Renewable Energy and Power Quality Journal
https://repqj.com/index.php/repqj/index
RE&PQJ, Vol. 23, No. 5, 2025

Study on Cooperative Optimisation Method for PCI Allocation in Power Load Management Communication Networks

Liyuan Sun[1], Koyu Wu[2,3*], Junwei Yang[2]

[1]Measurement Centre, Yunnan Power Grid Co., Ltd., Kunming, China
[2]LongShine Technology Group Co., Ltd., Wuxi, China
[3]Faculty of Information Engineering and Automation, Kunming University of Science & Technology, Kunming, China
[*]Corresponding author's email: 2976577860@qq.com

Abstract. With the rapid development of next-generation power systems, the complexity of load scheduling management for high-energy consumers in industrial parks has intensified, particularly under multi-stakeholder coordination challenges. Addressing the research gap in comprehensive optimisation of PCI conflicts, configuration ambiguities, and mod 3 interference within 5G New Radio (NR) environments, this study proposes a collaborative optimisation-based PCI allocation methodology. Incorporating PageRank values as weighting factors, a multi-objective model quantifies base station interference correlations. High-dimensional constraints are linearised via the Big-M method, while an enhanced genetic algorithm integrates greedy strategies, fixed indexing, and neighbourhood search to overcome local optima. Results demonstrate significant reductions in PCI-related impairments (conflicts, confusion, and mod 3 interference) compared to conventional methods, achieving stable transmission in 5G ultra-dense networks.

Key words. 5G communication networks, PCI planning optimization, Improved genetic algorithm, PageRank-based weighting

1. Introduction

A. Background

With the escalating global energy demands and mounting environmental challenges, effective energy system management has emerged as a pivotal research domain. Industrial parks, serving as primary energy consumption hubs, play an instrumental role in enhancing the operational efficiency of energy systems [1]. Within such contexts, load management is indispensable for optimising energy utilisation, ensuring grid stability, and fulfilling end-user energy requirements. This process entails the aggregation of load data from distributed appliances via metering devices and management terminals, which is subsequently transmitted to a centralised analytical platform. Based on the analysed data, consumption constraints are dynamically imposed. Notably, 5G private power networks are commonly employed to facilitate robust data transmission in these operational frameworks [2].

Given the inherent complexity of interference in non-orthogonal control regions between adjacent base stations [3], PCI allocation strategies are designed to mitigate Inter-Cell Interference (ICI) and enhance throughput in Physical Downlink Control Channels (PDCCH). Functioning as a fundamental physical-layer identifier, PCI enables base stations to differentiate cells during channel estimation and handover procedures [4,5]. While LTE networks historically restricted PCI availability to 504 unique identifiers-dictated by the configuration of Primary Synchronisation Signals (PSS) and Secondary Synchronisation Signals (SSS) [6] - 5G New Radio (NR) standards have expanded this range to 0–1007 [7]. This expansion, coupled with advanced synchronisation mechanisms and architectural innovations, supports dense cell deployments and refined interference management. Data transfer protocols and communication interfaces adhere to industry standards to ensure the integrity of load data transmission within these networks.

Nevertheless, PCI planning remains a highly intricate and mission-critical task, as optimal configuration is paramount to minimising interference while maximising transmission stability and throughput. Current PCI allocation strategies face three predominant challenges: PCI conflicts, arising when adjacent base stations are assigned identical identifiers; PCI confusion, occurring due to cyclical repetition of PCI patterns among neighbouring cells; Mod 3 interference, induced by overlapping co-channel coverage when adjacent cells share congruent PCI values mod 3.

As illustrated in Figure 1, each cell is assigned a unique Physical Cell Identifier (PCI) and a mod 3 value. Adjacent cells sharing the same frequency band are interconnected via lines, with the "adjacency" attribute strictly defined between directly linked pairs. For instance: PCI Conflict: Cells A and D exhibit identical PCI values and congruent mod 3 residues. When their signal strength difference falls below a predefined threshold δ, a PCI conflict arises due to overlapping coverage. Mod 3 Interference: Cells A and F share the same mod 3 value but demonstrate a signal strength difference exceeding δ. Despite non-overlapping coverage, this configuration induces mod 3 interference, degrading channel estimation accuracy. PCI Confusion: Cells B and C, both serving as co-frequency neighbours to the master cell i, are assigned identical PCI values. This redundancy results in PCI confusion, impairing handover reliability and network stability.

These scenarios underscore the criticality of systematic PCI allocation in mitigating interference-related anomalies. The interplay of PCI assignments, modulo 3 congruence, and signal strength thresholds necessitates rigorous optimisation to ensure seamless operation in power load management networks.

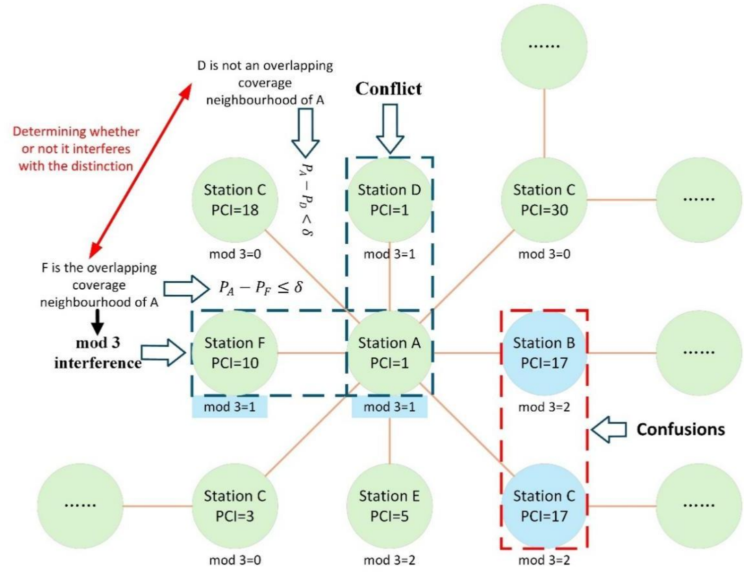

Figure 1. PCI conflict, confusion, interference schematic.

B. *Related Work*

With the progressive rollout of 5G networks, advancements in data rates and coverage capacity have driven researchers to focus on refining Physical Cell Identifier (PCI) allocation strategies within heterogeneous network (HetNet) environments integrating eNodeBs (eNBs) and gNodeBs (gNBs). For instance: Inoue et al. proposed a PCI detection method employing joint estimation of Secondary Synchronisation Signals (SSS), achieving high detection probabilities at 30 GHz carrier frequencies [8]. Li et al. developed a distributed PCI self-configuration and self-optimisation algorithm for LTE/5G dual connectivity, leveraging symmetric and asymmetric triangular cyclic algorithms for PCI reassignment [9]. Krishnaswamy et al. introduced a quasi-quantum stochastic graph colouring technique using virtual cell identifiers to optimise PCI allocation via heterogeneous intelligent controllers in large-scale networks [10]. Zeljkovic et al. devised two self-organising network (SON) algorithms-Weighted Automatic Neighbour Relation (ANR) and the ALPACA PCI allocation algorithm-to rapidly resolve PCI conflicts and confusion in high-density topologies [11]. Wu et al. designed a multimodal hybrid detection technique exploiting beam-scanning properties of 5G base stations to enhance PCI extraction accuracy [12]. Anrade et al. addressed technical and operational limitations of PCI allocation in practical scenarios through memetic algorithms (MA-WS and MA-NWS) to bolster network optimisation efficacy [13].

