

19th International Conference on Renewable Energies and Power Quality (ICREPQ'21)
Almeria (Spain), 28th to 30th July 2021
Renewable Energy and Power Quality, Journal (RE&PQJ)

Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, Volume No.19, September 2021



Robust-PCA Deep Learning for PQ disturbances classification using Synchrosqueezing Wavelet Transform

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Abstract. In this paper a Robust-PCA Deep Learning algorithm using Synchrosqueezing Wavelet Transform is proposed for PQ disturbances mutli-classification. The algorithm was implemented and programmed in MATLAB using custom code. This approach avoids white noise, outliers and overfitting phenomena. The Synchrosqueezing Wavelet Transform is performed and a Robust-PCA mapping is done. External data is necessary to perform the pretreatment for autoscaling. A Deep Feed Forward Neural Network is implemented with 5 layers, 3 of them are hidden layers with more than 1 million parameters to fit. The quality of the solution is validated by the cross validation of parameters, R2 and Q2. Moreover, mean square error (MSE), the root of the mean square error (RMSE), the mean absolute percentage error (MAPE), Akaike information criterion (AIC) and the Schwarz information criterion (SBC) are estimated. The adjusted R2 value is 0.989 and the RMSE obtained is 1.789. The value of R2 is 0.995. All these parameters are calculated over the

Keywords. Robust-PCA, Synchrosqueezing Continuous Wavelet transform, Discrete Wavelet transform, MATLAB, PQ disturbances, Power Quality.

1. Introduction

Power quality (PQ) refers to the cleansing of a voltage signal in power systems at the point of common coupling in consumer end. It should be disturbance-free in the electricity supply. Millions of electronic devices are connected to the grid and they can cause potential disturbances and deviations from the supplied ideal sinusoidal voltage waveform. PQ disturbances are defined as sources of voltage disturbances, which degrade and might damage modern devices [1]. Thus, to prevent this issue, electronic devices have included lately prevention mechanisms considering these PQ disturbances [2].

Two variables are mainly considered to categorize PQ disturbances in time domain: magnitude and duration. It is also possible to classify them as two different phenomena: steady-state and transient state. The usual PQ disturbances are sag distortion, voltage flickers, oscillatory transient, swell and interruption. Table I shows the PQ disturbances considered for the robust-PCA Deep Learning multi-

classification, where the parameters are randomly determined applying the given inequation. These disturbances were selected for its mathematical simplicity and the ease in the implementation.

However, the voltage variations imply to have different power characteristics. To deal with these disturbances, it is necessary to develop new methods capable of detecting and classifying such disturbances [3-4]. PQ disturbances definitions are stated in standards IEEE 1159 [5], IEC61000-4-30 and EN50160 [6].

Advanced signal analysis techniques are necessary to understand the role of the PQ disturbances. They are suitable to extract the information from voltage signal behind of these feature data. The most used technique is Fourier Transform (FT) for frequency domain. Obviously, this technique is especially recommended for periodic, stationary, and linear systems. For a continuous frequency time varying, it is possible to use the Short Time Fourier Transform observing a time window, but it hast a lot of limitations (STFT) [7]. Other advanced frequency techniques have been developed, trying to overcome the limitations of FT like Hilbert-Huang Transform (HHT), Wavelet Transform (WT) and Stockwell Transform (ST) [8][9].

The extracted features are used for detecting and classifying [10][11] the electrical events and PQ disturbances that could appear in the electric network. Moreover, in recent years, a lot of techniques were developed for PQ events recognition. For example, neurofuzzy system (NFS) classifier combined with HHT were performed with very good results [12]. The difficulties of the HHT algorithm are widely known. This is not the case of WT; therefore, some authors have developed an approach for the recognition PQ using WT and support vector machines (SVM) [13]. A derived WT technique is the Curvelet Transform (CV) which makes a projection of the signal extracted features over a 2D representation [14] applying SVM, so it is possible to recognize some PQ events. Other non-frequency-based techniques have been implemented successfully using compressive sampling method and reduced PQ disturbances signal dimensionally [15].

