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# Estimation of power components for non-sinusoidal currents and voltages regarded as power quality indices

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**Abstract.** This paper deals with the computation of power components of distorted signals seen as power quality indices. The proliferation of nonlinear devices into the power system results in nonlinear voltages and currents whereas this unfortunate trend is supposed to increase.
The prime tool for spectral analysis is the Discrete Fourier Transform, establishing the preliminary step in the power components computation of distorted signals. Power definitions proposed by Budeanu and afterwards by Shepherd and Zakikhani require spectral components as input. Windowing, leakage and other DFT properties and limitations influence significantly the assessment of power components.
The power definition proposed by Skopec and Stec is totally confined to signals represented in time domain. Similar idea was firstly outspoken by Fryze and entirely by-passed the issue of spectral components computation.
The authors made an attempt to apply practically different approaches to power components computation for non-sinusoidal currents and voltages.

## Key words

Power Quality, Fourier Transform, Reactive Power, Quality Indices

## 1. Introduction

The importance of power quality (PQ) issues is almost indisputable. Both, utilities and end users of electrical power are interested in maintaining all PQ indices within limits prescribed in standards and legal regulations. The motivation, not to violate limits has a strong technical foundation but even more important are economic consequences of electrical power delivered with unsatisfactory parameters [1].
Moreover, the number of nonlinear devices, e.g. power electronic converters, light sources, welders, arc furnaces drawing highly nonlinear currents is steadily increasing over time. Therefore, the harmonic distortion levels appear to be consistently growing. [2].
Harmonic components in voltage and current do not allow simple rules for active, reactive and apparent power computation allowed if only the pure fundamentals are present [1].
Budeanu proposed a definition of power components constituting an natural extension of the well-known definitions for systems with sinusoidal voltages and currents [3]. Distortion power introduced by him bridged the gap between active and apparent power.
Conceptually similar approach was presented by Shepherd and Zakikhani [4], but the equation defining power components were constructed differently.
Nevertheless, both approaches required the computation of harmonic components of current and voltage as a prerequisite for power components computation. The accurateness of spectral analysis is essential for power computation.
The Fourier Transform [5] is a widely used tool for the computation of spectral components. Usually, Discrete Fourier Transform (DFT) is applied practically[6]. Windowing, leakage and other DFT properties or features influence significantly the assessment of power components [7]. The limitations of accurate approximation of continuous signals with discrete spectral components are the main reason for uncertainties in computation of power components.
The new power definition proposed by Skopec and Stec [8] has been formulated in the time domain. No spectral analysis is required as a prerequisite. Although the idea of power computation in time domain was firstly introduced by Fryze [3], the proposal [8] is a significant step forward, especially in terms of reactive power compensation procedures.
This paper proposes an practical implementation of different approaches to the issue how to define and compute reactive power. The influence of spectral components on overall performance has been studied.

## 2. Proliferation of harmonics into the power network

The principal idea of harmonic proliferation into the network has been shown in Fig. 1.

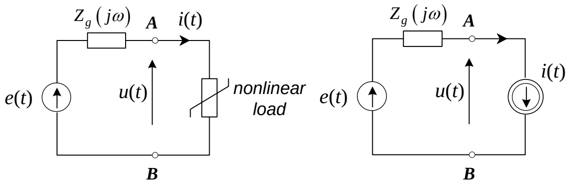


Fig. 1. Mechanism of harmonic proliferation into the network

Even if the voltage is sinusoidal the non-linear load (Fig 1. left) draws non-sinusoidal current. Simplifying the circuit with regard to the Thevenin theorem, the whole one phase network located left from A-B clamps can be represented with a single voltage source and impedance. The nonlinear load, located right from A-B clamps can be replaced with a corresponding current source. The nonlinear current affects impedances and causes non-sinusoidal voltage drop on elements of the system represented in Fig. 1 with a single impedance.