In complex network environments, conventional PCI allocation algorithms often exhibit limited adaptability and generalisability. Acedo-Herández et al. demonstrated the critical impact of meticulous PCI planning on uplink performance in LTE systems, underscoring the necessity of interference avoidance to maintain optimal control and data channel performance [14]. Abdulkareem et al. integrated matrix algorithms with graph colouring to optimise PCI reassignment [15], while Chen et al. proposed a multi-objective optimisation algorithm combining mutation and crossover operations to reduce conflict and confusion ratios [16]. Tu et al. enhanced clustering performance and PCI allocation efficiency via a fuzzy hierarchical clustering-based PCI assignment (FHC-PCIA) scheme using Euclidean distance metrics [17]. Ahmed et al. reviewed pivotal metaheuristic algorithms for network optimisation, highlighting the strong generalisation capabilities of greedy heuristics for large-scale problems with limited objectives and constraints [18].

In the domain of centralised PCI allocation, Liu et al. proposed an enhanced graph colouring algorithm wherein Operation, Administration, and Maintenance (OAM) servers comprehensively collect cell information to enable centralised PCI allocation [19]. This approach reduces the time complexity of PCI reassignment, thereby improving PCI utility. Building on this, Xu et al. introduced a hypergraph-based PCI self-configuration scheme [20]. The hypergraph colouring model enhances the adaptability of PCI allocation strategies, enabling conflict- and confusion-free assignments in dynamic network environments. Liu et al. further devised a PCI grouping and cell clustering algorithm for ultra-dense networks to mitigate PCI mod-k conflicts and confusion [21].

To provide a structured overview, Table 1 summarises key differences among referenced studies in terms of scenarios, management types, methodologies, and specific contributions. While prior works have advanced PCI allocation within specific domains and constraints, existing methods exhibit inefficiency in handling high-dimensional nonlinear constraints, struggle with the complexity of large-scale networks, and suffer from limited global search capabilities-often converging to suboptimal local solutions. This study addresses these shortcomings by constructing a comprehensive multi-objective optimisation model, integrating an enhanced genetic algorithm, PageRank-based weight allocation, and a PCI reuse balancing mechanism. These innovations significantly improve global optimisation capacity, computational efficiency, and practical applicability, delivering a more robust solution for 5G-enabled power delivering a more robust solution for 5G-enabled power load management.

Table 1. Summary of related works.

Reference	Scenarios			Mgmt. Type		Graph Colouring	Challenges	Contribution
	LTE	LTE-A	5G	Dist.	Cent.			
Inoue [8]			✓				Ensure high PCI detection at 30 GHz.	PCI detection via SSS sequence.
Li [9]			✓	✓			Manage PCI conflicts in ENDC with hybrid clustering.	Self-configuration and optimization in EN-DC.
Krishnaswamy [10]			✓			✓	Optimize SON with heterogeneous intelligent controller.	Quasi-quantum graph colouring.
Wu [12]			✓	✓			Improve PCI extraction across multiple 5G base stations.	Multi-mode mixed detection.
Andrade [13]			✓				Tackle PCI assignment with real-world constraints.	Hybrid memetic algorithm with warm start.
Acedo-Hernández [14]	✓						Minimize DM-RS collisions on PUCCH performance.	Focused PCI's role in data channel performance.
Abdulkareem [15]	✓					✓	Manage PCI reassignments in dynamic LTE.	Matrix-based algorithm for conflict-free PCI assignment.
Chen [16]	✓						Balance multiple PCI optimization goals using RL.	Multi-objective optimization with mutation and crossover.
Tu [17]		✓					Adapt PCI allocation to dynamic cell activity in HCNs.	Fuzzy hierarchical clustering for PCI allocation.
Ahmed [18]	✓			✓			Tackle optimization challenges in mobile network automation.	Multi-objective genetic algorithms.
Liu [19]	✓				✓	✓	Optimize centralized PCI assignment efficiency.	Enhanced graph coloring.
Xu [20]	✓					✓	Apply hypergraph colouring for LTE PCI self-configuration.	Hypergraph colouring for adaptable PCI allocation.
Liu [21]			✓		✓	✓	Mitigate mod-k conflicts in UDNs through clustering.	Reduced PCI mod-k conflicts in UDNs.
This Work			✓		✓		Addressing PCI planning challenges in high-density networks	PageRank-based PCI allocation algorithm

2. Methodology

Figure 2 shows the whole process of optimisation model construction in its entirety. The quantities of conflicts, confusions, and modulo 3 interferences within Measurement Report (MR) data serve as critical Physical Cell Identifier (PCI) metrics, forming the basis for three $N \times N$ matrices. Concurrently, decision variables (e.g., binary allocation indicators) and auxiliary indicator variables are integrated to formulate a PCI allocation optimisation model. When designating an optimised station as the master node, the PageRank algorithm is employed to quantify its topological correlations with neighbouring stations, thereby refining the weighting coefficients embedded within the objective function. This objective function, derived through systematic adjustments to the baseline model, is subsequently linearised via the Big M method to enhance computational tractability. To augment solution efficacy, the genetic algorithm is enhanced through three principal innovations: Greedy Heuristic Integration: A fitness-driven update mechanism prioritises high-quality solutions, accelerating convergence. Constraint-Aware Crossover and Mutation: Genetic operators are tailored to adhere to PCI allocation constraints, ensuring solution feasibility while expanding the search space. Computational Efficiency: Solution time is reduced through parallelised operations, and accuracy is improved via adaptive mutation probabilities. Central to this methodology is the construction of the MR frequency matrix, which synthesises conflict, confusion, and interference metrics into a unified framework. This matrix not only quantifies network performance degradation but also guides iterative refinements in PCI allocation strategies.

Figure 2 illustrates the PCI collaborative optimisation framework proposed in this study. The methodology initiates with data collection to construct a multi-objective optimisation model, followed by rigorous linearisation of high-dimensional nonlinear constraints through the Big-M method. Subsequently, the genetic algorithm population is initialised, and solution quality is evaluated via fitness functions. Iterative refinement is achieved through greedy strategy-informed crossover mutation operations integrated with neighbourhood search techniques until convergence criteria are satisfied. This methodology significantly enhances global search efficiency while ensuring solution space feasibility.

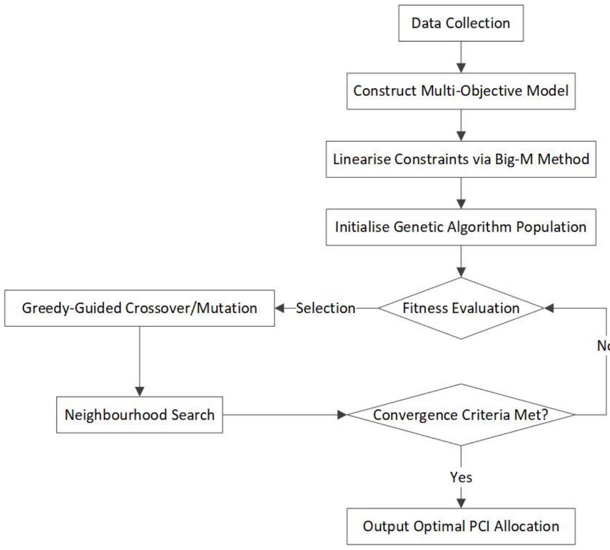

Figure 2. Flowchart of the PCI optimisation methodology.

3. Mathematical Models

A. Model Symbol

The symbols and meanings in the model developed are shown in Table 2.

Table 2. Description of symbols.

