Magnetic fields in Multiconductor Systems with Harmonic Currents

M. Pineda-Sánchez, L. Serrano-Iribarnegary

Department of Electrical Enginiering
E.T.S.I.I., Polyctechnic University of Valencia
Camino de Vera s/sn, 46022 Valencia (Spain)
phone:+34 963 877597, fax:+34 963 877599, e-mail: lserrano@die.upv.es, mpineda@die.upv.es

Abstract. Electric and magnetic fields produced by power transmision and distribution lines are receiving strong interest due to its biological effects and interference disturbances upon electronic and computer equipment. The presence of currents with harmonic contents, generated by variable speed drives and other power electronic devices worsen the problem. The accurate evaluation of the magnteic field in the vicinity of the line must take into account the non uniform distribution of currents inside the conductors, due to skin and proximity effects. The spatial distribution of induction lines down to the ground level must be plotted, to evaluate its effects in the body of a person situated below the line This paper proposes a two step procedure for solving the problem: first, the non uniform distribution of the currents in the conductors is calculated, applying Maxwell laws. Second, the distribution of the field is obtained, using a very fast and innovative approach, based on the application of the Bidimensional Fast Fourier Transform to solve a bidimensional convolution in the domain of the spatial frequency.

Key words

Harmonic currents, magnetic fields, transmission lines, convolution, biological effects.

1. Introduction

Electric and magnetic fields produced by power transmission and distribution lines systems are receiving strong interest due to its biological effects and interference disturbances [1].

The effects of low fequency electromagnetics fields can be very strong:: interferences with other systems, thermal effects in biological tissues, etc.. That's why it is of particular interest to analyze fields that have low frequency [2]. In this paper, the main goal is to accurately calculate the magnetic field due to a bifilar line, carrying currents with high harmonic contents, in the space occupied by a person situated directly below the line.

This study is carried out in two steps: first, the distribution of the current within the conductors of a phase is computed by the calculation of the electromagnetic field in the interior and the vicinity of the conductors. In a second step, the magnetic field in the air between the line and the ground is obtained.

Maxwell equations allow for the calculation of the magnetic field distribution *both* inside of the conductors and in the space surrounding them [3]. A general procedure to analyze the electromagnetic field under a quasi-stationary hypothesis in a multiconductor system has been proposed in [4]. Under the hypotesis of magnetic linearity, a matrix of self and mutual impedances, including eddy current and proximity effects, is calculated and inserted into circuit equations. In [5], the calculations of such inductances is carried out neglecting the effects of eddy currents, allowing for a fast calculation of the impedance matrix associated to the conductor system.

Magnetic linearity is assumed, since we are dealing with aerial lines. This allows for the calculation of the field distribution, both inside and outside of the conductors, applying the superposition principle: the space is discretized, in 2D, and the magnetic effects of all of the "discrete" pieces of the conductors are added in every point of space. This method was first introduced by Silvester [5], and has been in widespread use in calculating current distribution and frequency-dependent impedance of conductors with irregular cross-sectional shapes [6]. One difficulty with this method, as pointed out by Wang [7], is the huge number of subconductors needed to accurately model the distribution of the field. When extending this approach to calculate the field in the air below a power transmission line, in the range of tens of meters, the problem becomes unmaneageable.

In this paper, a new, original and extremely fast approach to obtain the distribution of the field, both in the air and in the interior of the conductors is followed. Field equations are discretized, and the effects of all the subconductors, in the form of a spatial 2D convolution, are accounted for in the spatial frequency domain. The use of the Bidimensional Fast Fourier Transform allow for a dramatic increase in the speed of the calculations. The magnetic potential is used for computation of the flux linkage of windings [8], as well as for calculation of the values of the field in the air. The total field in the space occupied by a person situated below the line is evaluated.

