20th International Conference on Renewable Energies and Power Quality (ICREPQ'22)
Vigo (Spain), 27th to 29th July 2022
Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, Volume No.20, September 2022

Decentralized Operation of an Isolated Microgrid with Storage Systems Using Multipliers with Alternating Directions

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Abstract. In view of the challenging task of the operation of microgrids, this work presents a distributed strategy to manage an isolated system by the optimal dispatch of active and reactive power. The ADMM (alternating directions method of multipliers) is adapted to solve two optimization problems, finding power profiles along the day for generators and storage systems, taking into account the power flow and demand coverage constraints. The joint dispatch allows the solution to minimize the generation cost and provide ancillary services such as voltage regulation and losses reduction. The method is tested in a study case with batteries and renewable sources to quantify the effects of the proposed strategy and compare with a centralized optimization.

Keywords: ADMM, Economic dispatch, Microgrid operation, Reactive dispatch.

1. Introduction

Microgrid operation is a challenging task, in all control layers, to manage distributed energy resources (DER), loads, and storage systems to supply the demand with appropriate quality of service. To deal with the operation goals, DERs have local energy management systems, which attend commands from a central controller in a coordinate way [1]. To guarantee a proper working in the microgrid, primary and secondary levels provides stability for active and reactive power through locally-controlled power converters. On the other hand, tertiary control defines optimal operation points according to technical and economic conditions, sending setpoints to lower levels to accomplish higher level requirements (e.g., supply demand, reduce operation costs, and provide ancillary services) [2]. In this sense, tertiary controllers could increase the microgrids efficiency, improve robustness, ease high penetration of DERs and users, and provide ancillary services to increase quality of service and reduce efforts in lower control levels [3].

Taking into account the complexity in the operation, some control algorithms are centralized, which present high efficiency and manage all information from microgrid's components. However, conventional centralized methods require high computational capacities and bandwidth to manage systems with a large number of users, generators or storage systems. Moreover, from a network perspective, the robustness is reduced due to the role of the central node, which hinders the required plug and play operation [4], [5]. In contrast, decentralized schemes facilitate implementation with local information and improve resilience of the grid to fails [6].

Other important challenge on the operation of isolated microgrids is maintaining stability and energy quality. For instance, voltage and frequency must be regulated in the face of the constant changes in demand and renewable resources [7]. Similarly, management systems must propend for efficiency and loss reduction with proper operation points in DERs. In all these processes, storage systems could mitigate demand variability by complementing generation in rush hours with optimal cycles of charging and discharging [8]. However, since batteries have constraints in the dynamics of the state of charge, storage management could be difficult within high level objectives such as cost reduction.

Dealing with the described problem, this work proposes a decentralized strategy based on ADMM (alternating directions method of multipliers) to manage an isolated microgrid. The operation optimizes the joint dispatch of active and reactive power of DERs, including storage systems and renewable generators in order to minimize costs, cover the changing demand, regulate voltage in each bus, and reduce active power losses. The method is tested in a simulation study case to analyse results and compare with a centralized strategy.

Next, Section 2 presents some notation and the power flow problem to introduce the optimization framework in Section 3. Then, the ADMM method is presented in Section 4 and the simulation results and analysis is stated in Section 5.

2. Preliminaries

A. Microgrids as graphs

A microgrid has buses (i.e., physical connection points for generators, loads, and transformers) interconnected by links that represent distribution lines. Thus, a microgrid can be modelled by an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N} = \{1, \cdots, N\}$ is the set of notes, and $\mathcal{L} \in N \times N$ is the set of arcs. Each node of the graph represents a bus, and the distribution lines are arcs, i.e., if buses i and j are linked by a distribution line, then $(i, j) \in \mathcal{L}$.

The nodes have some associated variables. For instance, all nodes have a voltage magnitude V_i (where i is the node index) and an angle θ_i. Moreover, if a node has a generator, it has additional variables associated: the active and reactive power generation, which are denoted by P_{G_i} and Q_{G_i}. respectively. Finally, if an electrical load is connected to a node, it has active consumption P_{L_i} and reactive consumption Q_{L_i}.

