

Voltage Waveform Comparison for Different PWM Modulation Strategies

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Abstract. A comparison between several commonly used PWM modulation strategies is made, namely Sinusoidal Regular-Sampled Asymmetrical PWM, Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection and Centred Asymmetrical Space Vector PWM, in order to determine which of them leads to a higher voltage waveform quality.

It is shown that the first PWM method causes higher harmonic losses and that although 3rd harmonic injection PWM spectrum contains few harmonic terms than Centred Asymmetrical Space Vector PWM, the latter shows less weighted harmonic distortion.

Key words

Voltage waveform, harmonic analysis, Asymmetrical Regular-Sampled PWM, 3rd harmonic injection, Centred Asymmetrical Space Vector PWM.

1. Introduction

Currently, AC drive systems with Pulse Width Modulated inverters have become an industrial standard. As a result of the research effort made in this field of Power Electronics several PWM switching strategies have been developed, aimed to achieve the highest output transfer ratio and the minimum low order harmonics (which cause undesirable effects on the motor side, such as harmonic losses, torque and current ripple, as well as audible noise).

Here, a comparison between several commonly used modulation strategies is established, in order to determine which of them leads to a higher voltage waveform quality. Simulations of the line-to-line inverter voltage waves are performed for three different PWM switching schemes, namely Sinusoidal Regular-Sampled Asymmetrical PWM, Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection and Centred Asymmetrical Space Vector PWM. Following, their harmonic spectra are obtained and their weighed TDH are calculated in order to compare the merit of the different considered modulation strategies.

2. PWM switching strategies

A. Suboscillation methods

This category of PWM switching strategies groups the several variations of the original standard sine-triangle algorithm, where the pulse patterns are generated by comparing a three-phase modulation reference with a triangular high-frequency carrier waveform [2], [3].

The digital implementation of this algorithm use a sampled and hold reference wave, receiving the name of Regular-Sampled PWM.

The reference modulation wave can be sampled once or twice within a carrier cycle, being then called Symmetrical or Asymmetrical Regular-Sampled PWM, respectively. Here, only the asymmetrical technique will be considered, because of the lower harmonic content of the resulting pulse pattern.

A.1. Sinusoidal Regular-Sampled Asymmetrical PWM

In this suboscillation technique a sinusoidal three-phase modulation reference wave is employed:

$$\begin{aligned}v_{ref\ a}(t) &= m \frac{E}{2} \sin(\omega_m t) \\v_{ref\ b}(t) &= m \frac{E}{2} \sin\left(\omega_m t + 2\frac{\pi}{3}\right) \\v_{ref\ c}(t) &= m \frac{E}{2} \sin\left(\omega_m t + 4\frac{\pi}{3}\right)\end{aligned}\quad (1)$$

where m is the modulation index ($0 < m < 1$), E is the DC link voltage and ω_m is the fundamental output frequency.

At Fig. 1 it is shown the reference sinusoidal wave (p.u) for the phase "a" of the inverter and also the corresponding pulse pattern generated by its comparison with the triangular carrier.

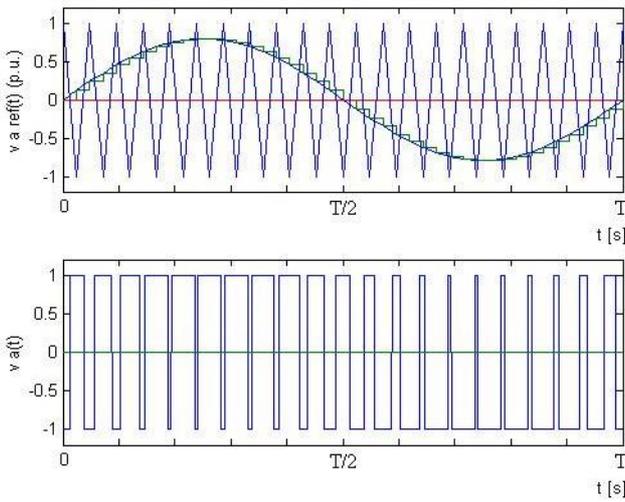


Fig. 1. Sinusoidal reference, carrier and PWM waveforms

A.2. Sinusoidal Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection

When a third harmonic is added to the sinusoidal reference waveform, as shown at (2) and at Fig. 2, the phase to ground voltages are distorted by the third harmonic, but the line to line voltages remain sinusoidal and are increased up to a 15% respect to pure sinusoidal modulation.

