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Experimental Measurement and Analysis of the Self and Mutual Inductances in Two Different Switched Reluctance Machines

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Abstract. This paper aims to present a comparative study among the self and the mutual inductances of a 6x4 switched reluctance machine prototype. Se same study was made with a French origin 8 x 6 machine. The second study confirmed the conclusions on the first one. Mechanical adaptations were made in the test bench to strongly block the rotor of the machines in any previously defined position in order to allow the experimental measurements. At each rotor angular position data were collected for different currents, step by step. The rotor position was also changed step by step from the aligned position to the position of minimum inductance. A data bank was constructed depending on both: rotor angular position and current. A mathematical model taking into account the mutual inductances is presented. Also are presented simulations results using the data got in the test bench. Results of simulations made with and without the mutual inductances values were compared. Surfaces that represent the inductances are presented for both machines. The results analyses show that the mutual inductances may have important influence in the simulation, design and, therefore, in the switched reluctance machines performance.

Key words

Switched reluctance machine, mathematical model, self and mutual inductances, experimental measurements.

1. Introduction

The mutual inductance effects are neglected in most of the technical literature on Switched Reluctance Machine (SRM). This is usual for both machine modeling and machine design. These mutual inductances are frequently quoted as too small to make the difference [9]. Despite this all the purpose of this paper is to evidence that the mutual inductances of a SRM must be object of a specific study, if not always, at least in many cases.

The reluctance machine is the unique family of electrical machines radically news in a century. It´s a doubly salient pole machine, with windings only in the poles of the stator and operates entirely based on magnetic attraction. Furthermore this kind of electrical machine has concentrated and independent windings for each phase of its stator. Usually only one phase is excited at a time. The SRM is a convenient application of an electronically switched trigger system usage in the industry. In spite of its few inherent disadvantage – and each of them may be mitigated - this kind of machine is the most cheap to fabricate and still one of the most reliable [1].

The SRM is also a versatile choice because it operates like a motor or generator through a simple commutation of its firing angles and it has a good performance in a wide speed range [2]. Due to it the SRM is considered for applications like starter motor-generator [4] and for wind power plants [5].

During the last decade the technical literature on SRM grew substantially and yet important investigation works are made in worldwide scale. It is to say that the SRM is been prepared to industrial and commercial applications.

As well known, the SRM requires a power converter to operate either as a motor or as a generator. A power electronics inverter must be used to operate grid connected. This need is similar to that of the induction generators. The excitation power normally comes from the grid itself [6]. A simplified control method of the SRM for generation in AC is proposed in [7] and the improvement of its performance may be obtained through the adjustment of the shooting angles as in [8].

As it was said above almost all the technical texts disregard the mutual linkage among the SRM phases. According to them this linkage is weak because the mutual inductances are small when compared with the self inductances of a phase. Since they are small they may be not taken into account in simulations, prototypes and projects. Even explanations about the equipment operation and performance become easier [9].

To neglect the mutual inductance effects introduces an important simplification in both the mathematics and the computational models of the machine. But some results presented in [10] showed that discrepancies and unexpected results were found when the mutual inductances were disregarded during tests conducted with a prototype.

This paper compares the mutual inductances values and behavior of a 6 x 4 SRM its phases self inductance. Detailed experimental measurements were made to find the self inductance and the mutual inductances of a prototype. They are presents here to illustrate the discussion.

2. Mathematical Model

Taking into account the mutual inductances effects a mathematical model for a 6 x 4 SRM can be that one presented in equation (1).

