

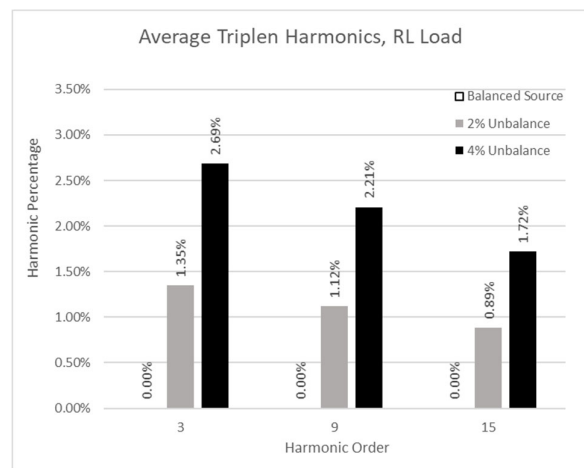


Fig. 12. Comparison of average values of triplen harmonics for the RL load.

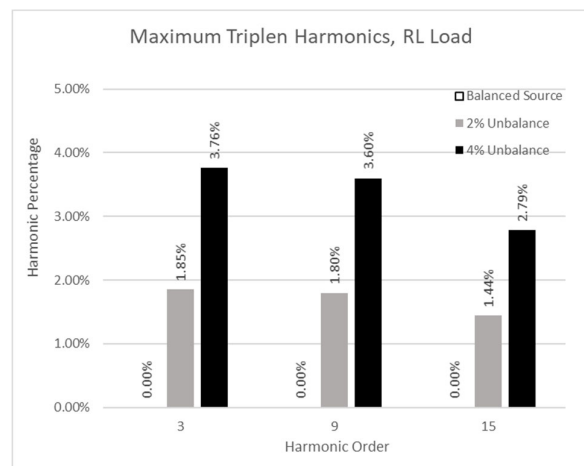


Fig. 13. Comparison of maximum values of triplen harmonics for the RL load.

## 5. Conclusion

Multiple simulations were performed to determine the effects of an unbalanced source on the harmonics produced by an uncontrolled six-pulse rectifier. The results of these simulations showed that source unbalance has an impact on the individual harmonic emissions and the THD. Every

unbalanced source produced significant triplen harmonics, while the balanced source did not produce any triplen harmonics. The average values of the non-triplen odd harmonics produced by the unbalanced sources were similar to the values produced by the balanced source, but there was a notable increase in the maximum values for both loads. Most importantly, there was a near linear increase in the average and maximum values for both the triplen and non-triplen harmonics when the source unbalance percentage was increased for both loads, albeit with a smaller overall increase for the non-triplens. This finding is considered to be the significant contribution in this work in that it could lead to a strategy for considering modifications to harmonic emission limits in the presence of supply system unbalance.

These simulations have shown that an unbalanced source can impact the resulting harmonics and that higher levels of voltage source unbalance have a larger impact on the resulting harmonics. Future work should include more simulations to determine correlations between specific types of source unbalance and the resulting harmonic emissions. This would potentially highlight how different source voltage magnitudes, phase shifts, and unbalance percentages impact the different harmonics and THD produced by an uncontrolled rectifier. Future work should also include performing similar simulations on source inverters in order to assess the impact of voltage unbalance and how it could affect the harmonic emissions of converters used in power generation.

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