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Early Detection of Voltage Instability in Distribution System utilizing Phasor Measurement Units

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Abstract. Power system automation leads to a great jump in detecting and solving the system problems. Due to the serious problems of system voltage collapse, an early prediction online voltage instability detector is discussed in this paper. The voltage instability detector utilizes the voltage and current phasor readings of connected phasor measurement units to the distribution terminals. The detector works according to two parallel algorithms. The first algorithm is a comparison for terminal current and voltage phasors with an offline calculated Look-up table, in which particle swarm optimization technique is used. The Look-up table represents the permissible maximum current and minimum voltage phasors for each terminal. The second algorithm is based on the online computation of the ratio of the Thevenin equivalent system impedance to the terminal load impedance. The simulated voltage detector is applied to two different unified power systems. It gives good and reasonable results.

Key words

Particle Swarm Optimization (PSO) technique, Phasor Measurement Unit (PMU), system Thevenin equivalent representation, voltage instability detector.

1. Introduction

Voltage instability problems in power systems have become, in many countries, one of the major concerns in power systems planning and operation [1]-[4]. These problems have become very complex due to the continuous growing on system interconnections and demand for electricity, and also due to economical and environmental constraints to properly expand the system. As a consequence, power systems have been operated closer to their maximum power transfer capability limits of the system transmission lines, with higher possibility of voltage instability events. Many countries have already reported cases of voltage collapse with great economical losses. This phenomenon is characterized by a progressive decline in voltage magnitudes, and occurs basically due to system inability to meet a growing demand for reactive power at certain buses in stressed situations. Load growth without a corresponding increase of transmission capacity has brought many power systems closer to their voltage stability boundaries, which leads to voltage stability problems increase. Furthermore the stable system contributes to reliability and reduction in system loss. For all these reasons the voltage stability problem has received a lot of attention not only from researchers but also from the industry. As a result many techniques have been developed to identify critical power system, buses and lines.

Voltage stability is serious concern to the electric utility industry. Several isolated and interconnected power systems are increasingly experiencing abnormal voltage problems, which can reach to voltage collapses or voltage failures leading to partial or complete blackouts. These voltage problems are mainly due to; (a) increased loading of transmission lines, (b) higher transmission impedance's, (c) insufficient local reactive supply, (d) shipping of power across long distances, and presence of induction motor loads, or HVDC systems, arc-furnaces, large welding machines or certain sensitive electronic devices, together with presence of 3-phase load unbalance or excessive amounts of harmonics penetrating in the network. Also voltage flickers may form certain voltage instability. The voltage dip, that occurs when the power system experiences sudden or gradual heavy loading or after excessive reactive power demand, is the main problem of voltage stability. It may lead to voltage oscillations or voltage collapse or both. Voltage instability occurs in four aspects, which are; voltage collapse, voltage oscillations, either modulated on each other, or voltage flickers. Voltage collapse is defined as a process by which voltage instability leads to very low voltage profile in a significant part of the system.

The new technologies of Phasor Measurement Units (PMUs) and Particle Swarm Optimization (PSO) can be utilized in voltage stability monitoring in distribution systems.

PMUs are now one of the recent measuring devices [5]-[8], that attract electrical engineering researchers. Due to the strong correlation between PMUs and the Global Positioning Satellites (GPS), PMUs began to spread widely after the great improvement in the satellite techniques and communications. Some researchers have paid attention to this great subject since the middle of

1980's, and there are others, everyday, who like to go through this way [6].

PMUs are used in many fields such as measurements, protection, control, observation,… etc. In measurements, it has unique ability to provide synchronized phasor measurements of voltages and currents from widely dispersed locations in an electric power grid, to be collected at control center for analysis. It is an electronic device that uses state-of-the-art digital signal processors that can measure 50/60Hz AC waveforms (voltages and currents) typically at a rate of 48 samples per cycle (2880 samples per second). The analog AC waveforms are digitized by an Analog to Digital converter for each phase. A phase-lock oscillator along with a GPS reference source provides the needed high-speed synchronized sampling with 1 microsecond accuracy. Additionally, digital signal processing techniques are used to compute the voltage and current phasors. Line frequencies are also calculated by the PMU at each site. This method of phasor measurement yields a high degree of resolution and accuracy. The resultant time tagged phasors can be transmitted to a local or remote receiver at rates up to 50/60 samples per second. PMUs come in different sizes. Some of the larger ones can measure up to 10 phasors plus frequency while others only measure from one to three phasors plus frequency [5].
It is suggested that PMUs can revolutionize the way of power systems monitoring and control. We can see this revolution in some power system researches that benefit this Wide Area Monitoring System (WAMS) technology in their studies.