Generally, the power consumption of load in the simplified circuit is a function of non-linear currents and voltages. The voltage drop over the impedance in Fig 1 and associated currents are also non-sinusoidal.

The power definitions for sinusoidal circuit dos not apply.

## 3. Fourier Transform

The Discrete Fourier Transform [6] is a well-known and widely used tool in spectral analysis. However, the transform parameters influence significantly the results of computed power. The basic dependences for spectral components are given below.

### A. Fourier series

The Fourier series of a periodic function has the form

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right) \quad (1)$$

or written in a short manner

$$x(t) = a_0 + \sum_{n=1}^{\infty} F_n \sin\left(\frac{2\pi nt}{T} + \psi_n\right) \quad (2)$$

where $F_n = \sqrt{a_n^2 + b_n^2}$ and $\mathrm{tg}\,\psi_n = \frac{a_n}{b_n}$.

### B. Fourier integral

In case of non-periodic functions we assume that the given function has one, infinitely long period. Starting from (1) and after modifications the expression for Fourier Integral may be written as

$$X(f) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt \quad (3)$$

In this particular case the spectrum of a signal is continuous.

### C. Discrete Fourier Transform

Numerical analysis is used in practice. The basic expression for discrete transform corresponds with (3).

$$X(m) = \sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi nm}{N}\right) + j\sum_{n=0}^{N-1} x(n)\sin\left(\frac{2\pi nm}{N}\right) \quad (4)$$

However, the signal is sampled (not continuous) and the number of collected samples and the number of spectral components is limited. The obtained components only approximate the original signal. Particularly, the number of samples, the sampling frequency and the frequency of the main component are in mutual relation and properly chosen extend the accuracy of estimation.

### D. Windowing

Windowing is a general technique for minimization of the leakage effect. Different types of windows help to reduce this side effect [5]. Most simple is the rectangular window (box-car window), constituting just a selection of samples sequence without any modification of them. Other windows reduce the discontinuity at the ends of the window, e.g. Hamming window (7) which is shown in Fig. 2.

$$w(n) = 0.5\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right) \quad (5)$$

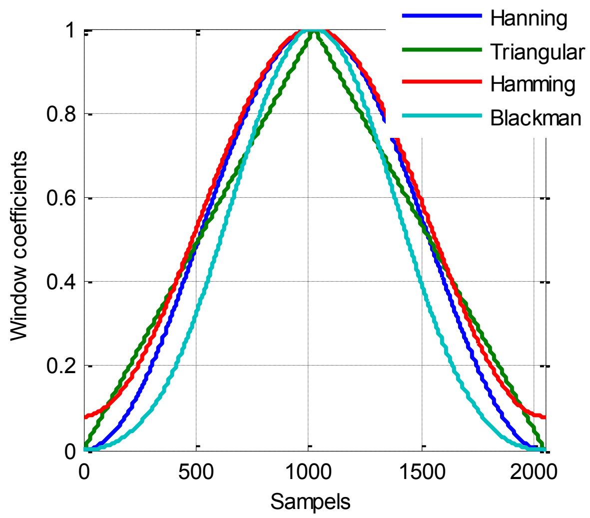


Fig. 2. Various window coefficients

## 4. Reactive power definition by Budeanu

The Budeanu approach was firstly published in 1927. This theory, also mentioned in a IEEE standard [9] defines the power through spectral components. Starting from the idea of active and reactive power for sinusoidal signals, the active power $P_B$ and reactive power $Q_B$ for all harmonic components has been defined as

$$P_B = \sum_{n=0}^{\infty} U_n I_n \cos\varphi_n \quad (6)$$

$$Q_B = \sum_{n=0}^{\infty} U_n I_n \sin\varphi \quad (7)$$

The difference between apparent power $S_B$ and the sum of active and reactive power was defined as distortion power $D_B$

$$D_B = \sqrt{S_B^2 - P_B^2 - Q_B^2} = \\ = \sqrt{\sum_{n=0}^{\infty} U_n^2 \sum_{n=0}^{\infty} I_n^2 - \left(\sum_{n=0}^{\infty} U_n I_n \cos\varphi_n\right)^2 + \\ -\left(\sum_{n=0}^{\infty} U_n I_n \sin\varphi_n\right)^2} \tag{8}$$

The apparent power is defined as

$$S_B = \sum_{n=0}^{\infty} U_n^2 \sum_{n=0}^{\infty} I_n^2 \tag{9}$$

The power computed for distorted signals is a useful index in power quality assessment.