$$\begin{aligned}
 v_{ref a}(t) &= m \frac{E}{2} \left(\sin(\omega_m t) + \frac{1}{6} \sin(3\omega_m t) \right) \\
 v_{ref b}(t) &= m \frac{E}{2} \left(\sin\left(\omega_m t + 2\frac{\pi}{3}\right) + \frac{1}{6} \sin(3\omega_m t) \right) \\
 v_{ref c}(t) &= m \frac{E}{2} \left(\sin\left(\omega_m t + 4\frac{\pi}{3}\right) + \frac{1}{6} \sin(3\omega_m t) \right) \\
 0 < m < 1.15
 \end{aligned} \quad (2)$$

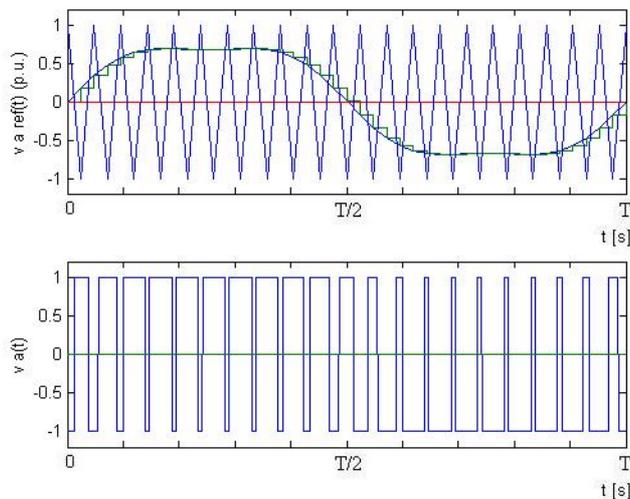


Fig. 2. Sinusoidal plus 3rd harmonic reference, carrier and PWM waveforms

B. Space Vector modulation

Space Vector modulation is conceptually different from the previously described suboscillation method; whereas the latter compute the switching instants separately for each of the inverter phases, the Space Vector PWM treats the inverter as a whole that can attain eight different possible states, represented by six active vectors $\mathbf{u}_1, \dots, \mathbf{u}_6$ of equal magnitude and $\pi/3$ phase shifted, and two zero vectors \mathbf{u}_0 and \mathbf{u}_7 , as shown at Fig.3, [1], [4].

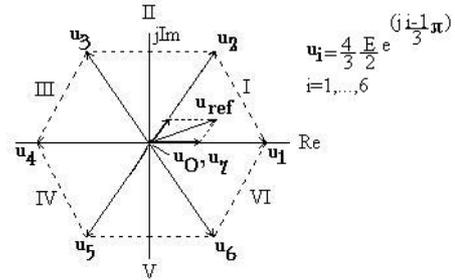


Fig. 3. Output voltage space vectors of a three-phase PWM inverter

Any desired voltage reference vector \mathbf{u}_{ref} can be obtained by decomposing it into components which lie along the active vectors that the inverter can generate. When only adjacent vectors are used in the decomposition, the transition from one vector to the next is performed by switching only one leg of the inverter, reducing in this way the number of switchings and the corresponding switching losses.

The duty ratios of the active vectors employed in the decomposition are determined in such a way that the average of the actually generated voltage vector equals the reference vector \mathbf{u}_{ref} :

$$\mathbf{u}_{ref} = \frac{t_i}{T_s} \mathbf{u}_i + \frac{t_j}{T_s} \mathbf{u}_j \quad (3)$$

where T_s is the switching period, \mathbf{u}_i , \mathbf{u}_j are two adjacent active vectors and t_i/T_s , t_j/T_s their corresponding duty ratios.

When the reference vector to generate lies inside the circle inscribed in the hexagon delimited by the six active vectors, the sum of the two duty ratios is smaller than one, and the remainder of the switching cycle is filled out by using zero vectors:

$$T_s = t_0 + t_i + t_j + t_7 \quad (4)$$

Optimal harmonic performance is obtained when the rest of the switching cycle is equally divided between the two zero vectors ($t_0=t_7$); this variation is referred to as Centred Space Vector PWM.

Again, the number of switchings is reduced when the transition from one switching period to another is made by toggling only one inverter leg, and this is accomplished by reversing the sequence of the employed vectors at every switching period, that is:

$$\underbrace{\mathbf{u}_0 \rightarrow \mathbf{u}_i \rightarrow \mathbf{u}_j \rightarrow \mathbf{u}_7}_{kT_s} \rightarrow \underbrace{\mathbf{u}'_7 \rightarrow \mathbf{u}'_j \rightarrow \mathbf{u}'_i \rightarrow \mathbf{u}'_0}_{(k+1)T_s} \quad (5)$$

being termed this practice as Asymmetrical Space Vector PWM.