In protection and control, it is used in many applications for measuring the synchronized phasor parameters needed for taking a decision or an action from the protection or control device.

This paper represents a simulation of new voltage stability detector. The detector utilizes the phasor readings of the PMUs, which are connected to the terminals of the distribution systems. The simulated voltage instability detector operates according to two parallel concepts. The first concept is comparing the PMUs readings with previous offline calculated Look-up table. PSO technique is used in the calculation of the Look-up table. The data of the Look-up table depends on the maximum current and minimum voltage phasor, which are permitted for each terminal before voltage collapse. The second concept is the online estimation of the Thevenin equivalent of the connected system, from the PMU readings. The Thevenin equivalent of the system contributes in voltage instability prediction of the system. The detector is applied to two unified power systems.

2. Thevenin Representation of the System

The simulated technique of the developed voltage instability detector is built on the Thevenin equivalent representation of the network to which the detector is connected. The Thevenin representation is considered to be the simplified reduced form of the network.

At each distribution terminal, the power network can be reduced to its Thevenin equivalent form, as shown in Fig.1. The voltage and current phasor diagram of the studied reduced system is illustrated in Fig. 2.

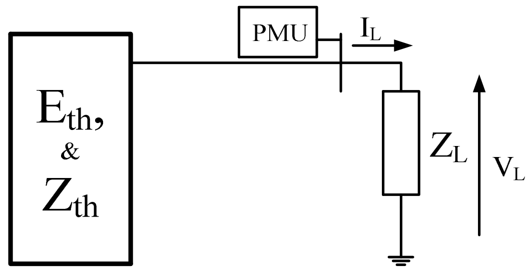

Fig. 1. Thevenin equivalent representation of the power system

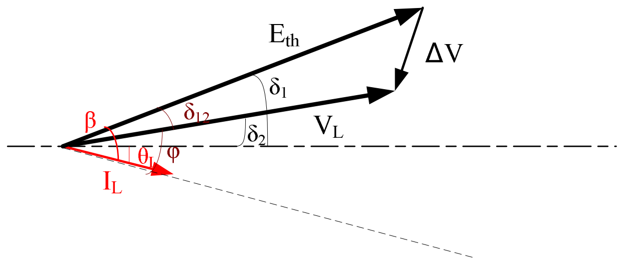

Fig. 2. The voltage and current phasors diagram of the Thevenin equivalent system

From Fig. 2, the relation between the load voltage, current and Thevenin voltage can be given as;

$$\mathbf{V}_L = \mathbf{E}_{th} - \mathbf{\Delta V} \tag{1}$$

$$\mathbf{\Delta V} = \mathbf{I}_L \mathbf{Z}_{th} \tag{2}$$

$$\mathbf{Z}_{th} = R_{th} + j\, X_{th} \tag{3}$$

where;

$\mathbf{V}_L$: is the terminal load voltage phasor.
$\mathbf{E}_{th}$: is the network Thevenin equivalent voltage phasor.
$\mathbf{Z}_{th}$: is the network Thevenin equivalent impedance phasor.

$$R_{th} << X_{th}, \qquad R_{th} \approx 0$$

$$\mathbf{Z}_L = Z_L \angle \varphi = R_L + j\, X_L \tag{4}$$

where;

$\mathbf{Z}_L$: is the load impedance phasor.
φ : is the load angle.

For maximum power transfer to the load terminal, the load impedance should be equal to the Thevenin equivalent impedance of the power network.

$$\mathbf{Z}_L = \mathbf{Z}_{th} \tag{5}$$

It is seen from the phasor diagram of Fig. 2 that;

$$E_{th} \cos \beta = I_L R_{th} + V_L \cos \varphi \tag{6}$$

$$E_{th} \sin \beta = I_L X_{th} + V_L \sin \varphi \tag{7}$$

where;

I_L	:	is the magnitude of the load current.
β	:	is the angle between the load current (I_L) and the Thevenin voltage equivalent (E_{th}).

