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Dispatch merit order- the place of renewable energy

Nuno Domingues

ISEL, Instituto Superior de Engenharia de Lisboa, Portugal

Rua Conselheiro Emídio Navarro, 1, 1959 – 007 Lisboa; ndomingues@deea.isel.ipl.pt

Abstract.

This paper presents results of a study concerning the generation strategic bids for a single hour.

In this study I incorporated the price and quantity bids.

I considered an elastic demand curve, approximated by an affine function, assuming that there is consumer's reaction and that the market price and the demand are related. Also, I consider the competitors reaction using a parameter that represents the conjectural variation.

I studied the market behaviour assuming that the market price is represented by a normal probability function.

I studied and compared the market behaviour for two price markets types, the MCP (Market Clearing Pay) and PAB (Pay As Bid), in two situations: without incorporating the externalities and taking account with the emissions.

Index Terms— Strategic Bidding, Generation Surplus, Conjectural Variation, Elastic Demand, Normal Price Distribution, Emissions.

1. NOMENCLATURE

- block i surplus: $m_i\,(.) = m_i\,(a_i\,, a_i^*\,, \lambda, \lambda_i^{sell}\,, P_{gi}^*)$

- block i production cost: a_i

- price strategic bid: a_i^*

- quantity strategic bid: P_{gi}^*

- block i selling price, λ_i^{sell}

- expected price assuming a rigid demand: λ

- maximum expected price assuming a rigid demand: $\lambda_{\max}$

- minimum expected price assuming a rigid demand: $\lambda_{\min}$

- expected price assuming an elastic demand: λ^θ

- maximum expected price assuming an elastic demand: $\lambda_{\max}^\theta$

- minimum expected price assuming an elastic demand: $\lambda_{\min}^\theta$

2. INTRODUCTION

It's desirable that the electricity market work in a perfect competition. However, due to the limited number of generation companies (lack of competitors), due to the high investment (one of the biggest barriers to new players), due to the long period of time taking from the planning to the exploration of a plant, the grid capacity and the transmission losses, the markets tend to work as an Oligopoly. Thereby, some companies can have a significant market share and make strategic bids to improve their profit.

The study of the market behaviour with the conjectural parameter, developed in 1924 by Bowley and in 1933 by Frisch, was used by several authors [3], [4], [5] but only to simulate oligopoly markets with linear bids and determining just one strategic bid.

The experience shows us that the normal distribution is the one that best represents the market prices [6]. When we consider a normal price distribution, the block surplus function is more complicated than when we consider an uniform price distribution. [1].

3. FORMULATION

I consider a market with several companies that bid by blocks, each block is identified by i. The block i surplus depends on both strategic bids: price and quantity. For each strategic bid it is assumed that all the companies want to maximize the surplus of each block separately.

I assume that the demand is elastic, allowing the price to change with the demand. Also, it is assumed that the market price depends on the demand, as illustrated in Fig. 1:

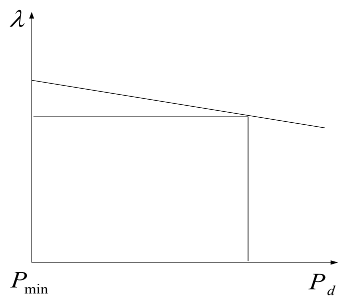

Fig. 1: Demand curve.

Thereby, the market price can be ruled by the equation:

$$\lambda = e - s(P_d - P_{\min}) \tag{1}$$

The value e is the maximum price when the demand is equal to the minimum quantity, $P_{\min}$, and isn't equal to $\lambda_{\max}$.