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ C_m \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & \omega\frac{\partial L_{12}}{\partial\theta} & \omega\frac{\partial L_{13}}{\partial\theta} & 0 & 0 \\ \omega\frac{\partial L_{21}}{\partial\theta} & R_2 & \omega\frac{\partial L_{23}}{\partial\theta} & 0 & 0 \\ \omega\frac{\partial L_{31}}{\partial\theta} & \omega\frac{\partial L_{32}}{\partial\theta} & R_3 & 0 & 0 \\ -\frac{\partial W_1^{co}}{\partial\theta} & -\frac{\partial W_2^{co}}{\partial\theta} & -\frac{\partial W_3^{co}}{\partial\theta} & D & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \omega \\ \theta \end{bmatrix} + $$

$$+\begin{bmatrix} L_{11} & L_{12} & L_{13} & 0 & i_1\frac{\partial L_{11}}{\partial\theta} \\ L_{21} & L_{22} & L_{23} & 0 & i_2\frac{\partial L_{22}}{\partial\theta} \\ L_{31} & L_{32} & L_{33} & 0 & i_3\frac{\partial L_{33}}{\partial\theta} \\ 0 & 0 & 0 & J & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{i}_3 \\ \dot{\omega} \\ \dot{\theta} \end{bmatrix} \tag{1}$$

where:

$x=1,2,3$	appoints a specific phase of the machine;
v_x	is the voltage in the phase x winding;
i_x	is the current in the phase x;
R_x	is the resistance of the phase x;
L_{xx}	is the self inductance of phase x;
L_{xy}	is the mutual inductances among phases x and y;
ω	is the rotor angular speed;
C_m	is the mechanical torque;
J	is the rotational inertia;
D	is the coefficient of friction;
θ	is the angular position of the rotor;
W_x^{co}	designates the co-energy of the phase x;
$\dot{g}$	is the time rate of any variable $g(t)$; and
t	is the time itself.

It is well known that the co-energy of any phase, the electromagnetic torque of the machine and its mechanical torque are given by equations (2), (3) e (4).

$$W_x^{co} = \int_0^i \lambda_x di_x \tag{2}$$

$$C_{emag} = \sum_{x=1}^{n} \frac{\partial W_x^{co}}{\partial\theta} \tag{3}$$

$$C_m - C_{emag} - J\frac{d\omega}{dt} - D.\omega = 0 \tag{4}$$

where:

C_m	is the mechanical torque;
C_{emag}	is the electromagnetic torque;
λ_x	is the linkage flux of phase x.

In order to simplify the mathematical treatment an equation of states can be provided here. Let [V], [R], [I], [L] and $[\dot{I}]$ be the matrices that appear in the equation (1) in the same order. If so it is, the corresponding equation of states for this type of machine is:

$$[\dot{I}] = [L]^{-1}[V] - [L]^{-1}[R][I] \tag{5}$$

Equation (5) is important to plan and to develop a computing model to simulate the behavior of the machine under the specified conditions.

Taking another path, from equation (1) it can be noticed that, based on the angular position Θ domain, the equation for the voltage in any phase winding follows the structure showed below:

$$\begin{aligned} v_1 &= R_1 i_1 + \frac{d\theta}{dt}\left(L_{11}\frac{di_1}{d\theta} + L_{12}\frac{di_2}{d\theta} + L_{13}\frac{di_3}{d\theta}\right) + \\ &+ \frac{d\theta}{dt}\left(\frac{\partial L_{11}}{\partial\theta}i_1 + \frac{\partial L_{12}}{\partial\theta}i_2 + \frac{\partial L_{13}}{\partial\theta}i_3\right) \end{aligned} \tag{6}$$

The third term in the equation (6) is the back electromotive force e induced in the winding of this phase, which feeds the load when the SRM operates as a generator, so that:

$$e = \frac{d\theta}{dt}\left(\frac{\partial L_{11}}{\partial\theta}i_1 + \frac{\partial L_{12}}{\partial\theta}i_2 + \frac{\partial L_{13}}{\partial\theta}i_3\right) \tag{7}$$

Once we have $\omega = d\theta/dt$ equations (6) and (7) also are showing that the machine performance, especially the back electromotive force, depends on its speed as can it was showed in the experimental results presented in [2].