The detector works according to two parallel alarming concepts. The first concept is comparing the adapted phasor readings of the PMU with a previously offline calculated Look-up table. The Look-up table presents the permissible maximum value of the load current, the minimum value of the load voltage, and the maximum angle between the load current (I_L) and the Thevenin voltage equivalent (E_{th}) with respect to each loading angle. The Look-up table is calculated utilizing a recent artificial intelligent technique, which is Particle Swarm Optimization (PSO) technique. The second concept is online calculation of E_{th}, Z_{th}, $Z_L$and the ratio between Z_{th} and Z_L (Z_{th}/Z_L).

3. Particle Swarm Optimization Technique (Offline Look-up Table Preparation)

Particle Swarm Optimization (PSO) is a stochastic, population-based evolutionary algorithm for problem solving [9]. It is a kind of swarm intelligence that is based on social-psychological principles. It provides insights into social behavior, as well as contributing to engineering applications. It is a powerful method to find the minimum of a numerical function, on a continuous definition domain [10].

PSO starts with some randomly distributed particles in searching space. Each particle moves with certain velocity (V) to certain position (X) trying to find a better position. After each movement, each particle compares its new and old position to find what is called its personal best (P_{best}). A comparison is held for all particles P_{best} to find the global best (G_{best}). Each particle determines its velocity and position according to:

$$V_{new} = w * V_{old} + c_1 * rand * (P_{best} - X) + c_2 * rand * (G_{best} - X) \quad (8)$$

$$X_{new} = X_{old} + V_{new} \quad (9)$$

where;

V	:	is the velocity component.
w	:	is the inertia weight.
rand	:	is random number between 0 and 1.
X	:	is the position of the particle.
c_1 , c_2	:	are the cognition and social factors, that are usually set to 2.0.
P_{best}	:	is the particle personal best position.
G_{best}	:	is the global best position of all particles.

P_{best} and G_{best} are determined according to a problem Cost Function (CF). In some problems there are constraints which should be considered in accepting or refusing the new particle position.

In the detector case, PSO is used to find the optimal values for the permissible maximum load current and minimum load voltage phasors ($\mathbf{I}_{Lm}$ and $\mathbf{V}_{Lm}$). From $\mathbf{I}_{Lm}$ and $\mathbf{V}_{Lm}$ the maximum β can be calculated for each loading angle φ. The solution is determined by satisfying the voltage study after applying the condition of maximum power transfer (illustrated in (1)-(7)). The voltage equations are considered to be the cost function of the detector problem.

The constraints of the detector problem are the maximum and minimum acceptable values for E_{th}, which are;

$$E_{thmax} = (V_L \cos \varphi) / \cos \beta \quad (10)$$

$$E_{thmin} = V_L , \quad (11)$$

where;

$$\tan \beta = (I_L Z_L + V_L \sin \varphi) / (V_L \cos \varphi) \quad (12)$$

The flowchart of the offline calculated look-up table algorithm, utilizing PSO technique, is clarified in Fig. 3.

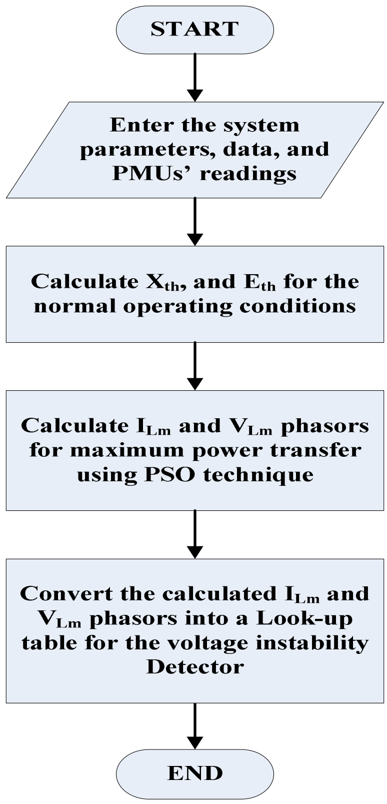

Fig. 3. Flowchart of the offline calculated Look-up table

4. Online Voltage Instability Detector

The main part of the voltage instability detector is the online early prediction of the voltage collapse of the connected terminal. The Thevenin equivalent voltage and impedance of the connected network are estimated online from the voltage and current phasors readings of the PMU. The load impedance is also calculated from $\mathbf{V}_L$ and $\mathbf{I}_L$. The Thevenin equivalent voltage and the ratio of the Thevenin impedance to the load impedance (Z_{th}/Z_L) can give a pre-indication to the voltage status and behaviour of the connected terminal. The increase of Z_{th}/Z_L indicates an obviously decrease in terminal voltage. The voltage collapse occurs when E_{th} becomes close to E_{thmax} and Z_{th}/Z_L tends to be equal 1.