The demand is:

$$P_d(P_{gi}, P_{\text{-}gi}) = P_{gi} + P_{\text{-}gi}(P_{gi}) \qquad (2)$$

Where $P_{\text{-}gi}$ is the aggregated opponents quantity strategic bid.
The value 's' is the slope of the demand curve and is associated with the consumer's reaction.
According to equation (1) and (2):

$$\lambda = e - s(P_{gi} + P_{\text{-}gi} - P_{\min})$$

thereby,

$$\frac{d\lambda}{dP_{gi}} = -s(1+\theta) \quad \text{with} \quad \theta = \frac{dP_{\text{-gi}}}{dP_{gi}} \qquad (3)$$

θ is a parameter which represents the conjectural variation. This parameter introduces the competitors reaction to the block i quantity strategic bid. When the block i changes is quantity bid the competitors change their quantity bid by $dP_{\text{-}gi}$.

It's assumed that θ is constant for each case study. Thereby,

$$\theta = \frac{dP_{\text{-gi}}}{dP_{gi}} = \frac{\Delta P_{\text{-gi}}}{\Delta P_{gi}} \qquad (4)$$

then

$$\frac{\Delta\lambda}{\Delta P_{gi}} = -s(1+\theta) \Leftrightarrow \lambda = K - s(1+\theta)P_{gi} \qquad (5)$$

K is the expected market price for the minimum value of P_{gi}. For different values of the parameter θ I have:

$$\lambda_n = K_n - s(1+\theta_n)P_{gi} \qquad (6)$$

which, assuming s constant, can be illustrated by Fig. 2:

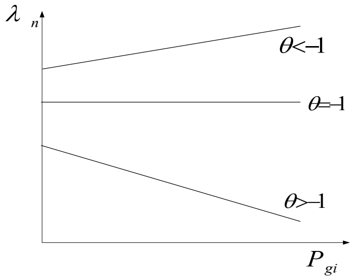

Fig. 2: Market price variation according to P_{gi} and θ

$d\lambda / dP_{gi}$ represents the influence of the quantity bid in the market price, according with θ. It's assumed that it is valid for all prices. Thereby,

$$\frac{d\lambda_{\min}}{dP_{gi}} = \frac{d\lambda_{\max}}{dP_{gi}} = -s(1+\theta) \qquad (7)$$

For θ, $\lambda^\theta \in [\ \lambda^\theta_{\min}, \lambda^\theta_{\max}\ ]$ and can be illustrated by Fig. 3.

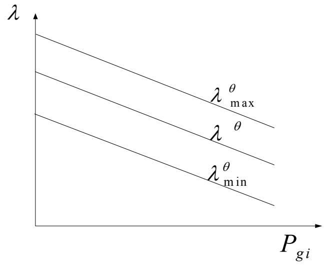

Fig. 3: Market Price reliable range according to P^θ_{gi}.

The market price with the conjectural variation approach is $\lambda^\theta = \lambda + \Delta\lambda$, where $\Delta\lambda = f(P_{gi}, \theta)$ is the market price difference to the market assuming a rigid demand.

$$\lambda^\theta = \lambda \ + s(1+\theta)\Delta P_{gi} \qquad (8)$$

It's defined ΔP^*_{gi} as

$$\Delta P^*_{gi} = P^{\max}_{gi} \text{ - } P^*_{gi} \qquad (9)$$

Thereby

$$\begin{cases} \lambda^\theta_{\max} = \lambda_{\max} + s(1+\theta)\Delta P^*_{gi} \\ \lambda^\theta_{\min} = \lambda_{\min} + s(1+\theta)\Delta P^*_{gi} \end{cases} \qquad (10)$$

The quantity strategic bid is $P^*_{gi} \in [\,0, P^{\max}_{gi}\,]$. Otherwise, I consider that the quantity strategic bid is the respective active restriction.
The selling price, λ^{sell}_i, depends on the quantities. In the MCP market, the active participant's payment is equal to the marginal price. In the PAB market, the active participant's payment is equal to their bid.
For $\theta > -1$, when the block reduces is quantity to P^*_{gi} the market reacts rising the marginal price to $\lambda^\theta = \lambda + \Delta\lambda$. In the MCP market, the block i will sell less quantity at a higher price. In the PAB market, the probability of dispatch of the block i is higher for the blocks that $a_i > \lambda^{MCP}_{\min}$. For both markets, there are a dispatch probability for the block i that $a_i > \lambda^{MCP}_{\max}$. Thereby, the determination of the strategic generation quantity and price bids leads to interesting dynamic market behaviour. Also, since $\theta \neq -1$, the demand will change and $P_d = f(\theta, P_{gi})$.
I studied the market behaviour assuming that the market price is represented by a normal probability function. With the normal probability function I assumed that the market price has higher probability to be in the middle of the reliable range [$\lambda_{\min}, \lambda_{\max}$], as shown by Fig. 4. The normal function has an error when it's limited by the range