Even in the same equation (6) one can note that, for each fixed angular speed, the middle term, which corresponds to the stored energy in the magnetic field of the machine, depends on the self and on the mutual inductances.

Furthermore equation (7) shows that the three terms inside the brackets depend on the currents in the phases and also on the self and mutual inductances rates of change with the angular position. At this point it is clear that it does not matter how big or small are the mutual inductances but how they behave in the angular position domain.

And so, according to equations (6) and (7), the investigation of the representativeness of the mutual inductances in a SRM operation depends not only on the corresponding relative values but also on their derivatives in the rotor angular position domain. Also it is worth here to observe that the currents in the three phases of this SRM have the same waveform, however they are out of phase. It is remarkable the fact that all of them have significant derivatives in the rotor angular position domain.

Obviously the machine mathematical model solution depends on this all. Thus, at once and for all, the SRM performance strongly depends on the phases currents values themselves and on their derivatives values in the θ domain.

3. Simulation Results

An experimental data bank of the phases self and mutual inductances was constructed from measures made in a prototype designed and built to operate as a Switched Reluctance Generator (SRG). These experimentally measured results are shown in the next section. The first and important thing to point here is that the data bank constructed also allows evaluating the self and mutual inductances and their differentials behavior. All of them were confirmed as a function of the angular position of the rotor and of the phase current. A specific computing model to simulate this machine behavior with the self and mutual inductances experimental data was constructed and the some results achieved are presented next.

Fig. 1 depicts the three currents in the 6 x 4 machine. They are out of phase as expected The first point to detach in Fig. 1 is that always there are two different currents flowing through different phases of the machine, that is, for this SRG there are always two simultaneous currents and not three or just one. Therefore, from equation (7), except if the inductance derivative is null, which were found not happening for this machine, always there will be a mutual inductance contribution to the back electromotive force value.

The self and mutual inductances experimental measures for the phases of this SRG are showed in Fig. 2. There the mutual inductances values are out of scale to make easy a visual comparison. The respective peaks are out of phase as expected, but their gaps in rotor angular position domain are smaller than the angle between the phases. The collected data showed that the gap between a self inductance and a mutual inductance is at about 13° for this prototype. Then there is mutual inductance with important slope concomitant with each phase current. So the second and the third terms inside the brackets in equation (7) should be evaluated before a detailed analysis of their effects on the SRG performance.

Using the computing model developed due also to the support carried by State University of Goias it was found that for this prototype the generated power simulated considering the mutual inductances was found about 16% greater than the value obtained from simulations where only the self inductances was took into account.

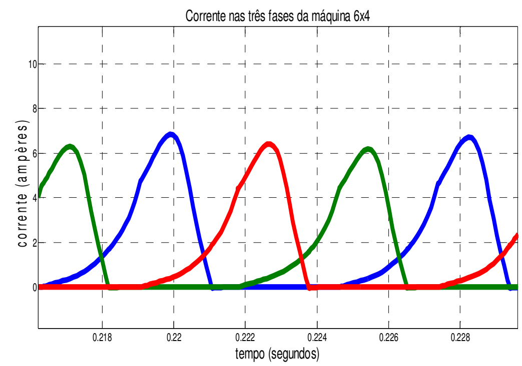

Fig. 1. Currents in the three phases of the 6x4 SRM prototype, operating as a SRG.

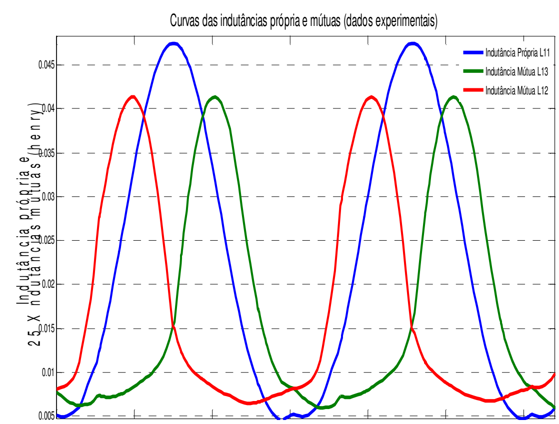

Fig. 2. Self and mutual inductances of the 6x4 SRM prototype.