For online estimation of the Thevenin equivalent of the circuit, R_{th} is assumed to be zero in (6), which leads to;

$$\beta = \cos^{-1} (V_L \cos \varphi / E_{th}) \quad (13)$$

The initial value of E_{th} is given by;

$$E_{tho} = (E_{thmax} + E_{thmin}) / 2 \quad (14)$$

E_{thmax} and E_{thmin} are calculated from (10) and (11) respectively. The initial value of β is calculated from (12).

The initial value of X_{th} is calculated from (7), where;

$$X_{tho} = (E_{th} \sin\beta - V_L \sin\varphi) / I_L \quad (15)$$

The algorithm of E_{th} and X_{th} calculation can be summarized as;

1- Enter the readings of load voltage and current phasors.
2- Calculate the initial value of E_{th}, X_{th} and β.
3- Calculate E_{th}^{i} according to;
   a- If the variation in Z_L is negative,
      i. If the variation in X_{th} is negative,
      $$E_{th}^{i+1} = E_{th}^{i} - \varepsilon$$
      ii. If the variation in X_{th} is positive,
      $$E_{th}^{i+1} = E_{th}^{i} + \varepsilon$$
   b- If the variation in Z_L is positive,
      i. If the variation in X_{th} is negative,
      $$E_{th}^{i+1} = E_{th}^{i} + \varepsilon$$
      ii. If the variation in X_{th} is positive,
      $$E_{th}^{i+1} = E_{th}^{i} - \varepsilon$$
   c- If Z_L is constant
      $$E_{th}^{i+1} = E_{th}^{i}$$
4- Calculate β^{i} and X_{th}^{i} from (13) and (15) respectively.
5- Repeat from step 1 for each new phasors readings.

The error ε is calculated from;

$$\varepsilon = \min(\varepsilon_{inf}, \varepsilon_{sup}) \quad (16)$$

$$\varepsilon_{inf} = \left| E_{th}^{i} - V_L^{i+1} \right| \quad (17)$$

$$\varepsilon_{sup} = \left| E_{th}^{i} - E_{thmax}^{i} \right| \quad (18)$$

The whole algorithm of the voltage instability detector is illustrated in the flowchart shown in Fig. 4.