[$\lambda_{\min}, \lambda_{\max}$] since it's defined for all domain. However, I assume that $8\sigma = \lambda_{\max} - \lambda_{\min}$, thereby the maximum error will be 0,006% [6].

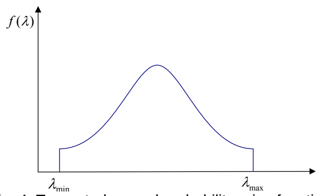

Fig. 4: Truncated normal probability price function

The price probability distribution is then

$$f(\lambda) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\lambda-\mu}{\sigma})^2};$$

$$f(\lambda) \sim N(\frac{\lambda_{\max} + 3\lambda_{\min}}{2}, \frac{\lambda_{\max} - \lambda_{\min}}{8})$$

Thereby we have

$$f(\lambda) = 4\frac{\sqrt{2}e^{J_x}}{\sqrt{\pi}(\lambda_{\max} - \lambda_{\min})}$$

where

$$J_x = -\frac{8(-2\lambda + \lambda_{\max} + \lambda_{\min})^2}{(\lambda_{\max} - \lambda_{\min})^2}$$

It's also defined an auxiliary variable

$$J = -\frac{8(-2a_i^* + \lambda_{\max} + \lambda_{\min})^2}{(\lambda_{\max} - \lambda_{\min})^2}$$

According to the production cost, the block surplus function is:

$$m_i(.) = \int_0^{+\infty} f(\lambda^\theta)(u(\lambda^\theta - \lambda^\theta_{\min}) - u(\lambda^\theta - \lambda^\theta_{\max}))(\lambda_i^{sell} - a_i)(P_{gi}^{\max} - dP_{gi})d\lambda^\theta$$

where $u(.)$ is the step function.

For the block i, the strategic bids are determined by the resolution of the following maximization problem:

$$\begin{array}{l} \max\limits_{a_i^*, P_{gi}^*} \quad m_i\ (a_i^*, a_i, \lambda, \lambda_i^{sell}, P_{gi}^*) \\ \quad s.t. \\ \qquad P_{gi}^* - P_{gi}^{\max} \leq 0 \\ \qquad P_{gi}^{\min} - P_{gi}^* \leq 0 \\ \qquad a_i - a_i^* \leq 0 \end{array}$$

I assume only the production limits restriction for the quantity strategic bid.

To avoid negative surplus for the block i, I assume that the price strategic bid is always $a_i^* \geq a_i$.

If the surplus function is concave and the restrictions are not active, the strategic bids can be determined by:

$$\begin{cases} \dfrac{\partial}{\partial a_i^*} m_i\ (.) = 0 \\ \dfrac{\partial}{\partial P_{gi}^*} m_i\ (.) = 0 \end{cases}$$

I consider that the strategic price bid of the block i doesn't influence the market price, thereby $\frac{\partial \lambda^\theta}{\partial a_i^{\theta *}} = 0$.

4. CASE STUDY

1) For the MCP market

In the MCP market, the selling price, λ_i^{sell}, is the marginal price, $\lambda^{\theta MCP}$. The price strategic bid a_i^* that maximize the expected block i surplus is a_i^{MCP}. The quantity strategic bid that maximizes the expected block i surplus is P_{gi}^{MCP}.