B. Power flow

The power flow is a feasible solution to the problem of satisfying the power demand of the loads, using generators through active and reactive power injections. The power flow model in polar form is defined as:

$$P_i(V,\theta) = P_{G_i} - P_{L_i}, \forall i \in \mathrm{N}, \tag{1}$$

$$Q_i(V,\theta) = Q_{G_i} - Q_{L_i}, \forall i \in \mathrm{N}, \tag{2}$$

where $P_i(V, \theta)$ and $Q_i(V, \theta)$ are active and reactive power injections or consumptions depending on their sign [9]. These variables can be calculated in terms of the node voltages and angles (V, θ) as:

$$P_i(V,\theta) = V_i \sum_{j \in i} V_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}\right), \forall i \in \mathrm{N}, \tag{3}$$

$$Q_i(V,\theta) = V_i \sum_{j \in i} V_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}\right), \forall i \in \mathrm{N}, \tag{4}$$

where G_{ij} and B_{ij} are the conductance and the susceptance of the line (i, j), θ_{ij} represents the angle difference between nodes i and j $(\theta_{ij} = \theta_i - \theta_j)$, and the notation $j \in i$ represents the set of nodes that are connected to the i^{th} node, including i.

The approximations for the energy constraints in a grid presented in (3) and (4) are often combined with an objective cost function establishing the optimal power flow (OPF) as appropriate operation points in microgrids [10], [11]. Next, we propose two joint optimization problems to deal with technical and economic objectives, providing, in addition, some ancillary services and storage management.

3. Active and reactive power dispatch considering storage systems

A. Active power dispatch considering storage systems

The active (or economic) dispatch determines the generation of each unit, including storage systems, that minimizes the operating cost. It is mandatory to satisfy the restriction of supplying the aggregate demand and the constraints related to the generation limits. Besides, storage systems require the management of charging/discharging actions to keep the state of charge (SOC) of batteries within operational limits.

Considering a microgrid with N buses, N_G generators and N_B storage systems, the economic dispatch must find the optimal power profile of the i^{th} generator during K time slots, i.e., $P_{G_i} = [P_{G_i}(1), \dots, P_{G_i}(K)]$; and the charging and discharging profile of the j^{th} storage system which we denote by $P_{B_{cj}} = \left[P_{B_{cj}}(1), \dots, P_{B_{cj}}(K)\right]$ and $P_{B_{dj}} = [P_{B_{dj}}(1), \dots, P_{B_{dj}}(K)]$, respectively. Notice that each profile has the dispatched powers for each instant of a programming horizon with K time slots. The active dispatch power is given by

$$\min_{P_{G_i}} F = \sum_{k=1}^{K} \sum_{i=1}^{N_G} F_i\left(P_{G_i}(k)\right) \tag{5}$$

Subject to:

$$\sum_{i=1}^{N_G} P_{G_i}(k) + \sum_{j=1}^{N_B} P_{B_{dj}}(k) - \sum_{j=1}^{N_B} P_{B_{cj}}(k) = P_L(k), \tag{6}$$

$$S_j(k+1) = S_j(k) + \eta_c P_{B_{cj}}(k) - \frac{1}{\eta_d} P_{B_{dj}}(k), \tag{7}$$

$$P_{G_{imin}} \le P_{G_i}(k) \le P_{G_{imax}}, \tag{8}$$

$$0 \le P_{Bcj}(k) \le P_{Bc_{jmax}}, \tag{9}$$

$$0 \le P_{Bdj}(k) \le P_{Bd_{jmax}}, \tag{10}$$

$$S_{jmin} \le S_j(k) \le S_{jmax}, \tag{11}$$

for all $i = 1, \dots, N_G, j = 1, \dots, N_B$, and $k = 1, \dots, K$. Here, for the instant k, $P_L(k)$ is the aggregate power demand, $F_i(P_{G_i}(k))$ represents the cost of the i^{th} generation unit, and $S_j(k)$ represents the state of charge of j^{th} storage system. The values $S_j(1)$ and $S_j(K+1)$ as the initial and final states of charge, respectively. Moreover, $P_{G_{imin}}$, $P_{G_{imax}}$, $P_{B_{cjmax}}$, $P_{B_{djmax}}$, S_{jmin}, and S_{jmax} are the operational limits of the generators and storage systems. Finally, η_c and η_d are the charging/discharging efficiencies.

The cost in every time slot is a typical quadratic function $F_i\left(P_{G_i}(k)\right) = a_i + b_i P_{G_i}(k) + c_i P_{G_i}^2(k)$, where a_i, b_i, c_i are cost coefficients of the i^{th} generation unit. Notice that (6) forced the systems to supply the demand, (7) describes the dynamics of the SOC of batteries, and (8) - (11) represent the limits of generators and storage systems.