In section 2, the discretisation of the line conductors is carried out, and the matrix of self and mutual inductances is calculated, as well as the frequency-dependent resistance of each phase. In section 3, the calculation of the magnetic field generated by the currents in the air down to the ground level is carried out by evaluating the convolution integrals in the spatial-frequency domain. The algorithms develped make intensive use of the Bidimensional Fast Fourier Transform of the spatial distribution of the field, wich allows for a dramatic reduction of the computation time and the memory requirements of the computer system. The frequency effects are considered with reference to a static converter (six-pulse rectifier), without neutral conductor, taking into account the 5^{th}, 7^{th}, and 11^{th} harmonics. Section 4 presents the main conclusions

2. Calculation of eddy currents and proximity effects in the line conductors

Maxwell equations fully characterize the behavior of the system. Due to the role of eddy currents and proximity effects, the following equations must be solved:

$$\begin{cases} \nabla x\vec{H} = \vec{J} \\ \nabla x\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \end{cases} \tag{1}$$

together with the constitutive equations

$$\vec{B} = \|\mu\| \vec{H} \quad \vec{J} = \|\sigma\| \vec{E} \tag{2}$$

Considering an infinite straight line following the z-axis, end effects are neligible, and a 2D field analysis can be performed (that is, all the space cross-sections are equivalent). We introduce a magnetic vector potencial such as $\nabla x\vec{A} = \vec{B}$, that will have only components in z-direction $\vec{A} = (0,0,A_z(x,y))$. Substituing in the above equations

$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial \nabla x\vec{A}}{\partial t} = \nabla x\left(-\frac{\partial \vec{A}}{\partial t}\right) \tag{3}$$

and, after integration, we get

$$\vec{E} = \left(-\frac{\partial \vec{A}}{\partial t}\right) + \nabla V \tag{4}$$

Substituing this value in (2) we get

$$\frac{\vec{J}}{\sigma} = \left(-\frac{\partial \vec{A}}{\partial t}\right) + \nabla V$$

and, as long as all the magnitudes have only z-component, this becomes an scalar equation in the z axis.

$$\frac{J}{\sigma} = \left(-\frac{\partial A}{\partial t}\right) + \nabla V \tag{5}$$

Let's integrate (5) along of one of the conductors of the line, between two points a and b separeted a distance l.

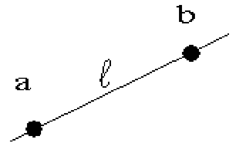


Fig. 1. Path of integration of Maxwell's equations

$$\int_a^b \frac{J}{\sigma} \cdot dl = \int_a^b \left(-\frac{\partial A}{\partial t}\right) \cdot dl + \int_a^b \nabla V \cdot dl$$

Assuming that J and A are constant within the line we get

$$\frac{J}{\sigma} \cdot \ell = -\frac{\partial A}{\partial t} \cdot \ell + (V_b - V_a)$$

If we consider a distance betweens points a and b of conductor of **l** = 1m, and assume that the voltage gradient is constant in the line , $\Delta V = \frac{(V_b - V_a)}{\ell} = cte.$,

$$\Delta V = \frac{J}{\sigma} + \frac{\partial A}{\partial t} \tag{6}$$

Under sinusoidal conditions, we use time phasors

$$\Delta\vec{V} = \frac{\vec{J}}{\sigma} + j\omega\vec{A}$$

The value of current density is $\vec{J} = \frac{\vec{I}}{S}$. So

$$\Delta\vec{V} = \frac{\vec{I}}{\sigma S} + j\omega\vec{A} \tag{7}$$

If the conductor has not neligible dimensions, neither the current density nor the potencial vector can be considered as uniforms. In this case, we follow the method of dividing the cross-section of the conductor in N identical slim square subconductors, as shown in Fig. 2. In each of this subconductors, the current density and the potential vector can be assumed constant. The sum of all the individual currents is the total current passing through the conductor

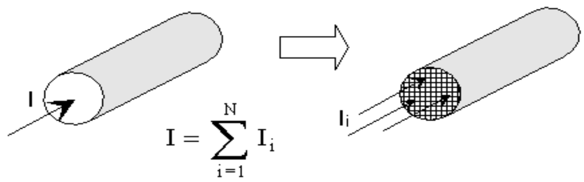


Fig. 2. Division of the conductor in N subconductors

The errors due to aproximating with squares an arbitrary geometry can be reduced to the desired level by refining the mesh. With this discretization, (7) becomes a system of N equations

$$\Delta\vec{V}_i = \frac{\vec{I}_i}{\sigma S_i} + j\omega\vec{A}_i \qquad i = 1..N \tag{8}$$

And, if all the squares are of equal dimensions, $\Delta x \cdot \Delta x$

$$\Delta\vec{V}_i = \frac{\vec{I}_i}{\sigma(\Delta x)^2} + j\omega\vec{A}_i \qquad i = 1..N \tag{9}$$

If only the k-th subconductor is supplied by a unitary current, the voltage drop in the j-th, ΔV_j, becomes the corresponding per-unit length impedance.

$$\vec{Z}_{kk} = \frac{1}{\sigma(\Delta x)^2} + j\omega\vec{A}_k \quad \text{self-impedance of conductor k}$$

$$\vec{Z}_{mk} = j\omega\vec{A}_m \quad \text{mutual impedance of conductors m and k}$$

Eq.9 can be rewritten as $[\Delta\vec{V}] = [\vec{Z}][\vec{I}]$

This approach can be extended to the case of multiconductor systems, single or polyphased, by iteratively appliyng the same procedure: feeding an unitary current into one of the subconductors of the system and calculating the voltage drop in all the others subconductors.