## 5. Reactive power definition by Shepherd and Zakikhani

The originally proposed definition of Powers [4] is used in this paper in a slightly simplified form. The assumption of identical harmonic component indices in current and voltage. The total apparent power $S_C$ can be resolved into two analytical components, instead of three as given in the original version of the algorithm. The first component is the active apparent power $P_C$ and the second one is the reactive apparent power $Q_C$. $P$ in (10) and (13) represents mean value of instantaneous power. It is the value measured by analogue wattmeter. $U$ stands for RMS of voltage.

$$P_C^2 = U^2 \sum_n^{\infty} (I_n \cos\varphi_n)^2 \geq P^2 \tag{10}$$

$$Q_C^2 = U^2 \sum_n^{\infty} (I_n \sin\varphi_n)^2 \tag{11}$$

The components $P_C$ and $Q_C$ sum to apparent power $S_C$ as follows

$$S_C^2 = P_C^2 + Q_C^2 \tag{12}$$

Due to the fact that $P_C^2 \geq P^2$ it is possible to extract active power from active apparent power introducing supplementary power $D_C$ [10] defined as

$$D_C^2 = P_C^2 - P^2 \tag{13}$$

## 6. Power definition by Skopec and Stec

The new power definition presented in [8] is based on the well-known active current $i_a(t)$ concept defined in time domain by Fryze [3]

$$i_a(t) = \frac{P}{U^2} u(t) = Gu(t) \tag{14}$$

where

$$G = \frac{P}{U^2} \tag{15}$$

is so called equivalent conductance of the load. As previously $P$ in (14) and (15) is the mean value of instantaneous power. It is the value measured by analogue wattmeter. Further on the index "F" is attached to so defined $P$ power to distinguish Skopec and Stec approach. The new theory [8] introduces the reactive current in time domain

$$i_b(t) = i(t) - i_a(t) \tag{16}$$

It is important to stress that the active and reactive current are orthogonal to each other. Reactive current is also orthogonal to the voltage $u(t)$. That feature justifies the name reactive current.

The orthogonality between $i_a(t)$ and $i_b(t)$ leads directly to the relationship between the norms of the mentioned currents

$$\|i\|^2 = \|i_a\|^2 + \|i_b\|^2 \tag{17}$$

Multiplying (17) by squared RMS value of voltage we get

$$S_F^2 = P_F^2 + Q_F^2 \tag{18}$$

where

$$Q_F = U\|i_b\| = U\|i - Gu\| \tag{19}$$

Equation (19) defines absolute reactive power [8].

## 7. Computational results

The presented approaches to the power definition problems are not contradictory. The computed results are obtained in a different manner. The Budeanu and Shepherd approach requires previous computation of spectral components in voltage and current. The method proposed by Skopec and Stec uses time domain information only.

Comparing eq. (8), (12) and (18) a simple relationship can be written

$$Q_A^2 = Q_B^2 + D_B^2 = Q_C^2 + D_C^2 \tag{20}$$

To illustrate the power computation simulated signals were used containing the fundamental and the harmonics of order 3, 5 and 7. Phase of the voltage harmonics was 0 and all current components were shifted by 60 degrees with regard to voltage. The waveform is shown in Fig. 3 and RMS values of spectral components in Fig 4 and Fig 5 respectively. The apparently simple signal has been chosen deliberately, to show the differences between different approaches to power component computation.