Parameters	Meaning
F_i	The carrier frequency of cell i
PCI_i	The Physical Cell Identifier (PCI) of cell i
a_{ij}	The number of conflicting measurement reports (MRs) between master cell i and its neighbouring cell j
b_{mn}	The number of measurement reports (MRs) affected by PCI confusion between cells m and n when both concurrently serve as neighbouring cells to cell k
c_{ij}	The number of measurement reports (MRs) associated with overlapping coverage between master cell i and its neighbouring cell j
cor_i^a	The correlation metric of cell i in PCI conflict-prone network environments
cor_i^b	The correlation metric of cell i in PCI confusion-prone network environments
cor_i^c	The correlation metric of cell i in mod-3 interference-prone network environments
w_p	Prioritisation weight during $\mathrm{PCI_p}$ configuration
Variables	
x_{pi}	If PCI value p is assigned to cell i, then $x_{pi}=1$; otherwise $x_{pi}=0$
SF_{ij}	The co-frequency indicator between cells i and j is defined as $SF_{ij}=1$ if they operate on the same frequency band, and $SF_{ij}=0$ otherwise
S_{ij}	The PCI identity indicator between cells i and j is defined as $S_{ij}=1$ if they share the same PCI, and $S_{ij}=0$ otherwise
M_{ij}	The mod 3 interference indicator between cells i and j is defined as $M_{ij}=1$ if a mod 3 conflict exists, and $M_{ij}=0$ otherwise

B. PCI Allocation Optimisation Model

This paper constructs a PCI allocation optimisation model to address PCI configuration challenges in power load management communication networks. The model incorporates: Binary decision variable x_{pi} determining PCI assignment to cell i Co-frequency indicator; SF_{ij} (1 if cells i and j share the same frequency band, 0 otherwise); PCI collision indicator S_{ij} (1 if identical PCI values exist, 0 otherwise); Mod-3 interference indicator M_{ij} (1 if mod-3 conflict occurs, 0 otherwise).

a_{ij} denotes the number of conflicting measurement reports (MRs) with cell i as the serving cell and cell j as the neighbouring cell; b_{mn} represents the number of confusing MRs where cells m and n simultaneously act as serving cells with another neighbouring cell; c_{ij} characterises the number of mod-3 interference MRs with cells i and j as serving cells and another neighbouring cell. The natural PCI planning model NP is formulated by calculating the increases in the number of conflicts ($a_{ij}+a_{ji}$), confusion ($b_{mn}+b_{nm}$), and mod-3 interference ($c_{ij}+c_{ji}$):

$$NP: \quad \min z=\sum_{i\in N^o}\sum_{j\in N_i} S_{ij}\left(a_{ij}+a_{ji}\right)SF_{ij}+\sum_{i\in N^o}\sum_{m,n\in N_i, m\neq n} S_{mn}\left(b_{mn}+b_{nm}\right)SF_{mn}+\sum_{i\in N^o}\sum_{j\in N_i} M_{ij}\left(c_{ij}+c_{ji}\right)SF_{ij} \quad (1)$$

In Equation (1), the first term represents the total number of conflicting MRs, the second term denotes the total number of confusing MRs, and the third term corresponds to the total number of mod-3 interfering MRs.

Where: N represents the set of all cells; N^o denotes the indexed set of cells requiring optimisation; F_i indicates the frequency points allocated to cell i ; PCI_i signifies the Physical Cell Identity value assigned to cell i .

1) Limitations and Constraints on Variables

$$\sum_{p\in P} x_{pi}=1, \quad \forall i\in N^o \quad (2)$$

$$\sum_{i\in N^o} x_{pi}\geq 1, \quad \forall p\in P \quad (3)$$

$$S_{ij}=\begin{cases}1, & \mathrm{PCI}_i=\mathrm{PCI}_j\\ 0, & \text{else}\end{cases} \quad (4)$$

$$M_{ij}=\begin{cases}1, & \mathrm{mod3}\left(\mathrm{PCI}_i\right)=\mathrm{mod3}\left(\mathrm{PCI}_j\right)\\ 0, & \text{else}\end{cases} \quad (5)$$

$$SF_{ij}=\begin{cases}1, & f_i=f_j \quad \forall i,j\in N^o\\ 0, & \text{else}\end{cases} \quad (6)$$

$$\mathrm{PCI}_i=\sum_{p\in P} p x_{pi} \quad \forall i\in N^o \quad (7)$$

$$x_{pi}\in\{0,1\}, \forall p\in P, i\in N^o \quad (8)$$

Equation (2) specifies that each cell is allocated exactly one Physical Cell Identity (PCI); Equation (3) indicates that PCI values may be reused across multiple cells; Equations (4)-(6) define the value constraints for the indicator variables S_{ij} , M_{ij} , and SF_{ij} respectively;

while Equation (8) establishes the value constraints for the decision variable x_{pi} .

2) Planning Principles for Data

$$\left|\mathrm{PCI}_i-\mathrm{PCI}_j\right|\geq 0, \text{ if } SF_{ij}=0, \forall i\in N^o, j\in N_i \quad (9)$$

$$\left|\mathrm{PCI}_i-\mathrm{PCI}_j\right|>0, \text{ if } SF_{ij}=1, \forall i\in N^o, j\in N_i \quad (10)$$

$$\begin{aligned}&\left|\mathrm{mod}\,3\left(\mathrm{PCI}_i\right)-\mathrm{mod}\,3\left(\mathrm{PCI}_j\right)\right|\geq 0\\ &\text{if } SF_{ij}=0, \forall i\in N^o, j\in N_i\end{aligned} \quad (11)$$

$$\begin{aligned}&\left|\mathrm{mod}\,3\left(\mathrm{PCI}_i\right)-\mathrm{mod}\,3\left(\mathrm{PCI}_j\right)\right|>0\\ &\text{if } SF_{ij}=1, \forall i\in N^o, j\in N_i\end{aligned} \quad (12)$$

$$\begin{aligned}&\left|\mathrm{PCI}_m-\mathrm{PCI}_n\right|\geq 0\\ &\text{if } SF_{mn}=0, \forall m,n\in N_i^+, i\in N^o\end{aligned} \quad (13)$$

$$\begin{aligned}&\left|\mathrm{PCI}_m-\mathrm{PCI}_n\right|>0\\ &\text{if } SF_{mn}=1, \forall m,n\in N_i^+, i\in N^o\end{aligned} \quad (14)$$

Equations (9)-(14) formalise three fundamental PCI allocation principles: (i) the Non-Conflict Principle (Eqs. 9-10) mandates unique PCI assignments for co-frequency adjacent cells while permitting reuse for frequency-diverse neighbours; (ii) the Non-Interference Principle (Eqs. 11-12) requires distinct PCI modulo values in co-located co-frequency sectors to maintain reference signal orthogonality; and (iii) the Non-Confusion Principle (Eqs. 13-14) enforces unique PCI allocation among co-frequency neighbours within a cell's neighbour relation table (NRT) while allowing PCI reuse for inter-frequency neighbours.

C. Objective Function

The objective function (15), derived through modification of the natural model, simultaneously minimizes: (i) the aggregate number of conflicting, confusing, and mod 3 interfering measurement reports (MRs) across all potentially affected cells, and (ii) PCI reuse frequency through preferential selection of reallocations with lower utilization counts. This dual optimization framework promotes rational PCI distribution network-wide while enhancing data transmission efficiency in power load management communication systems, subject to the constraints defined in Eqs. (9)-(14).

$$
\begin{aligned}
&\min z = \sum_{f \in F} z^f \\
&z^f = \sum_{i \in N^{of}} \sum_{j \in N_{i+}} \left(a_{ij} + a_{ji}\right) cor_i^a S_{ij} \\
&+ \sum_{i \in N^o} \sum_{m,n \in N_{i+}, m \neq n} \left(b_{mn} + b_{nm}\right) cor_i^b S_{mn} \\
&+ \sum_{i \in N^o} \sum_{j \in N_{i++}} \left(c_{ij} + c_{ji}\right) cor_i^c M_{ij}
\end{aligned}
\tag{15}
$$

The model employs the following notation: cor_i^a, cor_i^b, and cor_i^c represent the correlation coefficients for cell i under conflict, general interference, and mod 3 interference scenarios respectively; N^{of} otes the subset of cells operating on frequency f; N_{i+} defines the co-frequency neighbour set of cell i; N_{i++} characterises the overlapping-coverage neighbour set of cell i, where coverage areas intersect.