The resulting average line to neutral voltage for phase a of the inverter, as shown at Fig. 4, (being phases b and c $2/3\pi$ and $4/3\pi$ shifted, respectively), proves to be:

$$v_{a\text{ ave.}}(t) = \begin{cases} \frac{3}{2}m\frac{E}{2}\sin(\omega_m t), & 0 \leq \omega_m t \leq \frac{\pi}{6} \\ \frac{\sqrt{3}}{2}m\frac{E}{2}\sin\left(\omega_m t + \frac{\pi}{6}\right), & \frac{\pi}{6} \leq \omega_m t \leq \frac{\pi}{2} \end{cases} \quad (6)$$

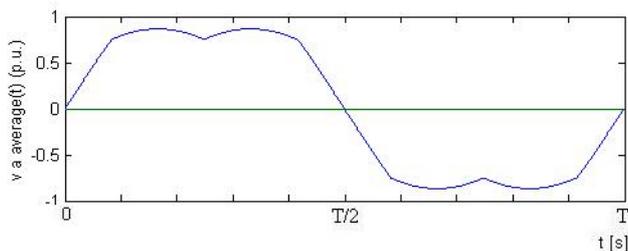


Fig. 4. Average line to neutral voltage for Asymmetrical Space Vector PWM

This averaged line to neutral waveform can be decomposed into a sinusoidal fundamental and a nearly triangular third harmonic component, which does not affect the line to line voltages and allows to extend the linear modulation range up to 1.15 ($0 < m < 1.15$), as in the case of 3rd harmonic injection suboscillation technique.

2. Simulation results

Simulations of the line-to-line inverter voltage waves corresponding to the three different considered PWM switching schemes are performed in order to determine which of them leads to a higher voltage waveform quality. Following, their harmonic spectra are obtained and their weighed TDH are calculated in order to compare the merit of the different modulation strategies.

Fig. 5 shows the harmonic spectra of the different line to line voltage waves (normalised against their fundamental component) corresponding to the following parameters:

- DC link voltage: $E=540$
- modulation index: $m=0.8$
- carrier frequency: $f_c=2000$ Hz
- modulation frequency: $f_m=50$ Hz

Table I lists the more significant harmonics (normalised against their fundamental component) for each modulation strategy and their corresponding weighted TDH (7), taking into account the first 150 harmonics.

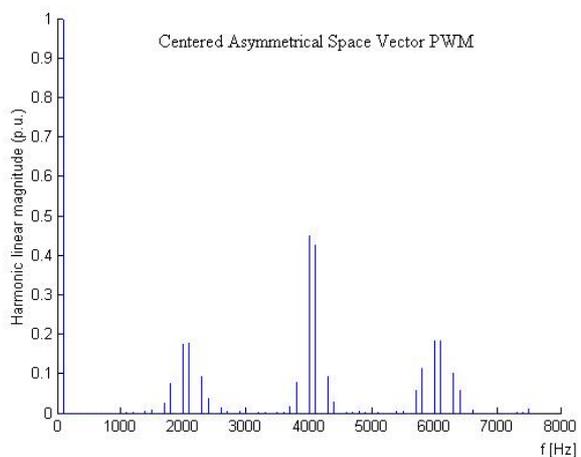
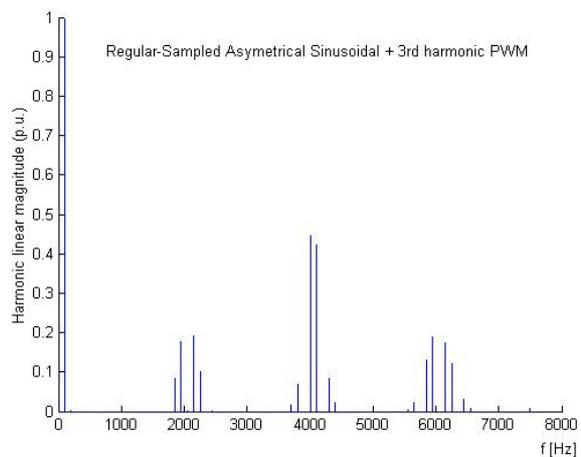
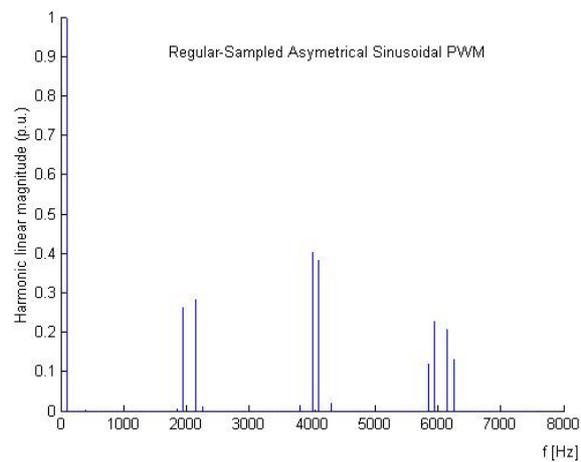