To calculate the impedance matrix one must calculate the magnetic field distribution in the points where the subconductors are located. Neglecting end effects, a circular conductor carring an unitary current creates a magnetic field whose lines of force are circles concentric to the conductor. Its value at a distance r of its center is obtained applying Maxwell equation $\nabla x\vec{H} = \vec{J}$. Integrating it we get

$$\iint_S (\nabla x\vec{H}) dS = \iint_S \vec{J} \cdot dS \Rightarrow \oint \vec{H} dl = I$$

and, considering a current of 1A, $H_\theta = 1/2\pi r$.

The magnetic vector potential A has only z component, and it is related to the induction components by the following equations, in cilindrical coordinates

$$\frac{1}{r}\frac{\partial A_z}{\partial \theta} = B_r \quad , \quad -\frac{\partial A_z}{\partial r} = B_\theta$$

So we get $B_\theta(r) = \mu_0 \cdot H_\theta(r) = \mu_0 \frac{1}{2\pi r} = -\frac{\partial A_z}{\partial r}$, and

$$A_z = -\frac{\mu_0}{2\pi}\ln(r) + C = -\frac{\mu_0}{2\pi}\ln(\sqrt{x^2 + y^2}) + C \tag{10}$$

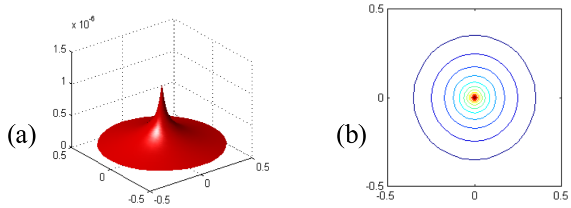


Fig. 3. Potential vector Az (a) and lines of induction (b) generated by a conductor placed in the origin feeded with a constant current of 1A

If the sum of all the currents equals 0, the summed effect of constant C of all the subconductors vanish. So we can assign it a value of zero without affecting the system response.

Mutual impedances between square subconductors

The distance r in (10) is replaced in this case by the geometric mean distance (GMD). It is approximated by the distance between their geometric centers, with an error that Arizon and Dommel [9] have shown to be of 0.655%. for the worst case (adjacent squares).

$Z_{jk} = j\omega\frac{\mu_0}{2\pi}\ln(\frac{1}{d_{jk}})$, where d_{jk} is the distance between the centers of the j-th and the k-th subconductors.

Self impedance of a square subconductor

The distance r in (10) is replaced in this case by the geometric mean radius (GMR). For a subconductor of side length Δx it can be computed using the integral

$$\ln(r_g) = \frac{1}{(\Delta x)^4}\int_0^{\Delta x}\int_0^{\Delta x}\int_0^{\Delta x}\int_0^{\Delta x} \ln\left(\sqrt{(x-u)^2 + (y-v)^2}\right) dx\,dy\,du\,dv$$

$r_g = 0.447(\Delta x)$, and the value of the self-impedance is

$$Z_{kk} = \frac{1}{\sigma(\Delta x)^2} + j\omega\frac{\mu_0}{2\pi}\ln(0.447 \cdot \Delta x)$$

Calculation of the per unit length impedance of the line

The distribution of the currents in a bifilar line formed by two parallel conductors of section 120 mm2, separated by a distance twice their radio, has been calculated. With a voltage drop per unit length of 1e-3 V, and currents of opposite sign in the conductors, the distribution of the currents is calculated for 50 and 250 Hz, the fundamental and 5th harmonics. The cell size is Δx = 1mm, that is, 120 cells per conductor. Values of the per unit length impedance of the line are given.