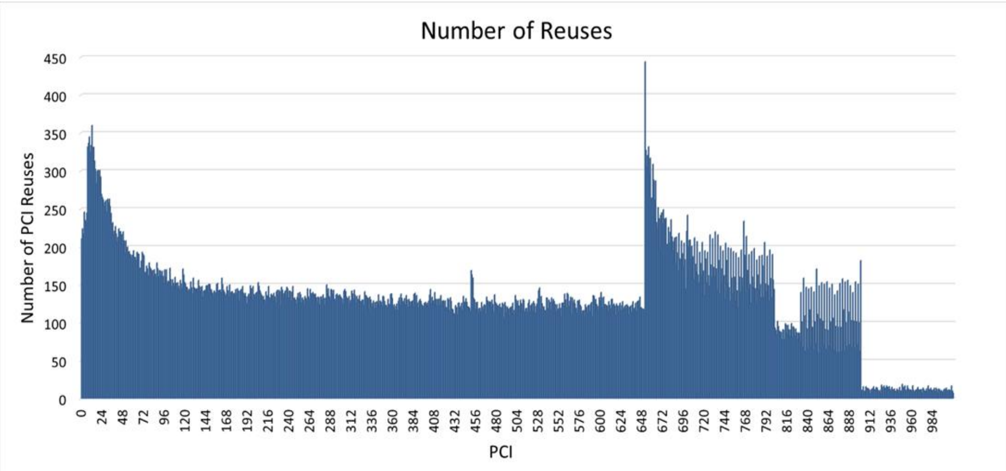

Figure 3. PCI reuse frequency statistics in a network-wide context.

Figure 3 presents a comprehensive analysis of the existing PCI allocation scheme, evaluating both: the minimal achievable counts of conflicting, confusing, and mod 3 interfering measurement reports (MRs) across all potentially affected cells, and PCI multiplexing patterns. The optimisation framework prioritises PCI reallocations with lower multiplexing frequencies, a principle formally incorporated through the derivation of objective function E_q . (16). This dual-criterion approach ensures network-wide PCI distribution efficiency while maintaining interference mitigation.

$$
\max \sum_{i \in N^o} w_p x_{pi} \tag{16}
$$

The weighting parameter w_p governs PCI selection priority, where w_p assumes higher values for less frequently utilised PCI values p in the current network configuration, and conversely. This frequency-dependent weighting mechanism promotes optimised PCI distribution across the network topology [22].

1) Adding Constraints

The PCI allocation framework incorporates critical decision constraints to mitigate areas exhibiting elevated conflict, confusion, and mod 3 interference levels. Threshold parameters σ_a, σ_b, and σ_c - determined through correlation analysis ranking – are established to bound the quantity of high-interference cells. These sequentially applied constraints, formulated below, ensure network-wide interference containment:

$$
\begin{aligned}
&\text{if } cor_i^a \leq \sigma_a \text{ then } \left|\mathrm{PCI}_i - \mathrm{PCI}_j\right| \leq 0 \\
&\forall i \in N^{of}, j \in N_{i+}
\end{aligned}
\tag{17}
$$

$$\text{if } cor_i^b \le \sigma_b \text{ then } \left|\mathrm{PCI}_j - \mathrm{PCI}_{j+1}\right| \le 0 \quad \forall i \in N^o, j \in N_{i+} \tag{18}$$

$$\text{if } cor_i^c \le \sigma_c \text{ then } \left|\mathrm{mod3}\left(\mathrm{PCI}_i\right) - \mathrm{mod3}\left(\mathrm{PCI}_j\right)\right| \le 0 \quad \forall i \in N^o, j \in N_{i++} \tag{19}$$

The constraint system operates as follows: Equation (17) explicitly prohibits PCI conflicts in high-conflict zones for cell i, Equation (18) implicitly prevents PCI confusion occurrences in high-confusion regions, while Equation (19) restricts mod 3 interference in areas prone to such interference. These constraints collectively enforce interference mitigation through spatial PCI allocation controls.

Figure 3 reveals an imbalanced PCI reuse distribution under the current allocation scheme, with certain PCIs being significantly underutilised while others are excessively reused. To address this disparity, we introduce a maximum PCI reuse threshold, formulated as follows:

$$1 \le \sum_{i \in N^o} x_{pi} \le M_p^r, \quad \forall p \in P \tag{20}$$

Equation (20) introduces M_p^r as the maximum permitted reuse count for any given Physical Cell Identity (PCI) value p, establishing an upper bound on PCI multiplexing frequency within the network architecture.

2) *Perform Linearisation*

While the model effectively addresses PCI allocation decisions for high-interference zones (encompassing conflict, confusion, and mod 3 interference), its non-linearity resulting from ordinal constraints necessitates linearisation via the big-M method [23], as detailed below, to facilitate computational tractability.

$$cor_i^a \le \sigma_a + \left(1 - \delta_a\right) M \tag{21}$$

$$\left|\mathrm{PCI}_i - \mathrm{PCI}_j\right| \le \left(1 - \delta_a\right) M, \ \forall i \in N^{of}, \ j \in N_{i+} \tag{22}$$

$$cor_i^b \le \sigma_b + \left(1 - \delta_b\right) M \tag{23}$$

$$\left|\mathrm{PCI}_j - \mathrm{PCI}_{j+1}\right| \le 1 + \left(1 - \delta_b\right) M, \ \forall i \in N^{of} \tag{24}$$

$$cor_i^c \le \sigma_c + \left(1 - \delta_c\right) M \tag{25}$$

$$\left|\mathrm{mod3}\left(\mathrm{PCI}_i\right) - \mathrm{mod3}\left(\mathrm{PCI}_j\right)\right| \le \left(1 - \sigma_c\right) M \quad \forall i \in N^{of}, j \in N_{i+} \tag{26}$$

$$\delta_a, \delta_b, \delta_c \in \{0,1\} \tag{27}$$

The auxiliary variables δ_a, δ_b, and δ_c dynamically regulate constraint strictness during processing, enabling: flexible adaptation to varying interference scenarios, and transformation of inherently non-linear constraints into linear form for computational tractability.

4. Improved Genetic Algorithm

Genetic algorithms (GAs), inspired by natural selection and genetic mechanisms, employ population-based search through genetic encoding, fitness evaluation, and iterative operations (selection, crossover, mutation) to converge towards optimal or near-optimal solutions. Renowned for their robust global search capabilities, problem-agnostic design, and inherent parallelism, GAs have been widely deployed in engineering design optimisation (e.g., mechanical parameter tuning for weight/cost minimisation), path planning (multi-criteria route optimisation), and machine learning (neural network hyperparameter tuning and feature selection). However, conventional GAs exhibit notable limitations when applied to PCI allocation in load management networks, including premature convergence to local optima, computationally intensive iterations, and suboptimal solution quality. To mitigate these constraints, this study proposes a suite of tailored enhancements to refine GA performance for PCI allocation tasks. The final realisation is shown in Figure 4.