Fig. 5. Simulated harmonic spectra for the different considered PWM switching strategies

$$\text{TDH}_{\text{weighted}}(\%) = \frac{\sqrt{\sum_{i=2}^{\infty} \left(\frac{V_i}{i}\right)^2}}{V_1} \cdot 100 \quad (7)$$

TABLE I. - Significant harmonic components corresponding to Fig. 5

Frequency [Hz]	Reg-Sampl. Sin. PWM	Reg-Sampl. Sin+3 rd h. PWM	Space Vect. PWM
1750	0,09%	0,06%	7,86%
1800	0,84%	8,71%	0,03%
1900	26,36%	18,01%	0,07%
1950	0,17%	0,02%	17,69%
2050	0,18%	0,02%	18,11%
2100	28,38%	19,48%	0,06%
2200	1,34%	10,24%	0,04%
2250	0,02%	0,06%	9,62%
3750	1,52%	7,27%	7,98%
3950	40,59%	44,80%	45,27%
4050	38,33%	42,60%	42,74%
4250	2,25%	8,59%	9,43%
5750	0,19%	0,13%	11,41%
5800	12,26%	13,44%	0,07%
5900	22,97%	19,09%	0,01%
5950	0,19%	0,03%	18,55%
6050	0,19%	0,03%	18,52%
6100	21,00%	17,82%	0,01%
6200	13,40%	12,6%	0,07%
6250	0,17%	0,13%	10,35%
TDH_{weighted}	1,21%	1,10%	1,09%

A visual inspection of Fig. 5 shows that the cleaner harmonic spectrum corresponds to Sinusoidal Regular-Sampled Asymmetrical PWM, whereas Centred Asymmetrical Space Vector PWM presents the most disperse spectrum. Nevertheless, as confirmed by their respective weighted TDH, the total harmonic distortion follows an inverse order.

Additional simulations have been performed for other frequency ratios and modulation indexes (Fig. 6), which confirm the previous statement.

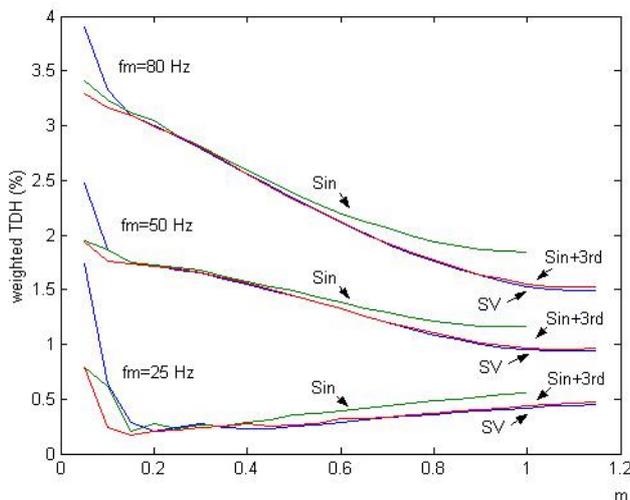


Fig. 6. Weighted TDH for $f_c = 2000$ Hz and the whole linear modulation range (Sin: Sinusoidal Regular Sampled Asymmetrical PWM, Sin+3rd: Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection, SV: Centred Asymmetrical Space Vector PWM)

From Fig. 6, it can be stated that the performance of Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection and Centred Asymmetrical Space Vector PWM is very similar for all the studied cases (as cited previously, the averaged line to neutral waveform of the latter method has also a triplen component in its decomposition), and is better than the corresponding to Sinusoidal Regular-Sampled Asymmetrical PWM.

3. Conclusions

The obtained results can be briefly summarized as follows:

- Sinusoidal Regular Sampled Asymmetrical PWM causes higher harmonic losses
- Regular-Sampled Asymmetrical PWM plus 3rd harmonic injection and Centred Asymmetrical Space Vector PWM have a very similar performance, although the following remarks can be made: 3rd harmonic injection PWM spectrum contains few harmonic terms than centred Space Vector PWM, but the latter shows less harmonic distortion (lower weighted TDH).

Due to its easy of digital realization and to its use of the space vector concept, Space Vector PWM exhibits as an excellent solution for high performance AC drives.

References

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