B. Reactive power dispatch to reduce voltage deviation and active power losses

Voltage regulation can be performed either by storage systems or by reactive dispatch of generator units. In the same way, power losses in a distribution network can be reduced by reactive power injections. These desirable services can be provided by an optimization strategy with the distribution losses and voltage deviations as the objective function:

$$\min_{V,Q} H = P_{loss} + \sum_{i=1}^{N}(1 - V_i)^2 \qquad (12)$$

Subject to power flow in (3) and (4), and

$$Q_{Gimin} \leq Q_{Gi} \leq Q_{Gimax}, \forall i = 1, \dots, N_G, \qquad (13)$$

$$V_{imin} \leq V_i \leq V_{imax}, \forall i = 1, \dots, N, \qquad (14)$$

$$\theta_{imin} \leq \theta_i \leq \theta_{imax}, \forall i = 1, \dots, N. \qquad (15)$$

The objective function (12) has two terms: The distribution losses, $\boldsymbol{P_{loss} = \sum_{i=1}^{N} G_{ij}(V_i^2 + V_j^2 - V_i V_j \cos\theta_{ij})}$; and a summation of penalties of the voltage deviations in the nodes from the p.u. measures. The constraints in (13) – (15) represent limits of reactive powers of each generator, and boundaries in voltages and angles for every node that can be defined according to regulatory policies in each region or country [12]. The decision variables in this case are the voltages and the reactive powers of the generators.

The voltages define a point that minimizes the objective function, while the generators allow the systems to reach these voltages through reactive power variation. The power flow equations (3) and (4) define the relationship between reactive power and the magnitude and angle of the voltages at the nodes. It is worth noting that the active powers are not decision variables since they are provided by the solution of the active power dispatch (5) to (11).

4. Decentralized operation based on ADMM

Alternating directions method of multipliers (ADMM) is an optimization technique that splits the global problem into subproblems that are individually solved. In addition, individual solutions interact in a coordinated way to finding a solution to the global optimization problem [13]. From a technical perspective, this method can be understood as a mix that takes advantage of the benefits of dual decomposition and the augmented Lagrangian method to improve convergence.

A. ADMM algorithm

Let us consider the following optimization problem with decision variables $\boldsymbol{x} \in \boldsymbol{R^n}$ and $\boldsymbol{z} \in \boldsymbol{R^m}$.

$$\min f(x) + g(z), \qquad (16)$$

$$\text{Subject to: } Ax + Bz = c, \qquad (17)$$

where $\in \boldsymbol{R^{p\times n}}$, $\boldsymbol{B} \in \boldsymbol{R^{p\times m}}$ and $\boldsymbol{c} \in \boldsymbol{R^p}$. Moreover, $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ are assumed to be convex.

The augmented Lagrangian of (16), (17) is

$$\mathcal{L}_p(x, z, y) = f(x) + g(z) + y^T(Ax + Bz - c) + \left(\frac{\rho}{2}\right)\|Ax + Bz - c\|_2^2, \qquad (18)$$

where $\boldsymbol{\rho > 0}$ is the penalty parameter, and the Lagrange multipliers are represented by $\boldsymbol{y} \in \boldsymbol{R^p}$. Assuming that strong duality holds between the primal and dual problems, then both problems' optimum $(\boldsymbol{x}^*, \boldsymbol{z}^*)$ is the same and occurs with the same Lagrange multipliers $(\boldsymbol{y}^*)$.

Notice that the augmented Lagrangian includes a quadratic term at the end. This regularization term improves the robustness and the convergence rate of the algorithm [13].

The ADMM algorithm iteratively solves the two-variables problem in (16), (17) as follows:

$$x^{k+1} := \underset{x}{\operatorname{argmin}} \mathcal{L}_p(x, z^k, y^k), \qquad (19)$$

$$z^{k+1} := \underset{z}{\operatorname{argmin}} \mathcal{L}_p(x^{k+1}, z, y^k), \qquad (20)$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c), \qquad (21)$$

where $\boldsymbol{\rho > 0}$ is the step size and penalty term. Notice that the ADMM algorithm has two minimization steps given in (19) and (20), and one updating step of the Lagrange multipliers given in (21). The augmented Lagrangian is sequentially minimized with each variable in turn, i.e., in (19), an optimal value is calculated which directs the solution towards the global optimum only in the x-direction. Equation (20) repeats the process directing the solution by z-direction. This leads the search for a global solution through alternating directions. Furthermore, the separation of $\boldsymbol{x}$ and $\boldsymbol{z}$ allows the decoupling or disaggregation of $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$.