Algorithm 1 Improved genetic algorithm

Input: Total number of PCI allocation; Station to be optimized PCI: n; Total clusters: $Npop$; Crossover operator: CR; Mutation operator: MR; Iteration: $t_{\max}$; Neighbourhood search number: NS;

Output: Optimal PCI allocation scheme:

Initialize Random PCI
Evaluate the value of the objective function for each total cluster
Selection of the optimal solution P_{best}
while $t \le t_{\max}$ **do**
  for $all\ i\ from\ 1\ to\ Npop$ **do**
    $P_{new} = Crossover(P_{best},\ P_i,\ CR)$
    $P_{new} = Mutate(P_{best},\ MR)$
    for $j\ from\ 1\ to\ NS$ **do**
      $P_{neighbour} = P'_{new}$
      if ! $Feasible(P_{neighbour})$ **then**
        end if
      if $Fitness(P_{new}) > Fitness(P_{best})$ **then**
        $P_{new} = P_{neighbour}$
      end if
    end for
    calculate P_{new}
    if $Fitness(P_{new}) > Fitness(P_{best})$ **then**
      $P_i = P_{new}$
    if $Fitness(P_{new}) > Fitness(P_{best})$ **then**
      $P_{best} = P_{new}$
    end if
  end for
  $t = t + 1$
end while

Figure 4. Improved genetic algorithms.

A. Setting a Fixed Index

This study introduces innovative fixed indexing for PCI and console zones, establishing a precision navigation framework within complex network architectures. The system enhances granular PCI and console management through: (i) optimised spatial organisation enabling rapid algorithmic deployment, and (ii) structured data processing pipelines for efficient operational execution.

$$P=\{p_1,p_2,\cdots,p_n\} \quad (28)$$

$$M=\{m_1,m_2,\cdots,m_k\} \quad (29)$$

$$I_p:P\to\{1,2,\cdots,n\} \quad (30)$$

$$I_m:M\to\{1,2,\cdots,k\} \quad (31)$$

Equation (25) defines the PCI set within the cyberspace domain, while Equation (26) specifies the console zone set. Equations (27) and (28) establish unique indexing functions for PCI values and console zones respectively, ensuring bijective mapping between physical network elements and their corresponding digital identifiers.

B. Setting the Fitness Function

The fitness function F is formulated as the summation of measurement report (MR) counts, providing a novel and operationally relevant metric to evaluate PCI allocation schemes [24]. This formulation quantifies each scheme's operational viability through its aggregate conflict, confusion, and interference MRs – where lower F values indicate superior interference mitigation capabilities. The function thereby intrinsically guides the search algorithm towards optimal solutions by prioritising schemes demonstrating robust performance across these critical network challenges, as formally specified below:

$$F=\sum_{i=1}^{n}\sum_{j=1}^{k}\left(C\left(p_i,m_j\right)+MR\left(p_i,m_j\right)\right) \quad (32)$$

The function $C\left(p_i,m_j\right)$ quantifies PCI conflict severity within master console area m_j, incorporating multidimensional interference parameters including signal interference strength and resource occupancy conflicts, while $MR\left(p_i,m_j\right)$ evaluates concurrent confusion and interference metrics for the same PCI-console pairing.

C. Greedy Algorithm

The greedy algorithm constitutes an efficient design paradigm for obtaining locally optimal solutions through iterative, myopic optimisation. Employing a top-down approach, it sequentially makes irrevocable decisions based solely on immediate optimality criteria—typically quantified through an objective metric—while disregarding global solution space exploration. Each iteration reduces the problem to a smaller subproblem, with the procedure terminating when successive iterations yield no further improvement [25].

$$G(t)=\{g_1(t),g_2(t),\cdots,g_l(t)\} \quad (33)$$

$$F\left(g_1(t)\right),F\left(g_2(t)\right),\cdots,F\left(g_l(t)\right) \quad (34)$$

$$g_{\text{best}}(t)=\arg\max_{g_i(t)\in G(t)}F\left(g_i(t)\right) \quad (35)$$

Equation (30) defines the iterable population, while Equation (31) specifies the associated fitness function values. Equation (32) implements an elitist selection mechanism that identifies the current highest-fitness individual through comparative iteration. This approach significantly accelerates algorithmic convergence while enhancing the probability of obtaining high-quality solutions under constrained computational resources.

D. Optimisation of Crossover and Mutation Operations

$$o_1=\{p_a[1:c],p_b[c+1:\text{end}]\} \quad (36)$$

$$o_2=\{p_b[1:c],p_a[c+1:\text{end}]\} \quad (37)$$

The mutation operation is governed by a predefined probability threshold p_m, with gene g_i in individual p undergoing mutation if and only if a randomly generated value r satisfies the condition $r<p_m$.

Throughout crossover and mutation operations, strict adherence to PCI allocation constraints is maintained, guaranteeing that all newly generated individuals satisfy both: the rigorous legality requirements of PCI configuration, and the practical operational feasibility constraints inherent to network deployment scenarios [26].

E. Introduction of Neighbourhood Search

The neighbourhood search technique operates on the current optimal solution x_{opt} within a defined radius δ. Each candidate solution x' is rigorously evaluated against the problem's constraint set, with x' replacing x_{opt} if and only if $f(x')<f(x_{\text{opt}})$. This iterative

refinement process progressively explores the solution space to identify globally optimal configurations.

As illustrated in Figure 5. The algorithm establishes a persistent mapping relationship between master consoles and their associated PCIs through consistent indexing. During iterative updates, each PCI maintains its originally assigned index, thereby preserving alignment with its designated target console throughout all optimisation stages.

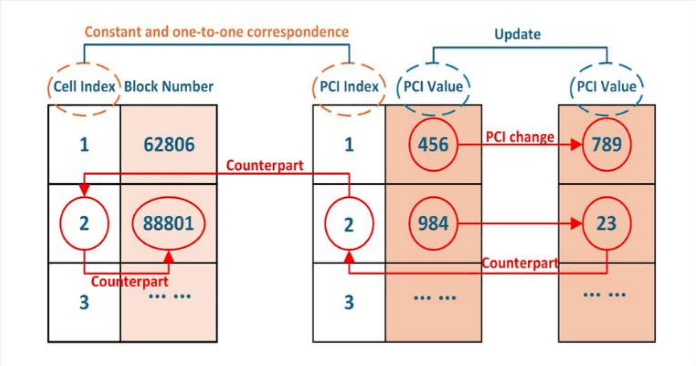

Figure 5. Schematic diagram of algorithm mapping relationships.

5. Simulation Experiments

This study addresses PCI allocation optimisation in power load management networks through controlled simulation of real-world deployment scenarios. Leveraging MATLAB's computational capabilities, we systematically evaluate four critical datasets: station parameters, neighbour relations, frequency dependencies, and conflict/interference matrices. The dual objectives comprise: validating the Improved Genetic Algorithm's (IGA) efficacy in mitigating PCI allocation impairments (conflicts, confusion, and mod 3 interference), and enhancing network transmission reliability and spectral efficiency through optimal resource allocation.

A. Data Correlation Analysis Based on PageRank

The PageRank algorithm, originally developed by Google co-founder Larry Page to quantify webpage importance [27], is adapted here to identify critical nodes in community networks. When applied to PCI planning for power load management systems, the algorithm evaluates: inter-station correlations when the optimised station serves as the master node, and the cascading impact of PCI reallocations across the network. This approach prioritises stations requiring conflict mitigation (Eq. 35) by analysing their connection degrees - where higher connectivity indicates greater conflict potential under current PCI allocations - thereby ensuring network stability.

$$PR(u)=\frac{1-d}{N}+d\sum_{v\in B_u}\frac{PR(v)}{L(v)} \quad (38)$$

The PageRank metric $PR(u)$ quantifies a station's network influence, where elevated values denote greater topological significance and consequently higher PCI allocation priority to prevent conflicts, confusion, or interference. The damping coefficient d (typically 0.85) models inter-station association attenuation, while N represents the total station count. The set B_u contains all stations linking to u, with each contributing station v's relative importance weighted by its own PageRank PR(v) $PR(v)$ and normalized by its out-degree $L(v)$.