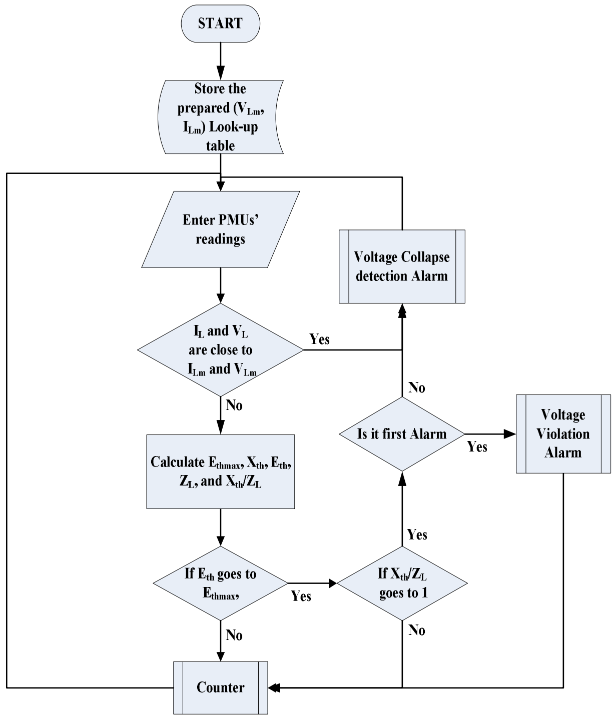

Fig. 4. Flowchart of the online voltage instability detector

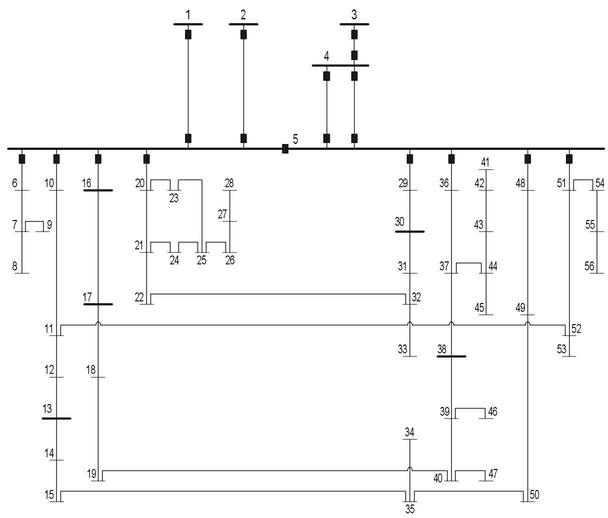

Fig. 5. Egyptian UEPN: 56- bus test system model

5. Studied Systems

The discussed detector is applied to two different unified distribution power networks. The first network is a system depicted from the unified electrical power network of Egypt. A part of the U.K. generic distribution network is the second network.

A. *A System depicted from the Distribution Network of the Egyptian Unified Electrical Power Network (UEPN)*

This system consists of 56 buses, which are connected as shown in Fig. 5. By applying the algorithm of the Look-up table offline calculations to the studied system, the maximum $\mathbf{I}_L$ and minimum $\mathbf{V}_L$ for each node can be determined. Table I represents I_{Lm}, V_{Lm} and β for different loading angle φ of some random selected nodes.

Table I is designed of main seven columns and ten main row sets. The first row shows the values of the loading angle φ. It varies from φ = zero (p.f. = 1) to $\varphi = 41.41^{o}$ (p.f. = 0.75). The other rows sets are designed of three subsets; the first sub-row is for the node V_{Lm}, the second sub-row is for the node I_{Lm}, and the third sub-node is for β. As shown from Table I, as φ increases both I_{Lm} and V_{Lm} decrease while β increases.

On the other hand, the online voltage instability detection algorithm is tested on the studied system. A sample of the results is shown in Fig. 6 and Fig. 7. The results is calculated for $\varphi = 25.84^{o}$ (p.f. = 0.9). Fig. 6 illustrates the behavior of Z_{th}/Z_L with the load current increase for node 36. Z_{th}/Z_L equals 1 when I_L equals to 2.85 of its rated value.

Table I. Samples from the offline calculated Look-up table for the UEPN of Egypt

Bus No. \ Φ		0	18.19	25.84	31.79	36.87	41.41
8	V_{Lm}	0.75	0.75	0.73	0.73	0.72	0.72
	I_{Lm}	2.2	2.16	2.11	1.94	1.88	1.81
	β	43.5	45.1	49.7	54.5	58.7	62.6
11	V_{Lm}	0.75	0.74	0.72	0.7	0.71	0.7
	I_{Lm}	2.18	2.17	2.13	1.98	1.94	1.9
	β	45.8	48.2	51.9	56.6	60.5	64.8
17	V_{Lm}	0.72	0.7	0.69	0.69	0.68	0.67
	I_{Lm}	2.4	2.37	2.35	2.26	2.15	2.06
	β	48.7	50.6	55.3	59.8	63.4	68.1
24	V_{Lm}	0.73	0.72	0.71	0.71	0.7	0.69
	I_{Lm}	2.2	2.16	2.11	1.94	1.88	1.85
	β	46.3	47.8	51.7	57.4	60.9	66.6
32	V_{Lm}	0.72	0.72	0.73	0.73	0.73	0.74
	I_{Lm}	2.1	1.95	1.86	1.8	1.75	1.72
	β	45.3	47.3	51.7	56.6	60.5	64.8
39	V_{Lm}	0.68	0.67	0.66	0.65	0.64	0.64
	I_{Lm}	2.5	2.46	2.42	2.36	2.28	2.21
	β	51.7	53.5	57.9	62.8	66.7	69.7
44	V_{Lm}	0.77	0.76	0.76	0.75	0.75	0.74
	I_{Lm}	1.95	1.91	1.87	1.84	1.83	1.8
	β	42.3	43.8	47.9	52.7	57.5	59.4
51	V_{Lm}	0.74	0.72	0.7	0.68	0.65	0.62
	I_{Lm}	2.3	2.27	2.24	2.21	2.21	2.15
	β	47.6	49.5	53.9	58.7	62.2	68.4
56	V_{Lm}	0.73	0.72	0.71	0.71	0.7	0.69
	I_{Lm}	2.2	2.16	2.11	1.94	1.88	1.85
	β	46.3	47.8	51.7	57.4	60.9	66.6