Based on the production cost, the strategic bids are

1ª) if $a_i > \lambda_{\max}^{\theta MCP}$

$$\begin{cases} a_i^{MCP} = a_i \\ P_{gi}^{MCP} \in [0, P_{gi}^{\max}] \end{cases}.$$

2ª) if $\lambda_{\min}^{\theta MCP} < a_i \leq \lambda_{\max}^{\theta MCP}$

First option

$$\begin{cases} a_i^{MCP} = \dfrac{\lambda_{\max}^{MCP} + \lambda_{\min}^{MCP} + 3a_i + 2s(1+\theta)P_{gi}^{\max}}{5} \\ P_{gi}^{MCP} = \dfrac{2}{5}P_{gi}^{\max} + \dfrac{\lambda_{\max}^{MCP} + \lambda_{\min}^{MCP} \quad 2a_i}{5s(1+\theta)} \end{cases}$$

Or second option

$$\begin{cases} a_i^{MCP} = a_i \\ P_{gi}^{MCP} = P_{gi}^{\max} + \dfrac{\lambda_{\max}^{MCP} \quad a_i}{s(1+\theta)} \end{cases}$$

According to the option that leads to higher expected surplus for block i.

3ª) if $a_i \le \lambda_{\min}^{MCP}$

$$\begin{cases} a_i \le a_i^{MCP} \le \lambda_{\min}^{\theta MCP} \\ P_{gi}^{MCP} = \dfrac{P_{gi}^{\max}}{2} + \dfrac{\lambda_{\min}^{MCP} + \lambda_{\max}^{MCP} - 2a_i^{MCP}}{4s(1+\theta)} \end{cases}$$

According to the strategic bids, the maximum expected surplus for block i is

a) if $a_i^{MCP} > \lambda_{\max}^{\theta MCP}$

$$m_i^{MCP\max}(.) = 0$$

b) if $\lambda_{\min}^{\theta MCP} < a_i^{MCP} \le \lambda_{\max}^{\theta MCP}$

$$m_i^{MCP\max}(.) = \frac{1}{8\sqrt{2\pi}}((e^{8} - e^{8(Jaux)^2})(\lambda_{\max}^{MCP} - \lambda_{\min}^{MCP}) +$$

$$+ (e^{\,8+8(Jaux)^2})(\frac{\lambda_{\max}^{MCP} \quad a_i}{s(1+\theta)} + \frac{2}{5}P_{gi}^{\max}) +$$

$$+ 2e^{8+8(Jaux)^2}\sqrt{2\pi}(\lambda_{\max}^{MCP} +$$

$$+ \lambda_{\min}^{MCP} - 2a_i)(ERF(2\sqrt{2}) + ERF\left[(\ \frac{16\sqrt{2}Jaux}{\lambda_{\max}^{MCP} \quad \lambda_{\min}^{MCP}}\right])))$$

where for the first condition

$$Jaux = \frac{3(\lambda_{\max}^{MCP} + \lambda_{\min}^{MCP}) - 6a_i + 4s(1+\theta)P_{gi}^{\max}}{5(\lambda_{\max}^{MCP} - \lambda_{\min}^{MCP})}$$

and for the second condition

$$Jaux = \frac{\lambda_{\max}^{MCP} + \lambda_{\min}^{MCP} - 2a_i}{\lambda_{\max}^{MCP} - \lambda_{\min}^{MCP}},$$

$ERF(X)$ is the integral of the Gaussian distribution, given by

$$ERF(X) = \frac{2}{\sqrt{\pi}}\int_0^X e^{-t^2}dt = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{+\infty}\frac{(-1)^n X^{2n+1}}{(2n+1)n!}.$$

c) $a_i^{MCP} \le \lambda_{\min}^{\theta MCP}$

$$m_i^{MCP\max}(.) = P_{gi}^{\max}(\frac{\lambda_{\max}^{MCP} + \lambda_{\min}^{MCP}}{2} \quad a_i)(ERF(2\sqrt{2}))$$

2) For the PAB market

In the MCP market, the selling price, λ_i^{sell}, is the price strategic bid, $a_i^* = a_i^{PAB}$. The quantity strategic bid that maximizes the expected block i surplus is P_{gi}^{PAB}.