B. ADMM for solving the active power dispatch with storage systems problem

Our purpose is to use ADMM to obtain an optimal solution for problem in (5) – (11) in a decentralized way. First, the inequality restrictions are included as equality restrictions through slack variables. In this case, we define a set $\boldsymbol{W}$ of slack variables as follows: $\boldsymbol{W} = \left[\boldsymbol{w_{min}^1(k)}, \cdots, \boldsymbol{w_{min}^{Nc}(k)}, \boldsymbol{w_{max}^1(k)}, \cdots, \boldsymbol{w_{max}^{Nc}(k)}\right] \forall\, \boldsymbol{k} = \boldsymbol{1} \dots \boldsymbol{K}$, where: $\boldsymbol{N_c}$ is the number of constraints; the $\boldsymbol{max}$ and $\boldsymbol{min}$ subscripts are related to the type of constraint: maximum or minimum. In order to turn inequality constraints (8) – (11) to equality constraints, it uses slacks variables as follows. For example, let $\boldsymbol{w_{max}}, \boldsymbol{w_{min}} \in \boldsymbol{W}$ be slack variables, then a constraint is modified according to:

$$P + w_{max} = P_{max}, \qquad (22)$$

$$-P + w_{min} = P_{min}, \qquad (23)$$

where: $\boldsymbol{P}$ is the decision variable, and $\boldsymbol{P_{max}}, \boldsymbol{P_{min}}$ are maximum and minimum limits.

Slack variables are auxiliary values that evolve in the process and facilitate reaching an optimal solution in a secondary role. In the iterative process, the variables in the set $\boldsymbol{W}$ must evolve maintaining the equality constraints and the feasibility of the solution. For the slack variables to keep feasibility, they exert changes in the objective function by adding a penalty function

$$f_+(w) = \begin{cases} C_+ & w < 0 \\ 0 & w \geq 0. \end{cases} \qquad (24)$$

The function $\boldsymbol{f_+(w)}$ works by increasing the value of the objective function a considerable amount $\boldsymbol{C_+}$, in case of obtaining negative values of $\boldsymbol{w}$ that could affect the feasibility of the solution.

Secondly, the Lagrangian $\mathcal{L}_\rho(\cdot)$ is defined by:

$$\mathcal{L}_\rho\left(P_{G_i}(k), P_{B_dj}(k), P_{B_cj}(k), Y, W\right) = F + F^+ + F_Y + F_{Y_\rho}. \qquad (25)$$

Here, F is the objective function in (5), F^+ is the summation of penalties functions related to each slack variables defined in (24). The terms F_Y and F_{Y_ρ} are related to the augmented Lagrangian, and are given by $F_Y = Y^T E$

and $F_{Y_\rho} = \frac{\rho}{2} E E^T$, where Y is the Lagrange multipliers vector and E is a vector with all constraints, defined as

$$E = \begin{bmatrix} \sum_{i=1}^{N_G} P_{G_i}(1) + \sum_{j=1}^{N_B} P_{B_d j}(1) - \sum_{j=1}^{N_B} P_{B_c j}(1) - P_L(1) \\ \vdots \\ \sum_{i=1}^{N_G} P_{G_i}(K) + \sum_{j=1}^{N_B} P_{B_d j}(K) - \sum_{j=1}^{N_B} P_{B_c j}(K) - P_L(K) \\ \eta_c \sum_{l=1}^{1} P_{B_cj}(l) - \frac{1}{\eta_d} \sum_{l=1}^{1} P_{B_dj}(l) + w_{max}^1(1) + S_j(1) - S_{jmax} \\ \vdots \\ \eta_c \sum_{l=1}^{K} P_{B_cj}(l) - \frac{1}{\eta_d} \sum_{l=1}^{K} P_{B_dj}(l) + w_{max}^1(K) + S_j(1) - S_{jmax} \\ -\eta_c \sum_{l=1}^{1} P_{B_cj}(l) + \frac{1}{\eta_d} \sum_{l=1}^{1} P_{B_dj}(l) + w_{min}^2(1) - S_j(1) - S_{jmin} \\ \vdots \\ -\eta_c \sum_{l=1}^{K} P_{B_cj}(l) + \frac{1}{\eta_d} \sum_{l=1}^{K} P_{B_dj}(l) + w_{min}^2(K) - S_j(1) - S_{jmin} \\ E_{min} \\ E_{max} \end{bmatrix} \tag{26}$$

First and second elements of vector $\boldsymbol{E}$ are related to the power demand balance for each one of the $\boldsymbol{K}$ time slots. The remainder of elements refer to maximum and minimum constraints of the SOC of batteries and the power limits of generators. In summary, the term $\boldsymbol{F_{Y_\rho}}$ is the summation of the 2-norm of the constraints multiplied by the penalty parameter $\boldsymbol{\rho}$.

Algorithm 1 presents the proposed ADMM algorithm for active power dispatch according to (19) – (21). The algorithm works iteratively until reaching the optimal power profiles for generators and the optimal charging/discharging actions profiles for batteries. It assumes that the optimal values are reached at the maximum number of iterations ($\boldsymbol{T_{max}}$), which was tuned experimentally considering that the Lagrange multipliers tend to be constant at $\boldsymbol{T_{max}}$.