rable 1. – FQ disturbances considered in the Robust-FCA Deep Learning Mutti-Classification.								
Symbol	PQ disturbance	Equation	Parameters					
PQd1	Sag	$y(t) = (1 - \alpha(u(t - t_1) - u(t - t_2))\sin(wt)$	$0.1 \le \alpha \le 0.9, T \le t_2 - t_1 \le 9T,$					
			$t_1 \le t_2$					
PQd2	Swell	$y(t) = (1 + \alpha(u(t - t_1) - u(t - t_2))\sin(wt)$	$0.1 \le \alpha \le 0.8, T \le t_2 - t_1 \le 9T,$					
			$u(t) = \begin{cases} 1 & t \ge 0 \\ 0 & t < 0 \end{cases}$					
PQd3	Interruption	$y(t) = (1 - \alpha \left(u(t - t_1) - u(t - t_2) \right) \sin(wt)$	$0.9 \le \alpha \le 1, T \le t_2 - t_1 \le 9T$					
PQd4	Flicker	$y(t) = (1 + \alpha_f \sin(\beta wt))\sin(wt)$	$0.1 \le \alpha_f \le 0.2, 5Hz \le \beta \le 20Hz$					
PQd5	Oscillatory transient	$y(t) = \sin(wt) + \alpha e^{-(t-t_1)/\tau} \sin(w_n)$	$0.1 \le \alpha \le 0.8, 0.5T \le t_2 - t_1 \le 3T,$					
		$(t-t_1)\big(u(t_2)-u(t_1)\big)$	$8ms \le \tau \le 40ms, 300 \le f_n \le 900$					
PQd6	Harmonic	$y(t) = \alpha_1 \sin(wt) + \alpha_3 \sin(3wt)$	$0.05 \le \alpha_3 \le 0.15, 0.05 \le \alpha_5 \le 0.15$					
		$\alpha_5 \sin(5wt) + \alpha_7 \sin(7wt)$	$0.05 \le \alpha_7 \le 0.15, \sum \alpha_i^2 = 1$					

Table I. – PQ disturbances considered in the Robust-PCA Deep Learning Multi-Classification.

Those techniques based on neural networks (NN) have been developed exponentially. They can be tagged as Machine Learning Algorithms. Artificial neural network (ANN), Deep Learning (DL) and Convolutional Neural Network (CNN) are the most commonly use strategies. NN have been applied in a lot of systems for multiclassification. Moreover, they have emerged as an important tool for classification and monitoring, being applied to a diverse classification tasks in industry, business and science. The input layer of ANN has the number of features obtained from some time-frequency transform of the electric signal, the hidden layer has a sigmoid activation function and, finally, the output layer is the classification layer [16][17].

This architecture is the base of DL. It has more than three layers. The results of this technique are quite promising, providing a solution to most classification problems. DL has been successfully applied in the areas of speech recognition, human face identification [34], computer vision and PQ disturbances classification and monitoring [18]. The latest proposed technique is CNN, which also has become a popular technique for classification. It is a biologically inspired technique and strongly relies on ANN. The CNN represents a model of human visual cortex. The applications of CNN are mainly for image recognition and video classification. Lately, this CNN have been applied to identify PQ disturbances. The successful application of this technique can be found in [19]. The limiting features for these techniques are the computation time and the complexity in algorithmic implementation. On the one hand, the algorithms used for signal analysis have problems such as mode mixing, intrinsic mode and mother wavelet, among others. In advanced signal analysis techniques, it is difficult to interpret the results because of their novelty [20]. On the other hand, when dealing with neural networks one must be especially careful with the training technique used. The backpropagation algorithm and its variations along with the number of layers, may force to use high performance computing (HPC) [21]. Therefore, they continue to be studied and well understood for researching purposes.

Herein, a custom PQ disturbances classification system coded in MATLAB is presented. In this application ten thousand synthetic signals with different PQ disturbances and white noise controlled are tested using [22]. Afterwards, the Synchrosqueezing Continuous Wavelet Transform (SCWT) [23] is calculated, and the signal is denoised. The signal denoised is normalized and scaled within the range values of the training data set. The robust-PCA is applied [24]. The robust PCA mapping is used as input signal along with the normalized and scaled values of the SCWT. The span time is configured by default and the train routine used is the backpropagation algorithm for Deep Learning scheme, which minimizes a continuous differentiable multivariate function.