For instance, consider a scenario with five base stations (A-E) whose interrelationships are illustrated in Figure 6. Applying Equation (38) with parameters $d=0.85$, $N=5$, and $L(v)$ representing the out-degree of node v, we assume an initial PageRank value of $PR(u)=0.2$ for all base stations. Taking cell A as an example, neighbouring cells B and C each have an out-degree of 2 (i.e., $L(B)=L(C)=2$). Substituting these values into Equation (38) yields an initial iteration result of $PR(A)=0.2$. Repeating this process iteratively generates the final PageRank values for all base stations, as presented in the table below.

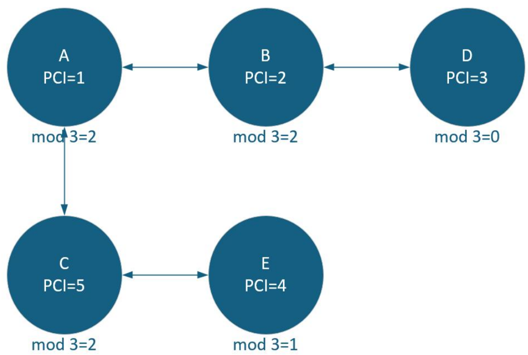

Figure 6. Relationship of the cells at the beginning.

Table 3. $PR(u)$ obtained after repeated iterations.

Cell	$PR(u)$
A	0.38
B	0.22
C	0.22
D	0.09
E	0.09

Consequently, Station A exhibits the highest $PR(u)$ value, thus warranting priority in PCI allocation. In contrast, Stations C and E demonstrate comparatively lower PageRank values, permitting relaxed constraints and PCI value reuse. The resultant optimised allocation scheme is illustrated in the accompanying Figure 7.

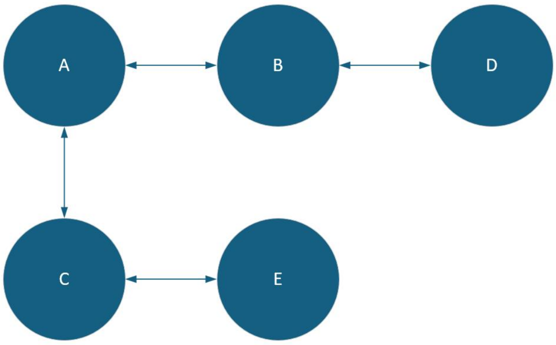

Figure 7. Optimised PCI allocation scheme.

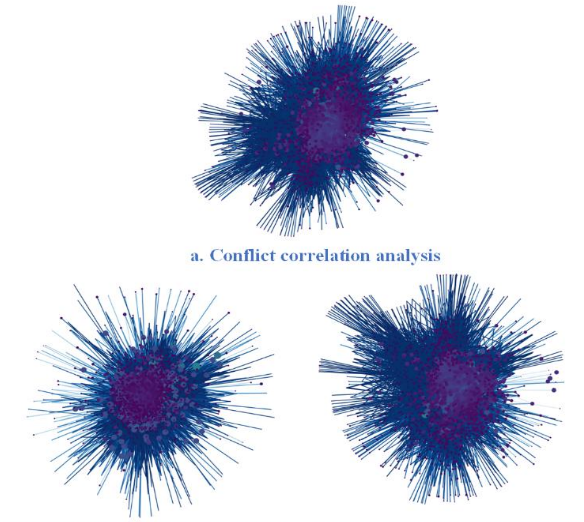

Figure 8. Analysis of PCI conflict, interference, and confusion correlations.

Figure 7 illustrates the optimised PCI allocation scheme: Base cell A serves as the critical node and is assigned a unique PCI; Cell B maintains mod 3 differentiation from A; Cell C, while sharing mod 3 equivalence with B, operates on a distinct frequency band; Cells D and E, as edge nodes, permit PCI reuse despite D's mod 3 coincidence with A, as they are non-adjacent.

In the given example, Station A exhibits the highest PageRank value, indicating its extensive influence across the network and thus necessitating priority PCI allocation. Crucially, when the network topology evolves, only PageRank recomputation-rather than full strategy redesign-is required to update the allocation policy. This mathematical approach transforms PCI allocation from an experience-based process to a data-driven methodology, effectively mitigating conflicts and interference while enhancing overall network performance. Particularly valuable in complex network architectures, this technique provides scientifically grounded guidance for PCI optimisation.

The PageRank algorithm, originally designed to assess webpage importance, has been adapted to optimize PCI allocation in base station networks. In this context, a station's PageRank value reflects its topological influence and connectivity strength within the network. Stations with higher values (indicating critical nodes) are prioritized for unique PCI assignments to prevent conflicts, while those with lower values allow resource reuse under relaxed constraints. The damping factor models signal attenuation characteristics, and iterative computations dynamically evaluate inter-station relationships. This approach transforms PCI allocation from an experience-based practice to a data-driven methodology, enabling adaptive policy updates in response to network topology changes while significantly enhancing overall network stability and performance.

Back to the paper. Figure 8. presents the derived correlation analysis of interference metrics when optimised cells function as master nodes.

Table 4. Selected pagerank analysis data.

Cell number	cor_i^a	cor_i^b	cor_i^c
62874	0.003177	0.003418	0.003626
84800	0.003157	0.003368	0.003133
…	…	…	…

Table 4 presents representative data for two sample stations, demonstrating that the correlation metrics cor_i^a, cor_i^b, and cor_i^c exhibit convergence toward characteristic values. For comprehensive analysis, these station-specific correlations must be aggregated into network-wide averages before being incorporated into the objective function.

B. Experimental Setup

The experimental framework employed randomised PCI allocation within the primary site area and its co-frequency neighbours during initialisation, generating 50 heterogeneous populations that comprehensively explore the solution space. Comprehensive data collection encompassed: station metadata (identifiers, location, power), neighbour relations (spatial interactions), frequency allocation patterns, and conflict/interference matrices derived from empirical measurements. This dataset accurately captures network dynamics while ensuring computational tractability. The parameters were configured with a crossover probability (CR) of 100%, a mutation probability (MR) of 3%, and 50 neighbourhood searches (NS) to optimise the search process.

C. Assessment Criteria

The experimental framework employs randomised PCI allocation within the primary site area and its co-frequency neighbours to generate 50 initial populations, ensuring diverse solution exploration. Three MR-derived matrices (conflict, confusion, and mod 3 interference) serve as quantitative evaluation metrics, where minimised values correspond to optimised network performance. The fitness function, defined as the aggregate sum of conflict, confusion, and interference MRs, provides a direct measure of allocation quality—lower values indicate superior schemes with reduced transmission impairments. This rigorous, data-driven approach integrates constraint satisfaction with practical network performance metrics, establishing a robust evaluation scheme within permissible value ranges. The fitness of each candidate solution is evaluated through a composite function incorporating: conflict/confusion/interference matrices, frequency relationships, and network impact metrics. Selection operations employing roulette or tournament mechanisms preferentially retain high-fitness individuals for reproduction.

Genetic operations proceed via: Single-point crossover: Exchanges PCI values between parent solutions at randomly selected loci. Constrained mutation: Introduces stochastic variations while maintaining PCI allocation legality.

Algorithmic refinement is achieved through: Elitist greedy selection of current optimum as reference. Neighbourhood search exploring proximate solutions through iterative evaluation and replacement.

The process terminates upon convergence criteria satisfaction, having systematically evolved solutions that minimise network impairments while satisfying all constraints.