Algorithm 1. ADMM for active power dispatch

1	**for** $\boldsymbol{t = 1}$ **to** $\boldsymbol{T_{max}}$
2	$\boldsymbol{P_{G_i}^{t+1}} \coloneqq \underset{\boldsymbol{P_{G_i}}}{\textbf{argmin}}\ \boldsymbol{\mathcal{L}_p}\left(\boldsymbol{P_{G_i}^t}, \boldsymbol{P_{B_{dj}}^t}, \boldsymbol{P_{B_{cj}}^t}, \boldsymbol{Y^t}, \boldsymbol{W^t}\right)$
	$\vdots$
3	$\boldsymbol{P_{G_{N_G}}^{t+1}} \coloneqq \underset{\boldsymbol{P_{G_{N_G}}}}{\textbf{argmin}}\ \boldsymbol{\mathcal{L}_p}\left(\boldsymbol{P_{G_{N_G}}^t}, \boldsymbol{P_{B_{dj}}^t}, \boldsymbol{P_{B_{cj}}^t}, \boldsymbol{Y^t}, \boldsymbol{W^t}\right)$
4	$[\boldsymbol{P_{B_{dj}}^{t+1}} \quad \boldsymbol{P_{B_{cj}}^{t+1}}] \coloneqq \underset{\boldsymbol{P_{B_{dj}}}, \boldsymbol{P_{B_{cj}}}}{\textbf{argmin}}\ \boldsymbol{\mathcal{L}_p}\left(\boldsymbol{P_{G_{N_G}}^t}, \boldsymbol{P_{B_{dj}}^t}, \boldsymbol{P_{B_{cj}}^t}, \boldsymbol{Y^t}, \boldsymbol{W^t}\right)$
	$\vdots$
5	$[\boldsymbol{P_{B_{dN_B}}^{t+1}} \quad \boldsymbol{P_{B_{cN_B}}^{t+1}}] \coloneqq \underset{\boldsymbol{P_{B_{dN_B}}}, \boldsymbol{P_{B_{cN_B}}}}{\textbf{argmin}}\ \boldsymbol{\mathcal{L}_p}\left(\boldsymbol{P_{G_{N_G}}^t}, \boldsymbol{P_{B_{dj}}^t}, \boldsymbol{P_{B_{cj}}^t}, \boldsymbol{Y^t}, \boldsymbol{W^t}\right)$
6	$\boldsymbol{W^{t+1}} \coloneqq \textbf{max}\left(\boldsymbol{0}, \underset{\boldsymbol{W^t}}{\textbf{solve}}(\boldsymbol{E^t})\right)$
7	$\boldsymbol{Y^{t+1}} \coloneqq \boldsymbol{Y^t} + \boldsymbol{\rho E^t}$
8	**end-for**
9	$\left[\boldsymbol{P_{G_i}^*}, \dots, \boldsymbol{P_{G_{N_G}}^*}\right] := \left[\boldsymbol{P_{G_i}^{T_{max}}}, \dots, \boldsymbol{P_{G_{N_G}}^{T_{max}}}\right]$
10	$\left[\boldsymbol{P_{B_{dj}}^*}, \dots, \boldsymbol{P_{B_{dN_B}}^*}\right] := \left[\boldsymbol{P_{B_{dj}}^{T_{max}}}, \dots, \boldsymbol{P_{B_{dN_B}}^{T_{max}}}\right]$
11	$\left[\boldsymbol{P_{B_{cj}}^*}, \dots, \boldsymbol{P_{B_{cN_B}}^*}\right] := \left[\boldsymbol{P_{B_{cj}}^{T_{max}}}, \dots, \boldsymbol{P_{B_{cN_B}}^{T_{max}}}\right]$

Algorithm 1 starts by loading parameters such as cost coefficients of generation units, load profile for 24 hours, and maximum and minimum operational limits of generators and batteries. Lines 2 and 3 refer to obtaining the optimal power profiles for generators by minimizing the augmented Lagrangian. In the same way, lines 4 and 5 obtain the optimal charging/discharging actions profile for batteries. It is remarkable that in lines 2 to 5 each agent (a generator or a battery) proposes an optimal solution to minimize a global function sequentially and individually.

Algorithm 1 is not completely distributed, but it is highly decentralized. This is because the solving process that each agent performs requires mostly local information but also some global information. This case uses gradient-based methods to solve problems in lines 2 to 5. This means that the method only requires local information related to their decision variables, avoiding knowing other variables which are constant under their point of view. However, the power balance constraint in (6) requires the complete state of generators and batteries.