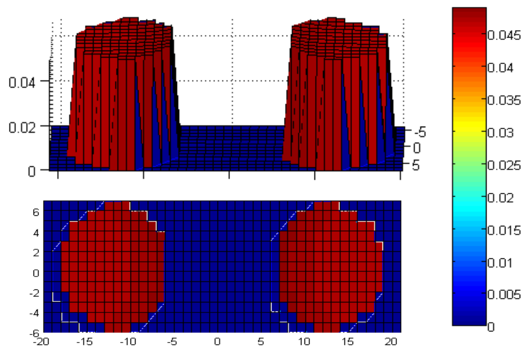


Fig. 4. Absolute value of currents in a conductor of 120 mm2 for a frequency of 50 Hz with a cell of 1x1 mm. $Z = (1.4041+1.0183i)\ 10^{-4}\ \Omega$

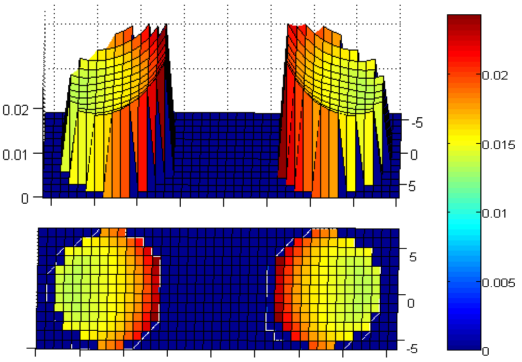


Fig. 5. Absolute value of currents in a conductor of 120 mm2 for a frequency of 250 Hz with a cell size of 1x1 mm
Z=(1.6020 +5.0087i) 10-4 Ω

By increasing the number of discretized points, we get more accuracy. With a cell size of 0.5 x 0.5 mm2 (480 cells per conductor), we get the following results

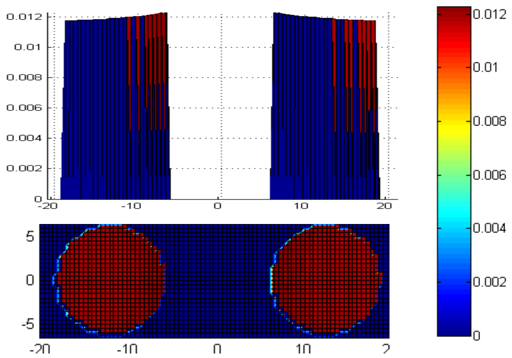


Fig. 6. Absolute value of currents in a conductor of 120 mm2 for a frequency of 250 Hz with a cell size of 0.5x0.5 mm.
Z=(1.4244 +1.0272i) 10-4 Ω

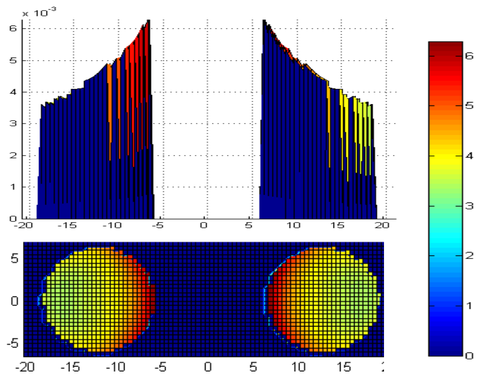
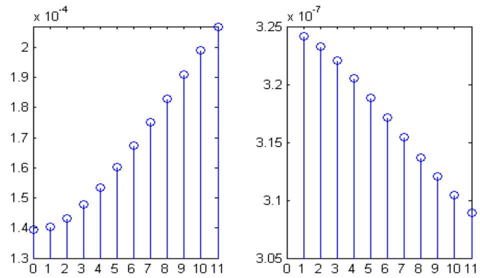


Fig 7. Absolute value of currents in a conductor of 120 mm2 for a frequency of 250 Hz with a cell size of 0.5x0.5 mm
Z=(1.6218 +5.0541i) 10-4 Ω

The evolution of the resistance (Ω) and inductance (H) per meter of the line as a function of the operating frequency, calculated up to the 11th harmonic of the currents (550 Hz) is shown in Fig. 8.



(a) (b)
Fig. 8. Per unit length (a) resistance (Ω) and (b) inductance (H) of the line for harmonic order up to 11th. Cell size of 0.5x0.5 mm

3. Magnetic induction in the vicinity of the line

To obtain the magnetic field in the air surrounding the line, we must calculate the magnetic potential vector in every point of the space. Its contour lines are the induction lines in the space surrounding the line. By superposition we get

$$A_z(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x',y')*A_{z0}(x-x',y-y')*dx'*dy'$$

This expression is a spatial convolution of two functions:

- I(x,y), the spatial distribution of currents,
- $A_{z0}(x,y)$, the potential vector generated by a single conductor situated on the origin of coordinates and feeded with a constant current of 1 A.