According to the production cost, the strategic bids are

1ª) if $a_i > \lambda_{\max}^{\theta PAB}$

$$\begin{cases} a_i^{PAB} \ge a_i \\ P_{gi}^{PAB} \in \left[0, P_{gi}^{\max}\right] \end{cases}$$

2ª) if $\lambda_{\min}^{\theta PAB} < a_i \le \lambda_{\max}^{\theta PAB}$

First option

$$\begin{cases} a_i^{PAB} = \dfrac{3\lambda_{\max}^{PAB} + 3\lambda_{\min}^{PAB} + 2a_i + 2s(1+\theta)\Delta P_{gi}^{PAB}}{8} \\ P_{gi}^{PAB} = 3\dfrac{\lambda_{\max}^{PAB} + \lambda_{\min}^{PAB} + 2s(1+\theta)P_{gi}^{\max} - 2a_i^{PAB}}{8s(1+\theta)} \end{cases}$$

Or second option

$$\begin{cases} a_i^{PAB} = \dfrac{\lambda_{\max}^{PAB} + \lambda_{\min}^{PAB} + 3a_i + 2s(1+\theta)P_{gi}^{\max}}{5} \\ P_{gi}^{PAB} = \dfrac{2}{5}P_{gi}^{\max} + \dfrac{\lambda_{\max}^{PAB} + \lambda_{\max}^{PAB} - 2a_i}{5s(1+\theta)} \end{cases}$$

According to the option that leads to higher expected surplus for block i.

3ª) if $a_i \le \lambda_{\min}^{\theta PAB}$

$$P_{gi}^{PAB} = P_{gi}^{\max}$$

If $a_i < \lambda_y^{\theta PAB}$ then $a_i^{PAB} = \lambda_{\min}^{\theta PAB}$.

If $\lambda_y^{\theta PAB} < a_i \le \lambda_{\max}^{PAB}$ then

$$a_i^{PAB} = \frac{3\lambda_{\max}^{\theta PAB} + 3\lambda_{\min}^{\theta PAB} + 2a_i}{8}$$

I define $\lambda_Y^{\theta} = \dfrac{5{,}5\lambda_{\min}^{\theta} - 3\lambda_{\max}^{\theta}}{2}$ as an auxiliary variable.

According to the strategic bids, the maximum expected surplus for block i is

a) if $a_i^{PAB} > \lambda_{\max}^{\theta PAB}$

$$m^{PAB\max}(.) = 0$$

b) $\lambda_{\min}^{\theta PAB} < a_i^{PAB} \leq \lambda_{\max}^{\theta PAB}$

$$m_i^{PAB\max}(.) = \frac{1}{250s(1+\theta)}(\lambda_{\max}^{PAB} + \lambda_{\min}^{PAB} + 2s(1+\theta)P_{gi}^{\max} -$$

$$-2a_i\,)(3(\lambda_{\max}^{PAB} + \lambda_{\min}^{PAB}) + 16s(1+\theta)P_{gi}^{\max})(ERF(2\sqrt{2}) -$$

$$-ERF(\frac{2\sqrt{2}(9(\lambda_{\max}^{PAB} - \lambda_{\min}^{PAB}) - 18a_i - 2s(1+\theta)P_{gi}^{\max}}{25(\lambda_{\max}^{PAB} - \lambda_{\min}^{PAB})}))$$

c) $\lambda_Y^{\theta PAB} < a_i^{PAB} \leq \lambda_{\min}^{\theta PAB}$

$$m_i^{PAB\max}(.) = \frac{P_{gi}^{\max}}{8}(\lambda_{\max}^{PAB} + 3\lambda_{\min}^{PAB} \quad 6a_i\,)(ERF(2\sqrt{2}))$$

d) $a_i^{PAB} \leq \lambda_Y^{\theta PAB}$

$$m_i^{PABma}(.) = P_{gi}^{\max}(\lambda_{\min}^{PAB} \quad a_i\,)(ERF(2\sqrt{2}))$$

5. Results

The results are for the following cases:

Case	s	θ
1	0.0001	-0.9900
2	1.0000	0.0000
3	1.0000	0.5000
4	1.0000	1.0000
5	1.0000	1.5000
6	1.0000	2.0000

Fig. 5: Case Study.