Lines 6 and 7 calculate the slack variables and the Lagrange multipliers. Line 6 updates the slack variables vector using the operation $\textbf{max}\left(\boldsymbol{0}, \underset{\boldsymbol{W^t}}{\textbf{solve}}(\boldsymbol{E^t})\right)$, which represents that if $\boldsymbol{W}$ has no positive value, it takes a value equal to zero to maintain the solution feasibility. In this case, there is a penalization in the objective function equivalent to $\boldsymbol{C_+}$ in Equation (24). Otherwise, $\boldsymbol{W^{t+1}}$ takes the outcome value for solving the corresponding equation in the matrix $\boldsymbol{E^t}$ for $\boldsymbol{W^t}$. Line 7 is the updating equation for Lagrange multipliers, which reach a stable value after a finite number of iterations if the objective function is convex. Finally, lines 9 to 11 ends the Algorithm 1 obtaining the optimal profiles of generators and batteries.

C. *ADMM for solving the reactive power dispatch enabling ancillary services*

Similar to the presented in the previous section, the inequality constraints in the problem (12) – (15) are expressed as equality constraints using slack variables. Then, the augmented Lagrangian is defined according to the cost function and the corresponding constraints, and finally, a solving algorithm is proposed.

First, the inequality restrictions in (13) - (15) are equality constraints through a set of slack variables $\boldsymbol{V}$ using (22), (23), and (24) as it is described in the previous section.

The augmented Lagrangian $\boldsymbol{\mathcal{L}_q}(\cdot)$ includes the objective function (12) and the constraints for the power flow and the technical boundaries in (3), (4), (13) - (15), such that

$$\mathcal{L}_q(V_i, Q_i, Z, V) = H + H^+ + Z_V + Z_q. \tag{27}$$

More explicitly, $\boldsymbol{\mathcal{L}_q}(\cdot)$ is composed of four terms, where $\boldsymbol{H}$ is the cost function (12), which is a summation of the active power losses and voltage deviations in terms of the node voltages ($\boldsymbol{V_i}$). The term $\boldsymbol{H^+}$ is the summation of penalty functions related to the set of slack variables $\boldsymbol{V}$ given in (24). The following terms are defined as $\boldsymbol{Z_V} = \boldsymbol{Z^T U}$ and $\boldsymbol{Z_q} = \frac{\boldsymbol{q}}{\boldsymbol{2}} \boldsymbol{U U^T}$, where $\boldsymbol{Z}$ is the Lagrange multipliers vector and $\boldsymbol{U}$ is a vector with all constraints given by

$$U = \begin{bmatrix} V_1 \sum_{j \in 1} \left(V_j \left(G_{1j} \cos \theta_{1j} + B_{1j} \sin \theta_{1j} \right) \right) - P^*_{G_1} + P_{L_1} \\ \vdots \\ V_N \sum_{j \in N} \left(V_j \left(G_{Nj} \cos \theta_{Nj} + B_{Nj} \sin \theta_{Nj} \right) \right) - P^*_{G_N} + P_{L_N} \\ V_1 \sum_{j \in 1} \left(V_j \left(G_{1j} \sin \theta_{1j} - B_{1j} \cos \theta_{1j} \right) \right) - Q_{G_1} + Q_{L_1} \\ \vdots \\ V_N \sum_{j \in N} \left(V_j \left(G_{Nj} \sin \theta_{Nj} - B_{Nj} \cos \theta_{Nj} \right) \right) - Q_{G_N} + Q_{L_N} \\ U_{min} \\ U_{max} \end{bmatrix} \tag{28}$$

The first terms of matrix (28) are related to the active power flow in (3). Note that this equation uses the optimal active power profiles found with Algorithm 1. Following rows of matrix $\boldsymbol{U}$ are related to the active power flow according to (4). Finally, $\boldsymbol{U_{min}}$ and $\boldsymbol{U_{max}}$ refers to power limits of generators and batteries.

Algorithm 2 proposes an iterative method to obtain the reactive power dispatch based on the ADMM. It works similarly to Algorithm 1, but it obtains the optimal profile of reactive power for generators and batteries by minimizing the augmented Lagrangian $\boldsymbol{\mathcal{L}_q}(\cdot)$ in (27). It is worth remarking that Algorithm 2 is completely distributed because it only needs local information to reach an optimal result. This is possible given that constraints are only in terms of variables from neighbor nodes as voltages and reactive loads.

