

Optimization for vibration analysis in rotating machines

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Abstract. A not stable mechanical movement transmission between systems produces equilibrium losses, such as a rotor of motors that are coupled in rotating machines. This can be studied as a disturbance “vibration” either as characteristic of the movement transmission due to controlled displacement over rotors, which transmits the movement. Therefore, in this research is presented an analysis for an optimal control of the rotor axis displacement that includes “vibration” as the part of the movement transmission. It implies mathematical modelling and specific sensors selections to correlate the vibration in this control task. Furthermore, in order to verify the proposed analysis, it was simulated and tested in a hybrid magnetic bearing system.

Key words. optimization, vibration, magnetic bearings, fast sensors.

1. Introduction

Rotating machines are quite important in movement transmission to develop mechanical work through machines. Notwithstanding, a big trouble in this task is given by the friction, owing to energy losses and consequently efficiency of the system is reduced too. Therefore, “Active Magnetic Bearing (AMB)” is used to avoid this problem. However, while the solution is more complex, the necessity to correlate or integrate more variables is the fact. This is the reason, why in this research is proposed the integration and correlation of many variables to get an optimal control. In this work, it was evaluated variable load hybrid AMB as the main system to study this task over rotating machines, for which usually the control is focused in its axis displacement. Furthermore, the AMB axis displacement control is more difficult, when a load position over the axis is variable. Nevertheless, it is not analyzed as the part of the control variable (the vibration implicit variable) that can be caused, because of disequilibrium of the reaction forces over the fixed points that support the gravity force of the AMB.

The vibration variable is considered as disturbance many times, but in this work it was analyzed as implicit part of the control variables, owing to this is a consequence of the dynamic of the system [2], [7]. For this reason, it was necessary to work with the Model Predictive Control through an identified model of the system as a multiple variable systems, which implied the selection of robust and faster sensors, as it was evaluated in this work the vibration sensitivity of sensors that are based in nanostructures.

This article is organized in following sections: “Materials and methods”, “Results” and “discussion”.

2. Methodology

A general system is represented by equation (1), in which “ x ” means space domain, “ f ” is a general function, that for natural behaviour of systems must to be nonlinear, “ u ” is the input variable and “ θ ” keeps information of the system parameters.

$$\dot{x} = f(x, u, \theta) \quad (1)$$

Also, if by equation (2)

$$y = h(x, u, \theta) \quad (2)$$

“ y ” gives information of the response of the system as dependence of a general nonlinear function “ h ”. Therefore, in order to get accurate responses, it was necessary to look for error equation, as consequence of the response signal comparison with the estimated signal, such as it is given by equation (3), in which “ g ” are functions of the parameters “ θ ”, “ F ” and “ E ” are smooth functions to reduce a general polynomial model, for derivatives “ p ” of fixed polynomial “ P ” at order “ n ”, for input variables “ u ”, by auxiliary variables “ j ” and “ k ”.

$$\sum_{j=0}^{n_1} \sum_{k=1}^{n_2} g_j(\theta) F_{jk}(u, y) P_{jk}(p) E_k(u, y) = 0 \quad (3)$$

Therefore, the costing function is given by equation (4)

$$J(\theta) = \sum_{j=0}^{n_1} \sum_{k=0}^{n_1} r_{jk} g_j(\theta) g_k(\theta) \quad (4)$$

Moreover, its solution looks for the “ θ ” matrix in order to achieve the physical parameters of the system. Therefore, it is achieved the estimated equation from the experimental data. That is the reason, why in figure 1 is depicted the system through block diagrams, in which all the input variables are stored in the matrix “ X ”, by other side the matrix “ Y ” stores all the output variables. Even though, it is necessary to get adaptation by adaptive coefficients as represented in this figure.

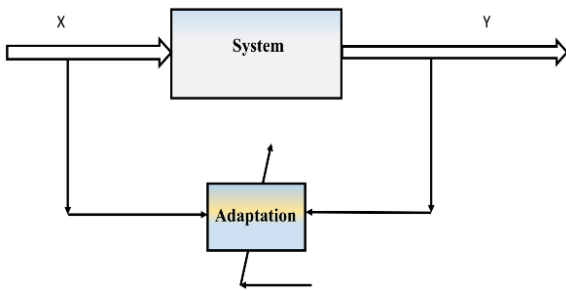


Fig. 1. Block diagram of the proposed system.