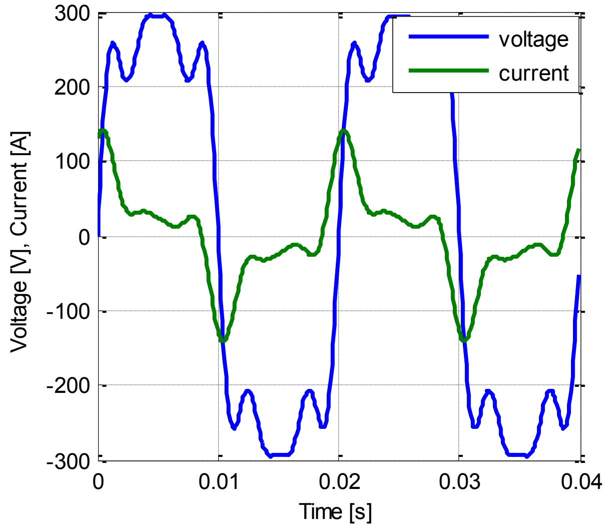


Fig. 3. Simulated current and voltage waveform

In an ideal case, 10 periods were sampled with the sampling frequency 10240 Hz. This frequency is usually used in power quality measurement devices, enabling FFT computation over 10 periods as prescribed in standards [11]. Samplings gives exactly 2048 points and therefore the spectral resolution is 5 Hz. The values in all Tables are in kW, kvar and kVA. The final column of every Table gives the values in var squared computed for (20).

*A. Fundamental frequency 50 Hz, rectangular window*

In the first case, the fundamental signal frequency was exactly 50 Hz. The computational results obtained for the above described case are summarised in Table I.

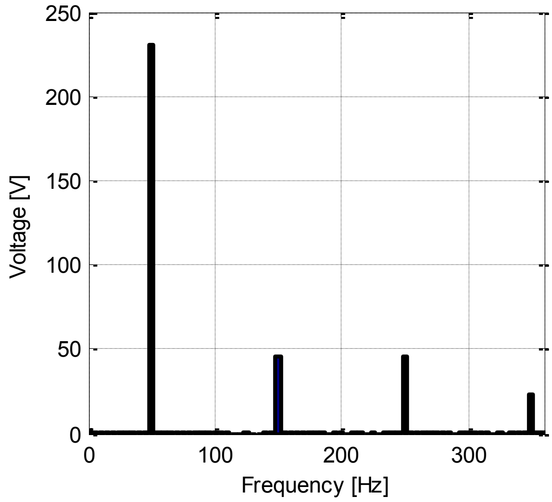


Fig. 4. Spectral components in the voltage

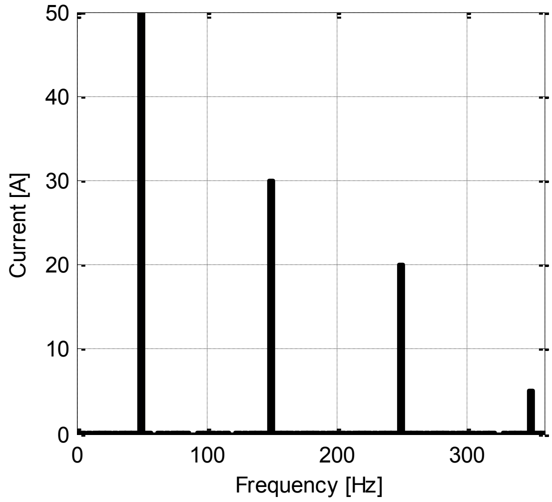


Fig. 5. Spectral components in the current

Due to the fact, that the fundamental frequency was exactly 50 Hz, whole 10 periods of the fundamental were covered with the measurement window. No leakage was observed and the power computation was exact. Therefore, with accordance to (20) all values in the last column are equal.