Algorithm 2. ADMM for reactive power dispatch

for $\boldsymbol{t = 1}$ **to** $\boldsymbol{T_{max}}$

$$\boldsymbol{Q^{t+1}_{G_i}} \coloneqq \underset{\boldsymbol{Q_{G_i}}}{\textbf{argmin}} \, \boldsymbol{\mathcal{L}_q} \left(\boldsymbol{Q_{G_i}}, \boldsymbol{Q^t_{B_{dj}}}, \boldsymbol{Z^t}, \boldsymbol{V^t} \right)$$

$$\vdots$$

$$\boldsymbol{Q^{t+1}_{G_{N_G}}} \coloneqq \underset{\boldsymbol{Q_{G_{N_G}}}}{\textbf{argmin}} \, \boldsymbol{\mathcal{L}_q} \left(\boldsymbol{Q_{G_{N_G}}}, \boldsymbol{Q^t_{B_{dj}}}, \boldsymbol{Z^t}, \boldsymbol{V^t} \right)$$

$$\boldsymbol{Q^{t+1}_{B_j}} \coloneqq \underset{\boldsymbol{Q_{B_j}}}{\textbf{argmin}} \, \boldsymbol{\mathcal{L}_q} \left(\boldsymbol{Q_{B_j}}, \boldsymbol{Q^t_{G_i}}, \boldsymbol{Z^t}, \boldsymbol{V^t} \right)$$

$$\vdots$$

$$\boldsymbol{Q^{t+1}_{B_{N_B}}} \coloneqq \underset{\boldsymbol{Q_{B_{N_B}}}}{\textbf{argmin}} \, \boldsymbol{\mathcal{L}_q} \left(\boldsymbol{Q_{B_{N_B}}}, \boldsymbol{Q^t_{G_i}}, \boldsymbol{Z^t}, \boldsymbol{V^t} \right)$$

$$\boldsymbol{V^{t+1}} \coloneqq \textbf{max} \left(\boldsymbol{0}, \underset{\boldsymbol{V^t}}{\textbf{solve}} (\boldsymbol{U^t}) \right)$$

$$\boldsymbol{Z^{t+1}} \coloneqq \boldsymbol{Z^t} + \boldsymbol{q U^t}$$

end-for

$$\left[\boldsymbol{Q^*_{G_i}}, \dots, \boldsymbol{Q^*_{G_{N_G}}} \right] := \left[\boldsymbol{Q^{T_{max}}_{G_i}}, \dots, \boldsymbol{Q^{T_{max}}_{G_{N_G}}} \right]$$

$$\left[\boldsymbol{Q^*_{B_{dj}}}, \dots, \boldsymbol{Q^*_{B_{dN_B}}} \right] := \left[\boldsymbol{Q^{T_{max}}_{B_{dj}}}, \dots, \boldsymbol{Q^{T_{max}}_{B_{dN_B}}} \right]$$

5. Simulation results

The microgrid simulated is a 3-nodes system in a voltage level of 4.16 kV (see Figure 1). In order to analyze the interaction between renewable energy and storage resources in an isolated mode includes a photovoltaic generator (G1), a batteries system (G2/B1), and a diesel generator (G3). The diesel unit is the slack generator that provides the voltage and frequency references. Table I details the capacities of each generator and the storage system, whose generation cost is not considered just to define a free of charge unit.

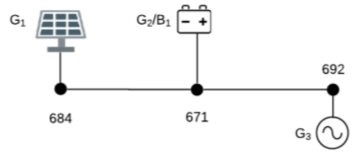

Fig. 1. Case study for a 3-nodes system.

Table I. Characteristics of the generators and the storage system

GENs.	Active power limits [kW]	Reactive power limits [kVAr]	Cost coefficients $(\boldsymbol{c_i}; \boldsymbol{b_i}; \boldsymbol{a_i})$
G1	[0, 400]	[0, 173]	0.00; 0.5; 0.5
G3	[40, 400]	[-100, 100]	0.01; 1.5; 0.5
G2/B1	[-500, 500]	[-193, 193]	-

Figure 2 shows the individual load profiles for each node of the active and reactive power and the aggregated profiles. The solar irradiance is a typical bell-shaped curve with a maximum at noon.

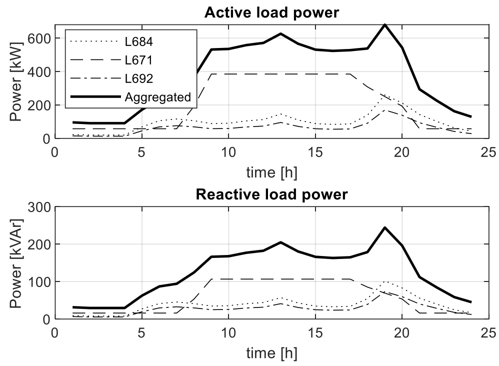

Fig. 2. Load profiles for the case study.