4. Experimental Results

Self and mutual inductances were measured in a 6 x 4 SRG prototype designed and built based on the experience accumulated in the Electric Drive Laboratory of the Federal University of Uberlandia and in the Electrical Machines and Drives Laboratory of the Pontifical Catholic University of Goias.

To lock the machine shaft in each desired angular position it was adapted a strong lock device in the test bench. After that, starting from 0.5 A, for each additional half A of current the self and the mutual inductances were measured from 0^0 (aligned position) to 46^0 (unaligned position), in steps of two degrees. The sequence of measures was made up to 10 A, which is the nominal limit for the phase current in this machine.

The data got were compared, analyzed and plotted. The final result obtained for the self inductance and the mutual inductances are shown in Fig. 3, Fig. 4 and Fig. 5.

Despite a very few discrepant points, as predicted and said above in this paper, the inductances showed up as functions of the current and of the angular position.

In Fig. 3 it can be seen that the self inductance follows the predicted pattern once more. It is symmetrical about the aligned position of the poles and shows the gradual saturation of the ferromagnetic material as the current increases. Although this experimental result was expected it is important to allow a comparison of the self inductance with the mutual inductances for each angular position and for each current value.

Fig. 4 shows the mutual inductance between phase 1 and the next phase to be excited. In the case of the SRG, it is the next phase in reverse direction of the machine rotation. One can see that the mutual inductance peak is 13^0 ahead of the self inductance peak. At this position the current in phase 1 is increasing and has a considerable slope. Also the mutual inductance presents a considerable slope. Hence the terms depending on the mutual inductances, on the current of the active phase and on their derivatives are not null or negligible. They contribute to the voltage and to the current values in these two phases. So, there is a link between the phases. This link has an influence on the global machine performance.

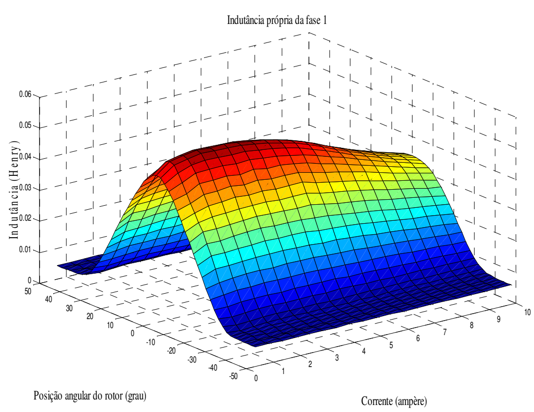

Fig. 3. Self inductance of the 6 x 4 SRM prototype.

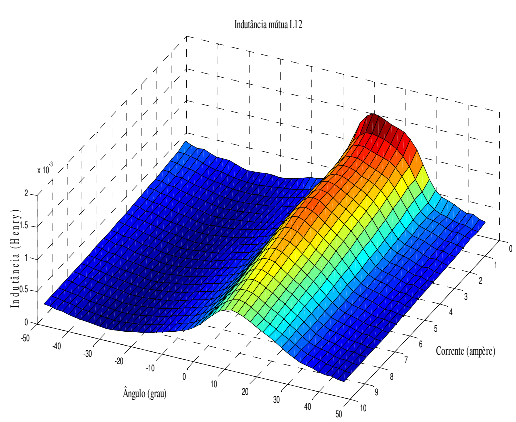

Fig. 4. Mutual inductance L_{12} of the 6x4 SRM prototype.