Table I. – Power computation for 50 Hz, no windowing

| | | | | |
|---|---|---|---|---|
| Budeanu | $P_B$ | 6.957 | $Q_B^2 + D_B^2$ | 172.15 e6 |
| | $Q_B$ | -12.050 | | |
| | $D_B$ | 5.189 | | |
| | $S_B$ | 14.851 | | |
| Shepherd Zakikhani | $P_C$ | 7.425 | $Q_C^2 + D_C^2$ | 172.15 e6 |
| | $Q_C$ | 12.861 | | |
| | $D_C$ | 2.594 | | |
| Skopec Stec | $P_F$ | 6.957 | $Q_F^2$ | 172.15 e6 |
| | $Q_F$ | 13.120 | | |
| | $S_F$ | 14.851 | | |

*B. Fundamental frequency 51 Hz, rectangular window*

The second case present an extreme situation, where the signal fundamental frequency is 51 Hz. There is a significant leakage and the computed power components differ from values given in Table I. RMS voltage values of not present 40 Hz and 60 Hz components were estimated at 20 and 24 volts respectively. Relatively unchanged are the values for the power components in time domain (bottom row of Table II).

Table II. – Power computation for 51 Hz, no windowing

| | | | | |
|---|---|---|---|---|
| Budeanu | $P_B$ | 7.099 | $Q_B^2 + D_B^2$ | 173.60 e6 |
| | $Q_B$ | -11.985 | | |
| | $D_B$ | 5.473 | | |
| | $S_B$ | 14.966 | | |
| Shepherd Zakikhani | $P_C$ | 7.690 | $Q_C^2 + D_C^2$ | 173.60 e6 |
| | $Q_C$ | 12.839 | | |
| | $D_C$ | 2.958 | | |
| Skopec Stec | $P_F$ | 6.962 | $Q_F^2$ | 171.90 e6 |
| | $Q_F$ | 13.111 | | |
| | $S_F$ | 14.845 | | |

*C. Fundamental frequency 51 Hz, triangular window*

Different windows has been applied to reduce the leakage effect. Results obtained for triangular window are presented. Although a reduction of amplitudes of unwanted spectral components has been reached, the multiplication of signals by a window resulted in an unfortunate phase shift. The 40 Hz and 60 Hz components reached ca. 2 volts only. The erroneous power components are given in Table III. Windowing was not applied for the last method, as it is defined in time domain.

Table III. – Power computation for 51 Hz, triangular window

| | | | | |
|---|---|---|---|---|
| Budeanu | $P_B$ | 9.290 | $Q_B^2 + D_B^2$ | 306.18 e6 |
| | $Q_B$ | -16.075 | | |
| | $D_B$ | 6.912 | | |
| | $S_B$ | 19.811 | | |
| Shepherd Zakikhani | $P_C$ | 9.908 | $Q_C^2 + D_C^2$ | 306.18 e6 |
| | $Q_C$ | 17.155 | | |
| | $D_C$ | 3.446 | | |
| Skopec Stec | $P_F$ | 6.962 | $Q_F^2$ | 171.90 e6 |
| | $Q_F$ | 13.111 | | |
| | $S_F$ | 14.845 | | |

## 8. Conclusion

Power definitions in circuit with non-sinusoidal currents and voltages are still a subject for discussions [12]. That fact must be taken into account by the computation of reactive power constituting an important power quality indicator. Given directly or in the form of a power factor.

Different definitions of reactive power were presented in the literature, both in frequency and time domain. The common frequency domain approach by Budeanu has some serious disadvantages. Various window functions are useful for spectral leakage reduction and therefore improve the values some power quality indices, e.g. total harmonic distortion THD. However, those methods cannot be applied for reactive power estimation, due to the phase shifting property and therefore erroneous power components computation.

Reactive power definitions, which are formulated in time domain only seem to be prone against computational inadequacies observed in frequency domain, such as leakage and frequency resolution. Therefore, the approach presented by Skopec and Stec [8] is an innovative continuation of Fryze ideas formulated in time domain. It is more suitable for smart metering applications.

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