Figure 3 shows the active power dispatch obtained with ADMM. Clearly, the cheap solar generator is dispatched to its maximum capacity, while the batteries, despite their free cost, are charged and preserved to provide cheap energy in the rush hours. The expensive diesel generator completes the rest of power to supply the aggregate demand in the system. The results are compared to the ones of a centralized method (interior-point method) without substantial difference between power profiles as shown in Table 2. This fact is particularly important since the centralized algorithm relies on complete information to reach the result, while the decentralized algorithm uses only local information to achieve the same performance.

As it is shown in Table 2, the optimal cost values are the same for both methods. However, the centralized process is less time-consuming than the decentralized one because ADMM needs more iterations to reach the optimal point. This occurs because ADMM updates variables one by one, whereas the centralized method updates all variables at a time. On the other hand, both methods maintain the power balance with a maximum error less than to 0.01% with respect to the maximum value of the demand.

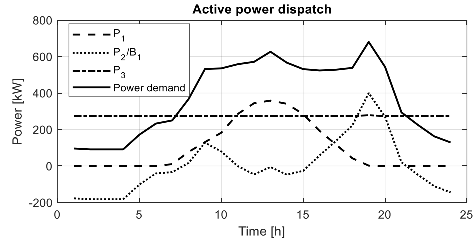

Fig. 3. Active power profiles dispatched by ADMM algorithm.

Table II. Performance of the methods

Method	Cost	Error	Time	Iter.
Interior-point	1.4509e+04	1.67e-14%	0.3637 s	41
ADMM	1.4509e+04	6.10e-05%	2.8110 s	100

The upper graph of Figure 4 shows the voltage profiles without considering the penalizing term of voltage deviation in the optimization process (12). In contrast, the bottom graph shows an important reduction of the voltage deviation (about 12%) considering the voltage penalizing term in the ADMM method. These voltage profiles are achieved by the reactive power dispatch shown in Figure 5, which take advantage of the cheap generators to supply the required reactive demand. This result reduces active power losses by 193.6 Wh in 24 hours which corresponds to 0.01% of the total energy consumed daily.

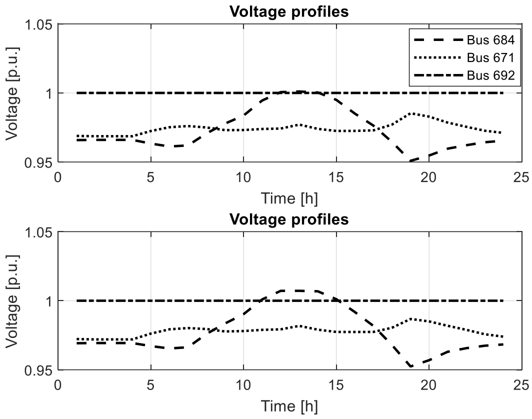

Fig. 4. Voltage profiles without (upper) and with (bottom) penalty term form voltage deviations.

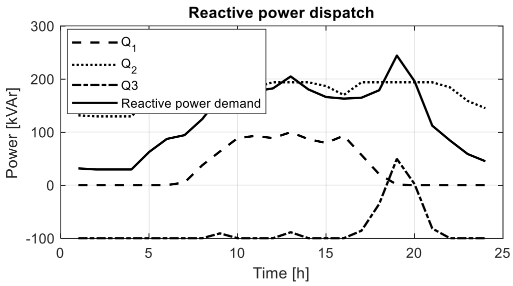

Fig. 5. Reactive power profiles dispatched by ADMM algorithm.

6. Conclusions

A decentralized algorithm based on ADMM has been proposed to obtain power profiles of generators and battery charging/discharging actions. The optimal active and reactive joint dispatch minimizes the cost of operation and provides ancillary services to reduce power losses and voltage deviation. Additionally, it is worth nothing that the decentralized algorithm exhibits a similar performance compared to a centralized algorithm. Indeed, both methods achieve the same optimal value. The simulation results in a study case show that the method improves losses and voltage regulation with the obtained optimal profiles in comparison to scenarios without joint dispatch. The algorithm is highly decentralized, which is appropriate to manage scenarios with numerous agents. The algorithm performance can be easily tested in larger microgrids as future work.

Acknowledgements

This work has been supported in part by “Proyecto Campus Verde Udenar”, BPIN 2020000100041, SGR Colombia; the Colombia Scientific Program within the framework of call named Ecosistema Científico, contract No. FP44842- 218-2018; Fundación CEIBA, contract No. 87067512; and the Research Department of Universidad CESMAG in Colombia. J. A. Aguado was partially supported by Junta de Andalucía (Spain) Project ref: P20_01164.

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