By discretization, this integral becomes

$$A_z[m][n]=\sum_{i=1}^{Nx}\sum_{j=1}^{Ny} I[i][j]\cdot A_{z0}[m-i][n-j] \qquad (11)$$

which gives the potential vector in every point of coordinates (m,n) of the space, generated by instantaneous currents in arbitrary points (i,j).

This expression has a serious drawback. The number of operations needed to evaluate it is proportional to $(Nx * Ny)^2$. If the line is considered, for example, at a height of 3m, and we want to obtain the potential vector in a square domain of 3m situated below it, we need, with a cell size of 0.5mm, Nx=Ny=6000, which gives a number of operations $O(6000^4)=O(1.3*10^{15})$. We have solved the problem in an original manner, as far as we know, namely, we have resort to a technique that, although has been developed and thought for time domain signals (radio, radar, etc.), nevertheless it can be formally extended to signals in the space domain: the frequency analysis. Thus, by performing a Discrete Fourier Transform in two dimensions (DFT2), the convolution of (11) can be expressed in the domain of the spatial frequency as

$$A\omega_z[u][v]=I\omega[u][v]\cdot A\omega_{z0}[u][v] \qquad (12)$$

where $A\omega_z, I\omega, A\omega_{z0}$ are, respectively, the DFTs of the discretized functions A_z, I, A_{z0}. There is a decoupling in the domain of spatial frequency. In the spatial domain, the potential vector in a point depends on the effects of the currents located in *every* point (11). On the contrary, in the spatial frequency domain, the value of the potential vector's harmonic of (u,v) order depends *only* on the value of the harmonics of the currents with the same order. The number of operations needed to evaluate the convolution in the spatial frequency domain reduces to O(Nx*Ny). In our case, $O(6000^2=3.6*10^7)$, which represents a reduction of 8 orders of magnitude!.

To calculate the DFTs of the spatial functions we use a very effective algorithm, the Fast Fourier Transform in two dimensions (FFT2). To recover the values of the functions in the spatial domain from the frequency one, we use the inverse DFT2 of the functions, which is calculate using the inverse FFT2 algorithm. Thus, the whole process has the following steps:

$$\left.\begin{array}{c}\text{FFT2}\\ Az_{00}[x][y]\Rightarrow A\omega_{z00}[u][v]\\ I[x][y]\Rightarrow I\omega[u][v]\end{array}\right\}$$

(1) Transformation into spatial frequency domain.

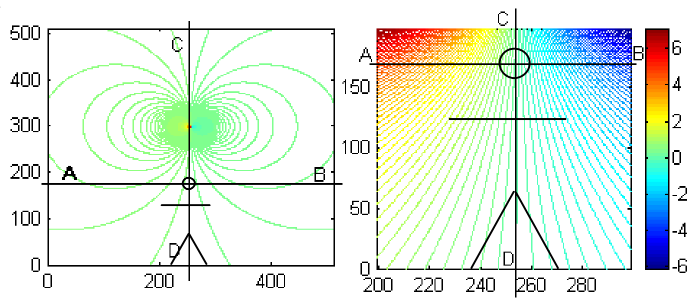
$$ZI\omega[u][v]\cdot A\omega_{z00}[u][v]=A\omega_z[u][v]$$

(2) Convolution.

$$\begin{array}{c}\text{IFFT 2}\\ A\omega_z[u][v]\Rightarrow A_z[x][y]\end{array}$$

(3) Transformation into spatial domain

The magnetic potential generated in the vicinity of the line depicted in the previous paragraph, when feeded by a current of 300 A, is the following one



(a) (b)

Fig. 9. Magnetic vector potential generated by a line situated at a height of 3m and feeded with 300 A in a the space around (a) and its value zoomed in the position of a person situated directly below it (b)

To calculate the induction in every point of the space, and from the definition of the magnetic potential vector, we have

$$\frac{\partial A_z}{\partial y}=B_x,\quad -\frac{\partial A_z}{\partial x}=B_y$$

As can be seen in Fig. 9, induction lines are practically vertical ones in the position of the person, so the value of the horizontal component of the induction, Bx , is negligible. Plotting Bx along the vertical line (CD) we get a value of zero for that component