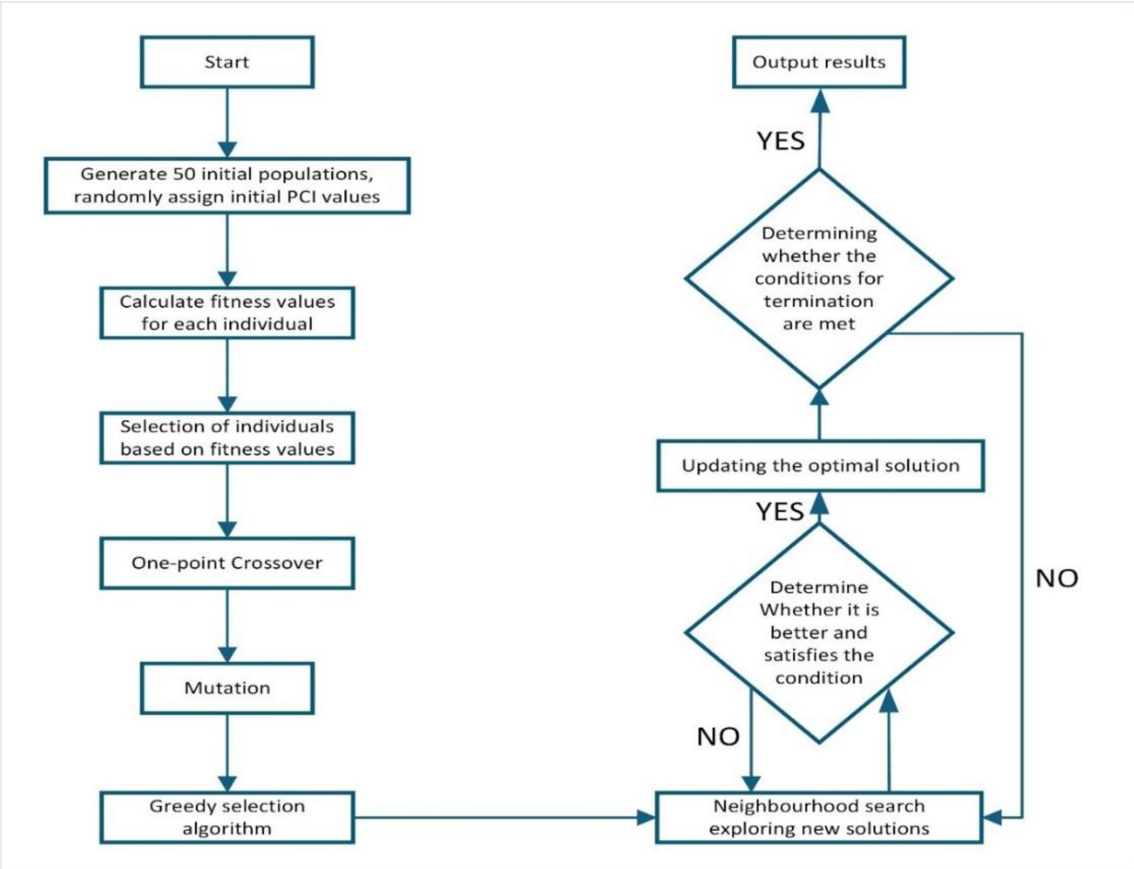

Figure 9. Experimental flow chart.

D. Presentation and Analysis of Result

The simulation results quantify the performance of the optimised PCI allocation scheme through three key metrics: (i) Conflict incidence. (ii) Configuration confusion events. (iii) Mod 3 interference occurrences. With corresponding measurement report (MR) counts tabulated below:

Table 5. Conflict confusion interference count.

	Frequency	**MR number**
PCI conflict	131	54125
PCI confusion	1337	11369464
Mod 3 interference	1895	97911085
Total	3363	109334674

Table 5 presents quantitative results for three critical PCI allocation metrics: conflict (131 occurrences), confusion (1,337 occurrences), and mod 3 interference (1,895 occurrences). The corresponding measurement report (MR) volumes were 54,125, 11,369,464, and 97,911,085 respectively. In aggregate, the dataset recorded 3,363 impairment events associated with 109,334,674 MRs.

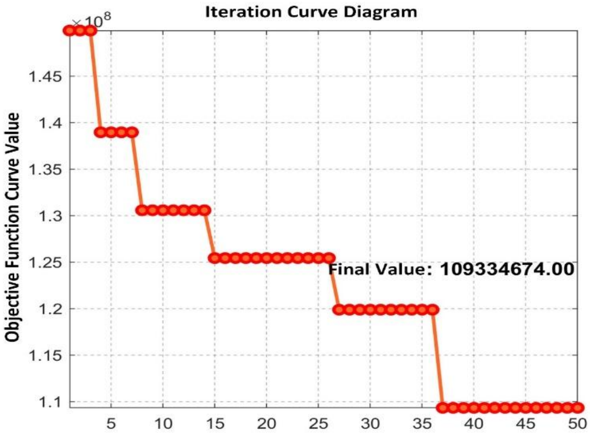

Figure 10. Iterative plot of the data.

Figure 10 demonstrates the algorithm's effective convergence, with the objective function asymptotically stabilising at 109,334,674.00. This reflects systematic reductions in PCI conflicts (131 instances), configuration confusions (1,337 instances), and mod 3 interference events (1,895 instances), while achieving minimisation of the total measurement report volume (109,334,674). The experimental results validate the operational stability improvements in power load management networks, demonstrating consistent efficacy in reducing PCI conflicts, configuration confusions and mod 3 interference occurrences, alongside minimisation of aggregate measurement reports. Comparative analyses confirm the proposed approach's superior performance in interference mitigation and network enhancement compared to conventional allocation methods, establishing its technical innovation and practical utility for PCI optimisation in critical infrastructure. The algorithm's robustness across diverse operational scenarios, combined with its performance advantages, positions it as an effective solution for complex PCI allocation challenges in power management systems.

6. Conclusion

This study proposes a collaborative optimisation-based PCI allocation methodology to address PCI conflicts, configuration confusions, and mod 3 interference in 5G NR power load management communication networks. The framework incorporates PageRank values as weighting factors to construct a multi-objective optimisation model that quantitatively evaluates base station correlation effects on interference patterns. To resolve high-dimensional nonlinear constraints, the model undergoes rigorous linearisation via the Big-M method, achieving significant computational complexity reduction. Furthermore, an enhanced genetic algorithm is developed by integrating greedy-guided crossover/mutation operations, fixed indexing mechanisms, and neighbourhood search techniques, collectively overcoming local optima convergence limitations while ensuring globally efficient solution exploration.

The simulation results demonstrate that the proposed methodology significantly reduces the total number of PCI-related impairments in 5G ultra-dense network deployments, achieving reductions to 3,363 incidents (comprising 131 conflicts, 1,337 configuration confusions, and 1,895 mod 3 interference events), with a corresponding cumulative measurement report (MR) volume of 109,334,674. Comparative analysis confirms the algorithm's marked superiority over existing PCI allocation approaches in mitigating conflicts, ambiguities, and interference, thereby effectively enhancing both operational stability and data transmission efficiency in power load management communication networks.

This study pioneers the integration of the PageRank algorithm into PCI allocation optimisation, establishing a quantitative framework for characterising inter-base station correlations. Through rigorous linearisation of high-dimensional nonlinear constraints via the Big-M method, computational complexity is substantially reduced. Furthermore, the enhanced genetic algorithm synergises greedy-strategy-driven crossover/mutation operations with neighbourhood search techniques, effectively overcoming local optima convergence limitations and achieving comprehensive global search capabilities. These innovations collectively provide a novel solution for PCI allocation planning in 5G NR environments, addressing critical gaps in multi-interference collaborative optimisation models.

Notwithstanding these advancements, persistent challenges in computational complexity and suboptimal convergence rates remain evident in large-scale network deployments. Furthermore, the current model does not account for adaptability to dynamic topological variations, such as base station addition/removal. Future investigations should prioritise: architectural refinements to enhance computational efficiency, incorporation of expanded real-world datasets to improve scalability for massive PCI allocation scenarios, and temporal adaptation mechanisms for time-variant network conditions.