2. Robust-PCA, Synchrosqueezing Wavelet Transform and Deep Learning

A. Robust-PCA

Classical Principal Component Analysis try to solve the minimization problem,

$$\min_{\substack{\text{subject to } rank(L) \leq k}} \|M - L\| \tag{1}$$

where $M = L_0 + N_0$ is a stack of data points as column vectors, L_0 is a low-rank matrix and N_0 is the perturbation matrix. The problem can be solved by seeking the best rank - k of L_0 and using the singular value decomposition (SVD). The best low rank is found when the noise N_0 is small and independent appropriately distributed as gaussian noise. Some problems arise in equation (1) because it requires solving the low-rank and sparse decomposition problem for matrices of extremely high dimension with ill-posed condition: non-unicity, numeric instability, and existence. In practical terms, this approximation makes it vulnerable with respect to outliers. In order to improve the optimization process in equation (1), the problem must be treated as weak formulated optimization problem. This mathematical concept was first introduced by Hadamard [25]. A problem is weakly formulated when the initial conditions and boundary

conditions are not well defined. Then, the problem can be solved by convex optimization, as follows,

$$\min_{\text{subject to}} \|L\|_* + \lambda \|S\|_1$$
 (2)

where $||L||_*$ is the nuclear norm, given by the sum of SVD. An estimate solution of the optimization given in equation (2) is possible using the Lagrange multipliers. Note that the $\ell_1 - norm$ can be used with the shrinkage operator. The regularization term "suction" the possible outliers enhancing the regular PCA.

B. Synchrosqueezing Wavelet Transform

Synchrosqueezing wavelet transforms was developed for analysing auditory signals [25]. This technique tries to tune a time-frequency representation. The behaviour is estimate it as a local value in $\mathcal{R}(t,\omega)$. The continous synchrosqueezing wavelet transform of a signal s(t) can be defined as,

$$W_{s}(a,b) = \int s(t)a^{-\frac{1}{2}}\overline{\psi\left(\frac{t-b}{a}\right)}dt$$
 (3)

where a and b are the scale and transportation values related with frequency and time, respectively. ψ is the mother wavelet, which extracts the instantaneous frequency. Using Plancherel's theorem, it is possible to rewrite equation (3),

$$W_{s}(a,b) = \int \hat{s}(\xi)a^{\frac{1}{2}}\overline{\hat{\psi}(a\xi)}e^{ib\xi}d\xi$$

$$= \frac{A}{4\pi}a^{\frac{1}{2}}\overline{\hat{\psi}(a\omega)}e^{ib\omega}$$
(4)

Since $\widehat{\psi}(\xi)$ is concentred in ω_0 , therefore, the value of wavelet in (a,b) must to be in $a = \omega_0/\omega$. So, the candidate for the instantaneous frequency $\omega_s(a,b)$ for the signal s(t) is,

$$\omega_s(a,b) = -i(W_s(a,b))^{-1} \frac{\partial W_s(a,b)}{\partial b}$$
 (5)

 $W_s(a, b)$ can be computed using the discrete values for a_k , $a_k - a_{k-1} = (\Delta a)_k$. The Synchrosqueezed transform $T_s(\omega, b)$ estimated for the ω_l centers, $\left[\omega_l - \frac{1}{2}\Delta\omega, \omega_l + \frac{1}{2}\Delta\omega\right]$, is defined,

$$T_s(\omega_l, b) = \Delta \omega^{-1} \sum_{a_k \le \frac{\Delta \omega}{2}} W_s(a_k, b) a_k^{-\frac{3}{2}} (\Delta a)_k$$
 (6)

The equation (6) defines completely the Synchrosqueezing Wavelet Transform and gives a concentrated instantaneous frequency in the time-frequency plane.

C. Deep Learning

The machine learning is fundamentally a non-linear optimization problem. Neural Networks specifically optimize over a compositional function,

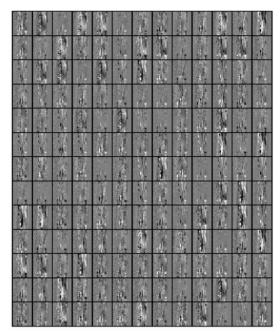


Fig. 1. General appearance of the hidden layer for proposed Deep Learning scheme.