According to analyze a rotating machine, it was studied an important part of them: their “rotor or shaft”. Rotor has an usual complication that was achieved in movement transmission, which is the friction that is caused by rotating movement of this inside bearings. However, it is reduced by AMB through the shaft displacement control. Even though, the vibration that was produced for movement transmission can be joined to the control algorithm as an input variable, not only as a disturbance that usually it is analyzed. For a general model [2], it is summarized forces over rotor of a rotating machine, “ M ” is the mass matrix, “ γy ” is the damping matrix coefficients, “ Ky ” is the stiffness matrix coefficients, “ Fg ” is the gravity force over rotor, “ F_R ” is the total reaction forces over rotor, “ F_C ” describes centripetal force effect. All of them described by equation (5).

$$M \frac{d^2 y}{dt^2} = \gamma_y \frac{dy}{dt} + K_y y + F_g + F_R + F_C \quad (5)$$

It was necessary to find correlation from experimental data with theoretically analysis, thus, it was joined every input variable “ u ” inside a matrix “ X ” for every output variable “ Y ”. This is the reason, why in equation (6) it was obtained the matrix coefficient “ β ” (estimation coefficients) in order to analyze the regression behaviour from “ X ” and “ Y ”.

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad (6)$$

Therefore, equation (7) gives the estimated matrix output variables.

$$\hat{Y} = X \hat{\beta} \quad (7)$$

Furthermore, by replacing equation (6) in equation (7) is obtained the expression of the estimated output variables as dependence on matrix of input variables. By other side, the output matrix can include some measured variables, because of the compromise to get the best estimation as it is represented by equation (8).

$$\hat{Y} = X(X^T X)^{-1} X^T Y \quad (8)$$

After to get the system response estimation (such as rotor displacement) and to achieve its parameters, it is necessary to control its displacement (rotor position) in order to study its dynamic through chosen variables, such as vibration and displacement.

The enhancement for the dynamic analysis in this research is given due to the selection of robust and faster sensors, which are based in nanostructures. Notwithstanding, another reason is supported through optimal adaptive predictive model, which helps in this task due to capacity to work with many variables and the dependence on auto corrections by weights that improve the main control target [6], [2].

In equation (9) is described the trajectory road “ R_s ” as input excitation “ r ” and auxiliary variables “ K_i ” [6], [2]. This transposed matrix (by the reference step in order to get only dependence of “ r ”) lets to achieve the optimal response that is looked for the system.

$$R_s^T = (111...11111111)r(k_i) \quad (9)$$

Thus, it was analyzed the costing function “ J ”, according to achieve the optimal desired position “ Y ” (the final response of the system) that is depending of the input variable matrix “ ΔU ”, which is more general than “ u ” owing to it contains variables as usually are rejected as disturbances: such as vibration. This costing function is proportionated by equation (10).

$$J = (R_s - Y)^T (R_s - Y) + \Delta U^T R \Delta U \quad (10)$$

From the last equation, it is expected that “ Y ” (as the response system) has the dependence on internal variables “ $X(k_i)$ ” and “ ΔU ” that were given by equation (11). In other side, “ Y ” depends also of their weights “ F ” and “ ϕ ”.

$$Y = FX(k_i) + \phi \Delta U \quad (11)$$

Otherwise, the equation (12) is the expansion after to replace equation (11) in equation (10) [2], [6].

$$J = (R_s - FX(k_i))^T (R_s - FX(k_i)) - 2\Delta U^T \phi^T (R_s - FX(k_i)) + \Delta U^T (\phi^T \phi + R) \Delta U \quad (12)$$

By deriving the costing function as the dependence on “ ΔU ”, it is obtained the equation (13).

$$\frac{\partial J}{\partial \Delta U} = -2\phi^T(R_s - FX(k_i)) + 2(\phi^T\phi + R)\Delta U \quad (13)$$

Therefore, through the equation (14) is obtained the optimal “ ΔU ”.

$$\frac{\partial J}{\partial \Delta U} = 0 \quad (14)$$

From which the optimal excitation signal “ ΔU ”, in order to find the optimal response “ Y ” is given by equation (15) and it must to be replaced in equation (11).