Acknowledge

This work was supported by the Science and Technology Project of China Southern Power Grid Co., Ltd., under Grants YNKJXM20240002 and the National Key Research & Development Program of China under Grants 2023YFB2407300.

References

[1] B. Turdybek, M. Tostado-Véliz, S.A. Mansouri, A. Rezaee Jordehi, F. Jurado. A local electricity market

mechanism for flexibility provision in industrial parks involving heterogenous flexible loads. Applied Energy, 2024, 359, 122748. DOI: 10.1016/j.apenergy.2024.122748

[2] Y. Gilaberte, L. Prieto-Torres, I. Pastushenko, Á. Juarran. Chapter 1 - anatomy and function of the skin. Nanoscience in Dermatology, 2016, 1-14. DOI: 10.1016/B978-0-12-802926-8.00001-X

[3] H.M. Yang, A.P. Huang, R.P. Gao, T. Chang, L.Z. Xie. Interference self-coordination: a proposal to enhance reliability of system-level information in OFDM-based mobile networks via PCI planning. IEEE Transactions on Wireless Communications, 2014, 13, 1874-1887. DOI: 10.1109/TWC.2014.030514.130447.

[4] P. Szilágyi, T. Bandh, H. Sanneck. Physical cell ID allocation in multi-layer, multi-vendor LTE networks. Mobile Networks and Management, 2013, 156-168. DOI: 978-3-642-37935-2_12

[5] F. Ahmed, O. Tirkkonen, M. Peltomäki, J.M. Koljonen, C.H. Yu, M. Alava. Distributed graph coloring for self-organization in LTE networks. Journal of Electrical and Computer Engineering, 2010, 2010(1), 402831. DOI: 10.1155/2010/402831

[6] M. Amirijoo, P. Frenger, F. Gunnarsson, H. Kallin, J. Moe, K. Zetterberg. Neighbor cell relation list and physical cell identity self-organization in LTE. IEEE International Conference on Communications Workshops, ICC, 2008, 37-41. DOI: 10.1109/ICCW.2008.12

[7] C.M. Li, C.W. Chiu, P.J. Wang. Low complexity physical layer cell identification for 5G new radio. Research Square, 2023. DOI: 10.21203/rs.3.rs-2979534/v1

[8] D. Inoue, K. Ota, M. Sawahashi, S. Nagata. Physical cell ID detection using joint estimation of frequency offset and SSS sequence for NR initial access. IEEE Conference on Vehicular Technology (VTC), 2021, 1-6. DOI: 10.1109/VTC2021-Spring51267.2021.9448662

[9] P.Z. Li, H. Yang, I. Kim, Z.Y. Liu, S.S. Li. Cluster-Based Hybrid Approach for PCI Configuration and Optimization in 5G EN-DC Heterogeneous Networks. Journal of Network and Systems Management, 2024, 32(2), 24. DOI: 10.1007/s10922-023-09799-0

[10] D. Krishnaswamy. Quasi-quantum PCI optimization in 5G networks. IEEE Conference on Vehicular Technology (VTC), 2021, 1-6. DOI: 10.1109/VTC2021-Spring51267.2021.9448985

[11] E. Zeljković, D. Gogos, G. Dox, S. Latré, J.M. Marquez-Barja. ALPACA: A PCI Assignment Algorithm Taking Advantage of Weighted ANR. Journal of Network and Systems Management, 2022, 30(2), 33. DOI: 10.1007/s10922-022-09644-w

[12] C. Wu, R. Shi, K. Deng. Reconnaissance and information extraction experiment on the multiple 5G base station signals overlapped in time-frequency-domain. International Conference on Communication Technology (ICCT), 2021, 858-864. DOI: 10.1109/ICCT52962.2021.9657945

[13] C.E. Andrade, L.S. Pessoa, S. Stawiarski. The physical cell identity assignment problem: a practical optimization approach. IEEE Transactions on Evolutionary Computation, 2024, 28(2), 282-292. DOI: 10.1109/TEVC.2022.3185927

[14] R. Acedo-Hernández, M. Toril, S. Luna-Ramírez, J.A. Fernández-Segovia, C. Úbeda. Analysis of the Influence of PCI Planning on the Physical Uplink Control Channel in LTE. Wireless Personal Communications, 2018, 98(1), 629-649. DOI: 10.1007/s11277-017-4887-7

[15] M.M. Abdulkareem, S.A. Yaseen, L.M. Abdullah. Matrix based graph coloring algorithm for LTE-PCI assignment and reassignment reduction. IEEE Control and System Graduate Research Colloquium (ICSGRC), 2017, 41-46. DOI: 10.1109/ICSGRC.2017.8070565

[16] L.L. Chen, P. Cheng, Y.T. Wang, W. Ye. Combining multi-objective evolutionary approach and machine learning to optimize PCI configuration in large-scale LTE networks. International Conference on Communication Engineering and Technology (ICCET), 2022, 32-39. DOI: 10.1109/ICCET55794.2022.00014

[17] S.S. Tu, M. Liu, M. Waqas, S. ur Rehman, R. Zhu, L. Liu. FHC-PCIA: a physical cell identification allocation method based on fuzzy hierarchical clustering for heterogeneous cellular network. IEEE Access, 2018, 6, 46976-46987. DOI: 10.1109/ACCESS.2018.2867066

[18] F. Ahmed, M.Z. Asghar, A. Imran. Combinatorial optimization for artificial intelligence enabled mobile network automation. Metaheuristics in Machine Learning: Theory and Applications, 2021, 663-690. DOI: 10.1007/978-3-030-70542-8_27

[19] Y.G. Liu, W.J. Li, H. Zhang, W.H. Lu. Graph based automatic centralized PCI assignment in LTE. IEEE Symposium on Computers and Communications (ISCC), 2010, 919-921. DOI: 10.1109/ISCC.2010.5546812

[20] H.T. Xu, X.W. Zhou, Y. Li. Model of hypergraph colouring for self-configuration in LTE networks. International Conference on Information Management, Innovation Management and Industrial Engineering, ICIII, 2011, 393-396. DOI: 10.1109/ICIII.2011.100

[21] F. Liu, Y. Meng, J.H. Chai, J. Hou, X.Y. Liu, J. Chen. A physical cell identity combinatorial optimal configuration scheme based on pcis grouping and cells clustering. International Conference on Advanced Infocomm Technology (ICAIT), 2023, 15-21. DOI: 10.1109/ICAIT59485.2023.10367289

[22] N. Nisan, T. Roughgarden, E. Tardos, V.V. Vazirani. Algorithmic Game Theory. Cambridge: Cambridge University Press, 2007. DOI: 10.1017/CBO9780511800481

[23] M. Cococcioni, L. Fiaschi. The Big-M method with the numerical infinite M. Optimization Letters, 2021, 15. DOI: 10.1007/s11590-020-01644-6

[24] C.R. Yang, Q. Qian, F. Wang, M.H. Sun. An improved adaptive genetic algorithm for function optimization. International Conference on Information and Automation (ICIA), 2016, 675-680. DOI: 10.1109/ICInfA.2016.7831905

[25] C.J. Colbourn. Efficient Greedy Algorithms with Accuracy Guarantees for Combinatorial Restrictions. SN Computer Science, 2024, 5(2), 247. DOI: 10.1007/s42979-023-02548-9

[26] E. Osaba, R. Carballedo, F. Diaz, E. Onieva, I. de la Iglesia, A. Perallos. Crossover versus mutation: a comparative analysis of the evolutionary strategy of genetic algorithms applied to combinatorial optimization problems. Scientific World Journal, 2014, 1, 154676. DOI: 10.1155/2014/154676

[27] L. Page, S. Brin, R. Motwani, T. Winograd. The PageRank citation ranking: Bringing order to the web. The Web Conference, 1999. https://api.semanticscholar.org/CorpusID:1508503