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{K} \left[-y_k^{(i)} \log \left(h_{\theta}(x^{(i)})_k \right) - (1 - y_k^{(i)}) \log \left(1 - h_{\theta}(x^{(i)})_k \right) \right] + \frac{\lambda}{2m} \left[\sum_{l=1}^{L} \sum_{j=1}^{K} \sum_{k=1}^{K} \left(\theta_{j,k}^{(l)} \right)^2 \right]$$

$$(7)$$

where J is the cost function and $h(x^i)$ is the activation function. The parameters are $\theta_{i,k}$ for each layer l. The train routine used is the backpropagation algorithm for Deep Learning scheme which minimizes a continuous differentiable multivariate function. The backpropagation algorithm uses Polack-Ribiere flavour of conjugate gradients. This technique is used to compute search directions and the approach for line search uses a quadratic and cubic polynomial approximation. To prevent overfitting, a regularization ℓ_2 is proposed. The best sparsity promoted coefficient (λ) is estimated using L-Curve, where the parameters norm is used as residual function. The L-curve is calculated as log-log plot and it is a useful graphical tool for displaying the variation of sparsity promoted coefficient (λ) and its residual. The Deep Learning scheme is presented in figure 2 as Deep Feed Forward Neurons. The number of layers is five, which three of them are hidden layers, the first one is the input layer and the last one is the output layer. A preprocessing step is performed on data. It is not a convolution, so it cannot be considered as convolution neural network, but a Deep Feed Forward Neural Network. The train set for the Deep Learning was performed using exclusively simulations data set. The simulation data set was created with the help of a MATLAB GUI developed in [22]. This application is intended for PQ simulations and FT, STFT and WT algorithms were implemented. Six PO disturbances were selected and randomly combined to generated over five cycles at 16 kHz, that is 1600 points. The number of simulations performed was five thousand trying to cover all possible real situations. However, real electrical quality measurements were not yet included.

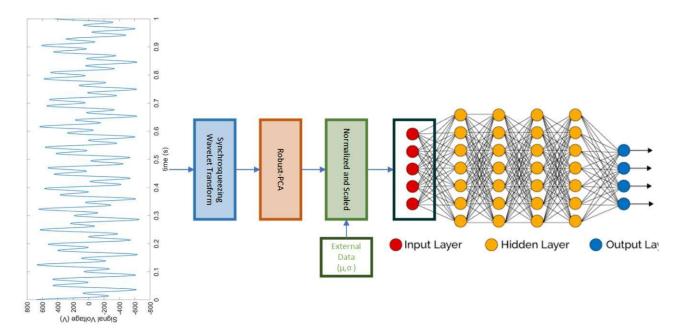


Fig. 2. General scheme for the multi-classification pipeline. Voltage signal input is transformed using SCWT. Afterward, a robust PCA algorithm is applied for cleaning the time – frequency plane. Then, a normalized and scaled operation is performed. Finally, the normalized and scaled values are the input of the Deep Feed Forward Neural Network.

3. Deep Feed Forward Neural Network Architecture

The number of hidden dense layers is five. The number of neurons per unit is 281,600 from the first to fourth layer and the last layer has six output neurons, one for each PQ disturbance to classify. Before of the first layer neuron input the data is pre-processed. First, a Synchrosqueezing Wavelet Transform is performed. The output is a matrix of 704 rows with 1600 columns. In order to clean de SCWT spectrogram, a robust — PCA is applied. This process filters the white noise and removes the outliers in the spectrogram. Afterward, a normalization and scaling step is applied for data curation. The normalized set the data as parts of unit and the scaled centers and balance in base of average and standard deviation.

In figure 1, it is shown how a Deep Learning hidden layer looks like. Each cell represents the weight for each

position of the SCWT matrix. The values are represented as black – white scale. The output layer is configured to set 0 or 1 for each PQ disturbance considered. In table I, it is shown the six PQ disturbances considered.