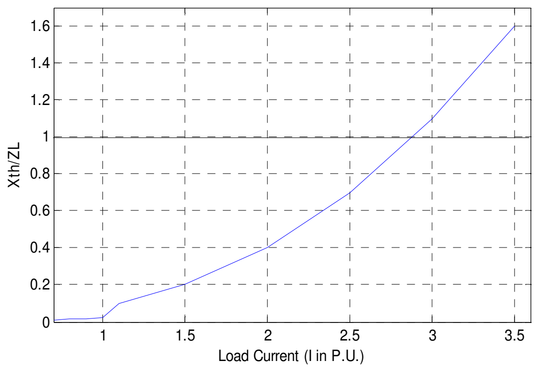

Fig. 6. The ratio of Thevenin impedance to load impedance versus the load current variation for node 36 in the UEPN of Egypt

The variation of V_L, E_{th} and E_{thmax} with I_L is clarified in Fig. 7. Both V_L and E_{thmax} decrease with I_L increase, while E_{th} increases. When I_L equals to 2.18 of its rated value, E_{th} is equal to E_{thmax}.

Finally, it is concluded that, as both E_{th} and Z_{th}/Z_L increase, the system tends to be in voltage instability mode. It leads to system voltage collapse.

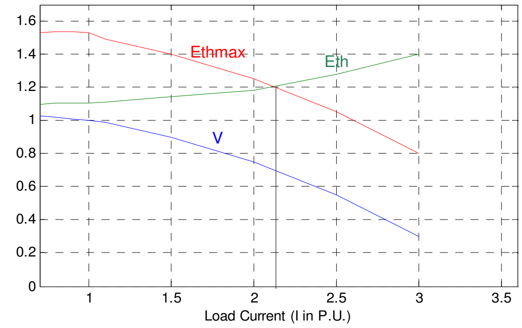

Fig. 7. The response of E_{th}, V_L, and E_{thmax} to the load current variation for node 36 in the UEPN of Egypt

B. A Part of the U.K. Generic Distribution Network (UKGDS) Model

The UKGDS is designed from 95 buses. The network diagram is shown in Fig. 8. Both offline and online algorithms are applied to the UKGDS model. The offline calculations of some random samples of nodes are illustrated in Table II. Table II is constructed similarly as Table I. The response of this system is similar to the Egyptian UEPN response.

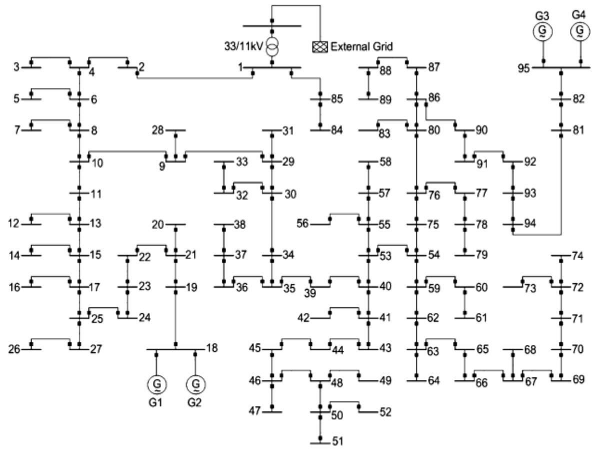

Fig. 8. UKGDS: 95-bus test system model [11].

Table II. Samples from the offline calculated Look-up table for the UKGDS

Bus No. \ Φ		0	18.19	25.84	31.79	36.87	41.41
9	V_{Lm}	0.81	0.8	0.78	0.77	0.76	0.76
	I_{Lm}	3.17	3.11	2.98	2.94	2.9	2.85
	β	46.4	49.3	54.1	58.7	62.9	66.2
24	V_{Lm}	0.79	0.78	0.78	0.77	0.76	0.76
	I_{Lm}	2.95	2.91	2.87	2.84	2.83	2.8
	β	47.3	48.8	52.9	57.7	62.5	74.4
35	V_{Lm}	0.83	0.81	0.81	0.8	0.79	0.79
	I_{Lm}	2.6	2.56	2.54	2.45	2.34	2.25
	β	48.7	50.6	55.3	59.8	63.4	68.1
44	V_{Lm}	0.83	0.82	0.82	0.81	0.8	0.8
	I_{Lm}	2.51	2.47	2.44	2.35	2.22	2.16
	β	46.3	47.8	51.7	57.4	60.9	66.6