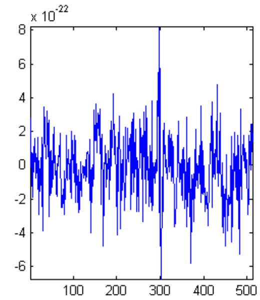


Fig. 9. Horizontal component of the induction (T) along the vertical line CD. Bx≈0

On the other hand, the plot of the vertical component of the induction, By, along the horizontal line AB, situated at a height of 1.70m above the ground, is given in fig. 10

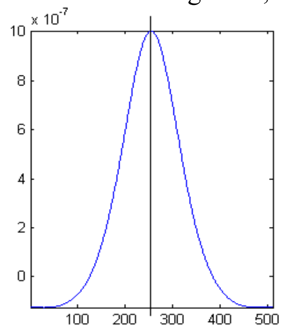


Fig. 10. Vertical component of the induction (T) along the horizontal line AB, situated at the height of the head

As can be seen, the strongest influence in the case of a person situated below the line is produced at the height of his head (average of 1.7m), and in the vertical line of symmetry. For the line carrying a current of 300A, this value is of 1μT (the magnetic field of the earth has a value between 30 and 70μT).

4. Conclusion

In this paper, the magnetic induction generated by a bifilar power transmission line has been calculated in the spatial points that can affect a person situated below of it. For that end, an innovative two step method has been employed, by using the magnetic vector potential as a fundamental magnitude for solving Maxwell equations both inside and outside the conductors: first, the distribution of the currents in the conductors of the line has been calculated taking into account skin and proximity effects; second, the magnetic field in the space of interest (5 x 5m) has been computed with a great resolution (0.5 mm) in the domain of the spatial-frequency, making use of very fast and memory saving algorithms based upon the properties of the Discrete Fourier Transform.

The results show that the main component of the induction that affects the body of a person is the vertical one, and its strongest value is located at the head of the person. This value has been computed for the case of a line with parallel massive conductors of 120 mm^2, separated a distance equal to twice their radio, and located at a height of 3m above ground. When carrying a current of 300A, the maximum value of the induction has be found to be of 1µT.

References

[1] CENELEC, "Human Exposure to Electromagnetic Fields, ENV50166-1(low frequencies) and ENV 50166-2 (high frequencies)", European Committee for Electrotechnical Standardization (CENELEC), Bruxelles, 1995.

[2] ICNIRP, "Guidelines for Limiting Exposure to Time-varying Electric, Magnetic, and Electromagnetic Fields (Up to 300 GHz)", Health Physics 74, 4, 494-522, 1998.

[3] O. Bottauscio, M. Chiampi, D. Chiarabaglio and M. Tartaglia, "Comparation between finite-element and traditional procedures for the prediction of Bushbar behavior", ETEP J, vol. 5, no. 4, pp. 233-238, 1995.

[4] M. Chiampi, D. Chiarabaglio and M. Tartaglia, "A general approach for analyzing power bushbar under a.c. conditions", IEEE Trans. Magn., vol. 29, pp. 2473-2475, Nov. 1993.

[5] M. Carrescia, F. Profumo and M. Tartablia, "Prediction of Magnetic Fields in Multiconductor Systems with Significant Harmonic Currents", IEEE Trans. on Industry Applications., vol. 36, no. 5, pp. 1206-1211, Sep. 2000.

[5] Silvester, P.: "Modal Network Theory of Skin Effect in Flat Conductors". Proc. Of the IEEE 54 (1966) no. 9, pp. 1147-1151

[6] Ametani, A, Fuese., I, "Approximate Method for Calculating the Impedances of Multconductors with Cross Sections of Arbitrary Shapes". Electr. Eng. In Japan 111 (1992) no. 2, pp 117-123.

[7] Wang Y, "A Precise Method for the Impedance Calculation of a Pwer Rail Taking into Account the Skin Effect and Complex Geometry", ETEP , vol. 10, no. 1, pp. 19-27. Feb. 2000.

[8] O. Biró, "Computation of the flux linkage of windings from magnetic scalar potential finite element solutions", IEE Proc –Sci. Meas, Technolo., vol 149, no. 5, pp. 182-185, Sep. 2002.

[9] De Arizon, P. and H. W. Dommel, "Computation of cable impedance based on subdivision of conductors." IEEE Trans. on Power Delivery, (1987), nº 2, pp 21-27.