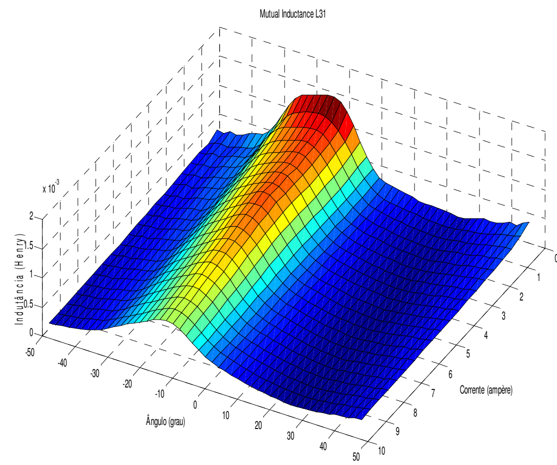

Fig. 5. Mutual inductance L_{13} of the 6x4 SRM prototype.

Owing to the symmetries $L_{xy}=L_{yx}$, which shows the existence of a relevant interplay between these two phases. This interplay repeats for the phase 2 and for phase 3 with respect to their own next phase making a cycle as the SRM turns.

The interaction between phases 1 and 3, when phase 1 is active is less representative because the peak of the mutual inductance L_{13} occurs out of the period of activity in this phase. This is due to the fact that the beginning of excitation of each phase is about -4.7^0 in relation to its aligned position and each one complete its cycle around the position 45^0 after the alignment. In this range both L_{13} and its derivative in the rotor angular position domain are small resulting in a small mutual effect.

Thus, the interaction between any two phases of the machine follows the feature described. A comparison based on the measurements data tables showed that the maximum values of mutual inductances are among 2.6% and 3.2% of the maximum value of the self inductance. It is a relation considerably bigger than that commonly found in the technical literature records.

Moreover, with regards to the gaps among the peaks of curves of self and mutual inductances only simultaneous measurements comparisons make sense. And they are different of those usually presented. In fact it was found in the tables that there are rotor angular positions where the mutual inductance reaches 10% of the self inductance.

It should also to be noted that Fig. 2, Fig. 4 and Fig. 5 indicate that the rates of change of the mutual inductances with respect to the rotor angular position are quite significant, which makes them important in equations (6) and (7).

Tests already done with an 8x6 French origin machine confirmed the dependence among the phases of the SRM. Its phases self and mutual inductances are showed from Fig. 6 to Fig. 9 next. The referred machine was manufactures by Radio Energie. Analyses done with the results follows the global pattern already presented sooner in this paper.

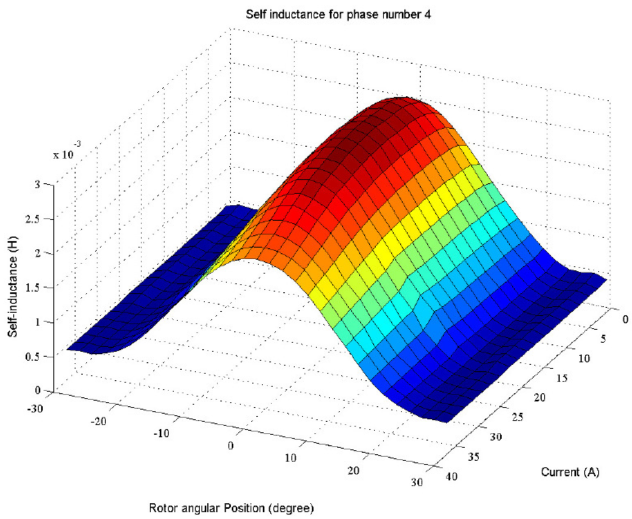

Fig. 6 Self inductance of a French 8 x 6 SRM phase.