$$\Delta U = (\phi^T\phi + R)^{-1}\phi^T(R_s - FX(k_i)) \quad (15)$$

Whereby, the optimal physical parameters “ θ ” are achieved by analyzing the costing function of equation (16), as the dependence on estimated “ θ ”, which are new consequence after to get new input estimated matrix “ Γ ” [5].

$$\frac{\partial J}{\partial \theta} = (Y - \Gamma\theta)^T W^{-1}(Y - \Gamma\theta) \quad (16)$$

Hereby, the parameters are showed by equation (17).

$$\theta = (\Gamma^T W \Gamma)^{-1} \Gamma^T W^{-1} Y \quad (17)$$

In summary, the procedure to obtain optimal parameters or variables is given by least square error analysis (costing function) [5], [6] and [2]. Nevertheless, as a recurrence algorithm owing to get adaptive coefficients, the technique “Least Mean Square” (LMS) algorithm is applied in this research. The error analysis “ E ” is improved, because it is achieved the optimal weights matrix “ W ” and weight “ μ ” as depicted by equation (18).

$$W(n+1) = W(n) + \mu X(n)E(n) \quad (18)$$

Finally, it is obtained the total response matrix that is showed by equation (19), in which every optimal analysis helped to get an optimal control of the chosen control variables.

$$\bar{Y} = W_i^T X_i \quad (19)$$

In this section is necessary to explain vibration as input variable of the system, because of vibration that is obtained as the part of not balanced energy transmission through shafts of rotating machines. For this reason, while it is possible to measure the vibrations in different points of the movement transmission, it is possible to get information to enhance optimal control response of the system, such as the displacement control. However, it is important to remember that vibration transmission has the dependence on its frequency and amplitude changes, that depends of the material and geometrical characteristics between contact surfaces, because by other side this vibration transmission can reduce values and the registered information can not get good results in the main analysis.

Theoretical equation for waves transmission is given by equation (20) as it studied by [8], in which “ c ” is the sound speed, “ P ” and “ q ” are the space pressure, where it is propagated the wave on space “ x ” at instance “ t ”.

$$\frac{1}{c^2} \frac{\partial^2 (P'(x,t) - q(x,t))}{\partial t^2} - \nabla^2 (P'(x,t) - q(x,t)) = q(x,t) \quad (20)$$

Thus, its proposal solution is given through equation (21), in which “ P ” is the solution and “ p ” is the function as the dependence on “ x ” and wave physical parameters “ σ ” [8].

$$P(x,t) = p(x,\sigma)e^{j\sigma t} \quad (21)$$

By other side, by equation (22) it is described a summation of “ P ” until component “ N ” as the instance solution in every “ t ”; which depends on parameters “ σ ” and “ γ ” [8].

$$P(x,\gamma) = \sum_{n=0}^N p(x,\sigma)a_n(\gamma) \quad (22)$$

Therefore, as it is known theoretically: the behaviour of the wave over systems can be joined to analyze its dynamic. Such as by equation (23) is described a general polynomial model to study dynamic of systems [5], that is useful to achieve many identification models. Nevertheless, in order to get a control system of this research, as it was explained in sections above, “vibration” is not considered as a disturbance. It is considered as the part of the implicit variables of the system. This is the reason, why in equation (23), in which “ P ” is the derivative for every order “ n ” in every change of the output array of system output signals “ y ” as the consequence of the input excitation signals “ u ”, for every package (as matrix) of physical parameters “ a ” and “ b ”. So, in the error analysis is included the vibration.

$$P^n y(t) + \sum_{j=1}^n a_j P^{n-j} y(t) = \sum_{j=1}^n b_j P^{n-j} u(t) + e(t) \quad (23)$$

Where the solution error analysis is suggested by the equation (24)

$$e_n(m) = \sum_{k=m}^{n+m} \alpha(k,m,\theta_a) V(k) \quad (24)$$

In the context of the equation (25) is obtained:

$$\alpha(k,m,\theta_a) = C_{k-m} \sum_{j=0}^n a_j (jkw_0)^{n-j} \quad (25)$$

Thereby, all the mathematical analysis was proposed to explain how to achieve an optimal position (the displacement control) in rotating machines. In which the vibration is not rejected as a simple disturbance. This is summarized by an algorithm to prove this research. Hence, this algorithm is described in figure 2 that depends on the correct weights of the achieved matrix after to identify correctly the system by mathematical procedure that is explained in the section above.