Robust – PCA is applied to the spectrogram to preventing outliers and signal noise. This technique uses the shrinkage operator, which make a multivariate discrimination. All spectrogram sequence is presented in figure 3. The external data are the average and the standard deviation of the training set. With the average and the standard deviation, it is possible to do a pretreatment method, called autoscaling, equation (8).

$$\widetilde{x}_{ij} = \frac{x_{ij} - \overline{x}_i}{s_i} \tag{8}$$

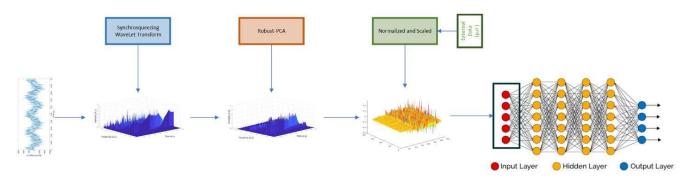


Fig. 3. General scheme for the multi-classification with a spectrogram sequence visualized. To the Voltage Signal input, it is applied a white gaussian noise and then it is transformed using SCWT. Afterward, a robust PCA algorithm is performed. The scale and normalization are done, which it is the input to the Deep Feed Forward Neural Network.

3. Results

Given the data set, five thousand simulation data are splatted in three pieces. The first piece is called the training set, this is a usual piece. The second piece of this data is called cross-validation set. The cross-validation is used to check the model, where it is possible to change the neuron numbers of each hidden layer and check the variability of the sparsity promote coefficient. Finally, the last data set, it is called test set. Within this test set, it is possible to evaluate the generalized error committed. The typical ratio for splitting the data set are 60% for training set, 20% for cross-validation set, and 20% for the test set.

The quality of the Robust-PCA Deep Learning multiclassification using Synchrosqueezing Wavelet Transform is validated by the cross validation of parameters, R2 and Q2, in order to make the comparison with different Deep Feed Forward Neural Network variations. The RMSECV value is 1.40. The mean square error (MSE), the root of the mean square error (RMSE) and the mean absolute percentage error (MAPE) were calculated yielding the results in the table II. Moreover, the Akaike information criterion (AIC) and the Schwarz information criterion (SBC) is estimated, obtaining good multiclassification for the proposed model. The adjusted R2 value is 0.978.

Table II. – Evaluation parameters obtained for Robust-PCA Deep

Learning Multi-Classification.

R2	R2 Adj.	MSE	RMSE	MAPE	AIC	SBC
0.995	0.989	2.88	1.30	0.130	222	119

To evaluate the generalized error, the Robust-PCA Deep Learning Multi-Classification was evaluated with a test set. On the one hand, the R2 parameter obtained is 0.989. On the other hand, the RMSE obtained is 1.789. As expected, the generalized error is greater than the error obtained using training set or cross-validation set.

4. Conclusion

In summary, we have developed novel multi-classification PQ disturbances predictor, which improves the accuracy of the PQ disturbances classification with respect to others proposal models. The PQ disturbances classification system is coded in MATLAB. To perform the PQ disturbances multiclassification, the Synchrosqueezing Continuous Wavelet Transform (SCWT) is used. The resulting spectrogram is fed into the Robust-PCA, which can filter the noise and remove the outliers. The Robust-PCA mapping is scaled and normalized using external data.

The Deep Learning backpropagation algorithm used is programmed entirely in MATLAB. The optimization was performed using an ASUSTeK Intel Xeon Gold 5220 64bits with NVIDIA GeForce RTX 3090 and 128 GB RAM server. The computation time for training the Deep Learning Schema is the limiting issue because the backpropagation algorithm must train more than one million parameters for the light scheme. It is highly

recommendable to use high performance computing servers and parallel computing to train the data set.

For the future work, it is advisable to use real data both for training and for calculation of the generalized error. Moreover, to get a better model it is necessary to extend the L-curve for the sparsity promote coefficient estimation. A benchmark with others framework, like Keras and Tensorflow, could be used for performance measurement and result comparison. Finally, a more complete dataset of PQ disturbances could be used.

Acknowledgement

This work was supported by project PGC2018-098813- B-C33 (Spanish "Ministerio de Ciencia, Innovación y Universidades"), and by European Regional Development Funds (ERDF).

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