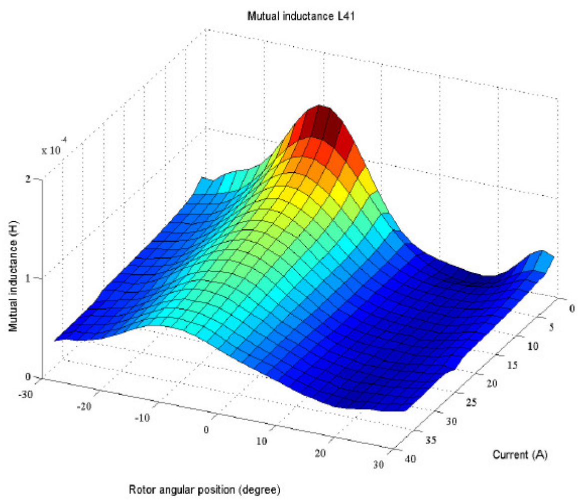

Fig. 7 Mutual inductance L41 of a French 8 x 6 SRM.

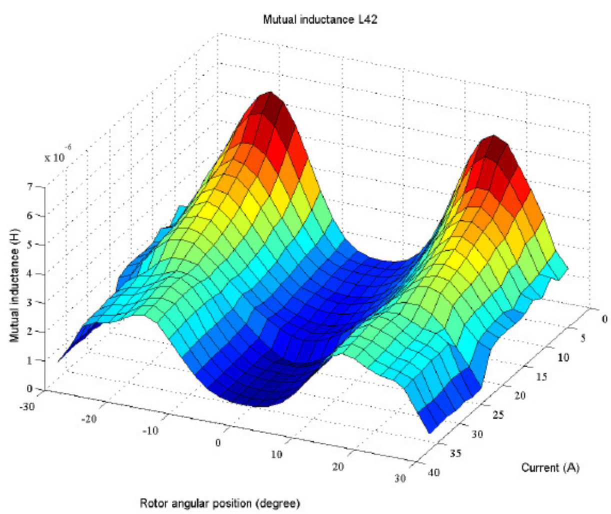

Fig. 8 Mutual inductance L42 of a French 8 x 6 SRM.

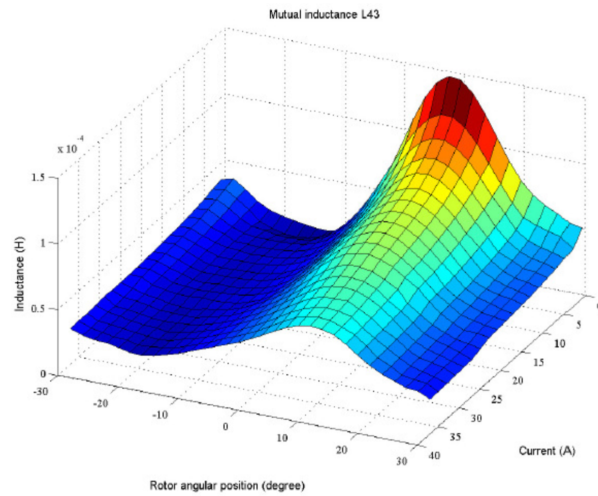

Fig. 9 Mutual inductance L43 of a French 8 x 6 SRM.

5. Conclusions

Two Switched Reluctance Machines of different manufactures and configurations were tested concerning about their self and mutual inductances. Results got allow concluding that the effects of mutual inductances are important and must be considered in their mathematical model and simulation. The mutual inductances cannot be discarded hastily in a switched reluctance machine design. Preliminary measurements and analyses must be done to get more about their influence on the machine performance. The SRG global performance may strongly depend on the phase currents values themselves side by side with their derivatives values in the θ domain. Simulations made taking into account the values of these inductances got experimentally from a 6 x 4 SRG prototype showed that, under the same operating conditions, it is expected 16% more power than the prediction made without the mutual inductances. After plotting the experimental results they showed up that the gap among the curves of self and mutual inductances of the 6 x 4 prototype is about 13^0. The differences among their peak values are among 2.6 and 3.2% of the maximum value of self inductance but rises substantially when only simultaneous values are compared. Measurements made in the second machine were presented and confirm the results discussed in details concerning the first tested prototype. All the results presented here are showing that a major dependence among the phases can characterizes the SRM and so must not be neglected.

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