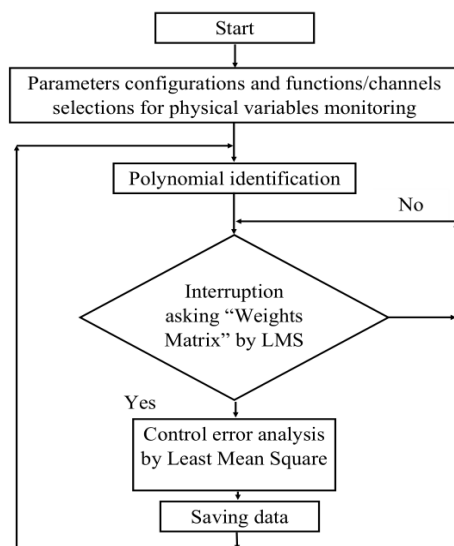


Fig. 2. Flowchart of the main algorithm.

3. Results

In this section is described the evaluation of the mathematical model and main control algorithm that was studied above. To make tests it was used a motor system that needs to work between 1000RPM to 2000RPM and produces noise between 40dB until 80dB approximately. In figure 3 is showed the setup of the system that is composed by a personal computer in which it was executed the main algorithm and an acquisition card the model USB 6008 from the company National Instruments. This card received signals from position and vibration sensors (from ARDUINO and BALLUF companies). Furthermore vibration sensor prototypes [2] elaborated from Anodic Aluminum Oxide (AAO) with particles of iron inside its holes [2]. Also it is showed the motor that transmits the movement to the rotor and the AMB system, which has a load over the rotor in order to test the performance of the main algorithm while the load change position over the rotor.

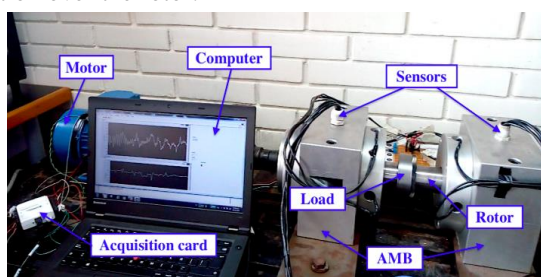


Fig. 3. Setup configuration for the AMB evaluation.

In figure 4 is depicted two vibration surfaces that were measured around one fixed point, in which it was received reaction forces and while there is not equilibrium, it is produced vibration. The surface “Cn” (the vibration around one fixed point of the Active Magnetic Bearing) is caused while it is not applied the control algorithm. However, the surface “Cb” is achieved while the control algorithm is executed. Therefore, it is reduced in 5 decibels (approximately). The not uniform vibration transmission through the AMB holders support is proposed here, because of waves that can not transmit the same power (by amplitude and frequency) while there are

different solids between them, or not uniform contact surfaces, even could be controlled one of the main vibration sources, the not uniform vibration transmission will stay as it is depicted in figure below.

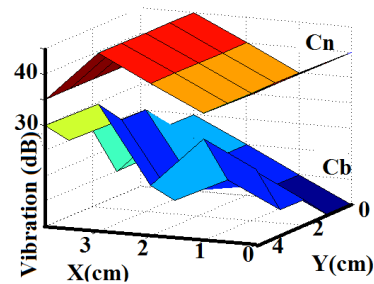


Fig. 4. Vibration effect under controlled and not controlled rotor displacement.

Therefore, the vibration can be reduced while it is achieved a rotor displacement control. Thus, the figure 5 shows the time domain of vibration while it is increased the motor speed percent. The vibration was achieved in its voltage equivalent that is showed in this figure. It is possible to see that is obtained more changes in amplitude, when time increased (owing to rotor speed increased). However, it tends to achieve periodical behavior again, owing to the displacement control.