50	V_{Lm}	0.83	0.81	0.81	0.8	0.79	0.79
	I_{Lm}	2.56	2.51	2.49	2.4	2.29	2.2
	β	47.6	49.8	54.5	58.7	62.5	67.1
60	V_{Lm}	0.78	0.77	0.76	0.76	0.75	0.75
	I_{Lm}	3.15	3.1	2.98	2.89	2.8	2.72
	β	51.7	53.5	57.9	62.8	66.7	69.7
77	V_{Lm}	0.8	0.79	0.79	0.77	0.76	0.76
	I_{Lm}	2.95	2.91	2.87	2.84	2.83	2.8
	β	47.5	48.8	52.9	57.9	62.6	74.4
86	V_{Lm}	0.81	0.8	0.78	0.77	0.76	0.76
	I_{Lm}	3.17	3.11	2.98	2.94	2.9	2.85
	β	46.4	49.3	54.1	58.7	62.9	66.2
94	V_{Lm}	0.79	0.79	0.78	0.77	0.77	0.76
	I_{Lm}	3	2.98	2.93	2.89	2.84	2.8
	β	44.6	46.9	50.2	54.4	59.8	64.6

Fig. 9 and Fig. 10 show the behavior of the voltages and the impedances with the load current variation as seen from bus 54 side at 0.9 lagging p.f. The variation of Z_{th}/Z_L with I_L is represented in Fig. 9. It is seen that $Z_{th}/Z_L = 1$, when $I_L = 3.74\ I_{rated}$. From Fig. 10, it can be detected that $E_{th} = E_{thmax}$ when $I_L = 2.97\ I_{rated}$.

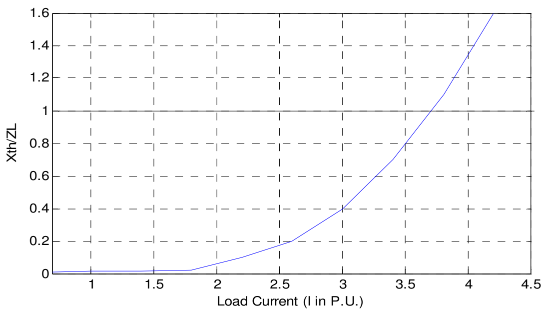

Fig. 9. The ratio of Thevenin impedance to load impedance versus the load current variation for node 54 in UKGDS

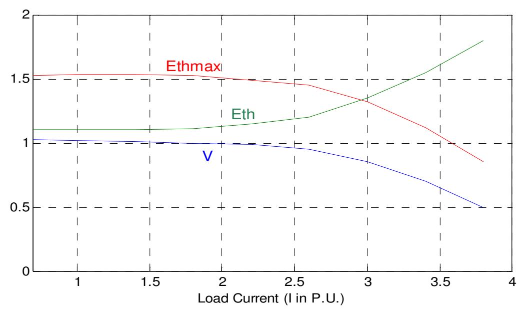

Fig. 10. The response of E_{th}, V_L, and E_{thmax} to the load current variation for node 54 in UKGDS

6. Conclusion

Early prediction of voltage collapse has great advantages in reducing lot of reliability and economical problems. An early voltage instability detector, based on PMUs readings which are connected in the distribution system, is simulated in this paper. The simulated detector depends on two parallel concepts for voltage collapse prediction, to give faster and precious response. These concepts are clarified in the main two simulated alarming parts of the detector algorithm. In the first part, the Look-up table offline calculated values for I_{Lm} and V_{Lm} phasors are compared with the online PMUs readings. The values of I_{Lm} and V_{Lm} phasors are computed using PSO technique. The objective function of the PSO depends on the system Thevenin equivalent as seen from the load terminal, which is connected to the PMU. The second main part of the algorithm is the online calculation of the ratio of the Thevenin impedance to the terminal load impedance (Z_{th}/Z_L) of the network, utilizing the readings of the connected PMU. The variation in Thevenin voltage is also used in voltage instability prediction. The Thevenin equivalent of the system contributes in voltage instability detection of the system.

The simulated voltage instability detector is applied to two unified power systems. The first system is a system depicted from the distribution network of the Egyptian unified electrical power network. The second system is a part of United Kingdom distribution power system. The simulated detector gives effective results. It reacts with the systems changes in good and reasonable behavior.

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