The emissions of a coal power plant are 1000kg/MWh [7], thereby the cost of introducing the emission externality is 20 €/MWh.

The results were obtained for the following values:

- for $a_i = 15$, coal technology without taking account with the externalities;
- for $a_i = 35$, coal technology taking account with the externalities;
- $\lambda_{\min} = 22$; $\lambda_{\max} = 38$.

The results are in the Appendix.

6. Conclusions

I assume that the companies have price and quantity strategic bids to maximize their surplus. According to the tables in appendix, we can see that the influence of all technologies is bigger in the MCP market than in the PAB market, when I assume a normal price distribution. The demand satisfied is lower and the market price is higher in the MCP market than in the PAB market.
Also, when the emission externality is introduced as a production cost, the surplus is lower. Therefore, the market can work as an incentive for sustainability.

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II. Biography

Nuno A. S. Domingues (b. 1972) received the Licenciate (5-year) degree in Electrical Engineering from ISEL (2005) and Master degrees in Electrical Engineering and Computer Science from IST (2008). Presently he is a PhD candidate at FCT with his thesis on energy, sustainability and efficiency.

Presently he works as a Labour Engineer at ISEL (2001-2008) in Engineering Systems.

His topics of research include electricity markets modelling and simulation, energy systems, sustainability, efficiency.

I. Appendix

$a_i = 15$	MCP Market						PAB Market					
	Bid		Surplus		Market Price		Bid		Surplus		Market Price	
Case	Price	Quantity	Per unit	Total	Minimum	Maximum	Price	Quantity	Per unit	Total	Minimum	Maximum
1	15,00	10,00	15,00	149,99	22,00	38,00	28,13	10,00	10,84	108,38	22,00	38,00
2	15,00	8,75	16,25	142,18	23,25	39,25	28,13	10,00	10,84	108,38	22,00	38,00
3	15,00	7,50	18,75	140,62	25,75	41,75	28,13	10,00	10,84	108,38	22,00	38,00
4	15,00	6,88	21,25	146,08	28,25	44,25	28,13	10,00	10,84	108,38	22,00	38,00
5	15,00	6,50	23,75	154,37	30,75	46,75	28,13	10,00	10,84	108,38	22,00	38,00
6	15,00	6,25	26,25	164,05	33,25	49,25	28,13	10,00	10,84	108,38	22,00	38,00
	15,00	5,63	41,25	232,02	48,25	64,25	28,13	10,00	10,84	108,38	22,00	38,00

Table. 1: Market behaviour without externalities

$a_i = 35$	MCP Market						PAB Market					
	Bid		Surplus		Market Price		Bid		Surplus		Market Price	
Case	Price	Quantity	Per unit	Total	Minimum	Maximum	Price	Quantity	Per unit	Total	Minimum	Maximum
1	35,00	10,00	0,004	0,04	22,00	38,00	35,25	10,00	0,002	0,02	22,00	38,00
2	37,00	2,00	2,78	5,56	30,00	46,00	35,50	4,69	0,04	0,19	27,31	43,31
3	37,00	2,67	6,00	16,00	33,00	49,00	35,50	5,00	0,11	0,55	29,50	45,50
4	35,00	2,25	10,50	23,62	37,50	53,50	36,25	5,16	1,20	6,17	31,69	47,69
5	35,00	2,40	14,00	33,60	41,00	57,00	37,50	5,25	2,46	12,94	33,88	49,88
6	35,00	2,50	17,50	43,75	44,50	60,50	38,50	5,31	3,74	19,84	36,06	52,06

Table. 2: Market behaviour with emission externalities