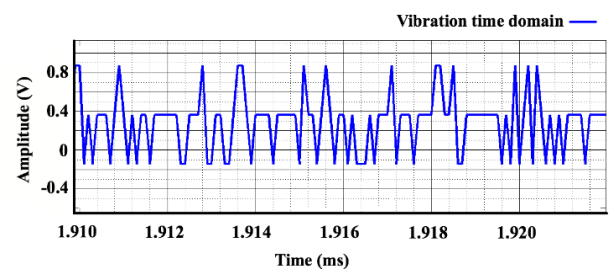


Fig. 5. Time domain for vibration of the system, in which the rotor is supported by the control.

Thereby, it is possible to analyze changes in frequency domain too. For this reason, the figure 6 shows the frequency domain of vibration (of the surface that supports the rotor) while it is increased the motor speed percent. It is possible to see that for every interval of 1kHz it is obtained more picks of decibels. However, it tends to achieve periodical behavior again owing to the displacement control.

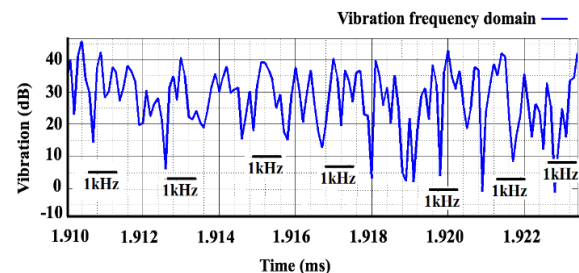


Fig. 6. Frequency domain for vibration of the system, in which the rotor is supported by the control.

Furthermore, the figure 7 shows all the control variables, in which during 120 miliseconds it was increased 5 times

(sequentially) the motor speed that was joined to the AMB shaft, as it's depicted by red color "step" curves. As controlled displacement responses were obtained the shaft position control through green and violet curves, from which (the position of sensors that were fixed in opposite side from each to other) the shaft displacement was controlled even it was increased the rotor speed.

Furthermore, it was evaluated the implicit control that was achieved by vibration sensors (as the knock analysis) that were positioned almost at same place in one reaction holder of one electromagnet of the AMB system. Thus, by the black color curve (that was achieved by ARDUINO vibration sensor), the correlation testing with the blue color curve (that was achieved by a designed vibration sensor based in nanostructures of AAO) was very close that helped to verify that vibrations sensors based in nanostructures need to be robust and faster to improve the main control response of the system.

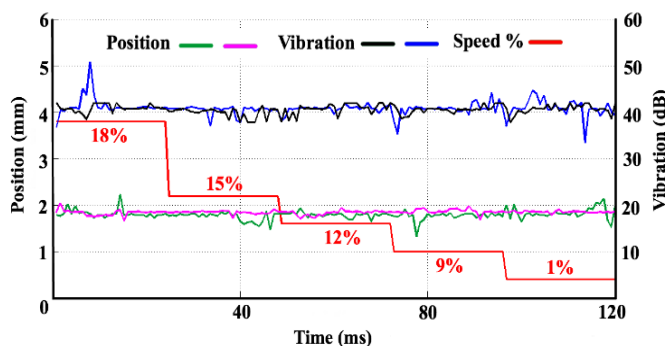


Fig. 7. Controlled variables.

4. Conclusion

It was achieved a good performance in control position for a rotating machine that was based in Active Magnetic Bearing, it because the analysis of the vibration (that was produced as the consequence of reaction forces disequilibrium over the controlled system: motor/rotor/rotating machine) was introduced as the part of the main rotor position control algorithm.

It was possible to design a robust and faster control algorithm that was based in Model Predictive Control, owing to a surface analysis of the position control variables (including vibration). That analysis was supported by sensors based in nanostructures to correlate vibration measurement in fixed reaction points to look for better equilibrium of the system in parallel to the position control. It means, this type of algorithms need faster and robust sensors that are in researching stage nowadays, due to systems tend to be more integrated, whereby geometrical characteristics in internal design of sensors/actuators. These geometrical characteristics help to integrate the main mathematical model of the main control algorithm.

It is suggested to correlate the implicit vibration control that was achieved with a sound consequence, due to rotating machines produce sound more than 40dB in operating work. Therefore, as it is estimated the mathematical model of transmitted vibration, this support

can help to estimate an adaptive sound signal to attenuate produced sound around the rotating machine.

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