Evaluation of the Harmonic Impact in an Electrical System Using Polynomial-Regression Method

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Abstract. This paper presents a method to determine the individual harmonic influence of multiple harmonic-producing loads in an electrical power system. This method is based in the development of statistical models from measurement data using the polynomial-regression technique. A real case study is presented in order to show the performance of the proposed method.

Key words

Harmonic-producing loads, least-square method, Polynomial Regression, correlation factor.

1. Introduction

Determine the individual harmonic contribution of multiple harmonic-producing loads in an electrical power system is an important task for the management of the voltage harmonic distortion. However, due to the variety of harmonic sources present in electrical networks, it isn't an easy task to accomplish since the harmonic currents in power grids interact in such way as to cause increase or decrease of the harmonic distortion levels.

According to [1], there are two classes of methods to quantify the harmonic contribution of loads: the model-based method and the data-based method. The model-based method aims in the development of equivalent circuits to evaluate the impact of different harmonic sources. And the data-based method aims in the development of models from measurement data that describe the cause-effect relation of these data.

In [2], it was proposed a method based in the development of linear models from measurement data using linear regression techniques that describes the relation between the harmonic voltage in any point of an electrical system and the harmonic current of a harmonic-producing load.

This paper presents a method to determine the individual harmonic contribution of multiple harmonic-producing loads using the polynomial regression.

2. Regression Analysis

Regression analysis is a statistical method used to build models that describe the behaviour of a random variable (known as response or dependent variable) due to the variation of another variable or a set of variables, known as regressor or explanatory variables.

There are many methods for regression analysis where the simplest and the most common is the simple linear regression.

A more general form of the simple linear regression is the polynomial regression method since the regression model is represented by a kth-order polynomial equation instead of the first-order polynomial equation.

A. *Linear Regression*

A simple linear regression model is a regression model in which the regression equation is linear as shown in equations (1) and (2)

$$Y = \beta_0 + \beta_1 X + \epsilon \quad (1)$$

$$\epsilon \sim N(0; \sigma^2) \quad (2)$$

where: Y is the dependent variable
X is the regressor
β_0 and β_1 are the linear model's coefficients
ϵ is the error of the model

The linear model's coefficients are estimated through the least square method using the equations (3) and (4)

$$\widehat{\beta_0} = \bar{y} + \widehat{\beta_1}\bar{x} \qquad (3)$$

$$\widehat{\beta_1} = \frac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \qquad (4)$$

B. Polynomial Regression

A polynomial regression model is a regression model in which the model is represented by a kth-order polynomial equation as shown in equations (5) and (6).

$$Y = \beta_0 + \beta_1 x + \cdots + \beta_k x^k + \epsilon \qquad (5)$$

$$\epsilon \sim N(0; \sigma^2) \qquad (6)$$

In this model, x is the fixed value of the regressor variable X, and the parameters β_0, β_1, and β_k are the kth-polynomial coefficients, and ϵ is the error of the model. According to [4], a polynomial regression method can cause an increasing in the flexibility of the relationship between the dependent variable and the regressor.
The least square method is used to estimate of the kth-order polynomial coefficients through equation (7).

$$\hat{\beta} = (X'X)^{-1}X'y \qquad (7)$$

C. Analysis of Variance

Analysis of variance (ANOVA) is a method used to evaluate the significance of the regression model. This method is similar to hypothesis test where some inferences are made about the parameters of the population distribution.
According to [6], ANOVA decompose the total variability in the response variable into meaningful components as the basis of the test as shown in equation (8).

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 \quad (8)$$

The equation (8) can be rewritten as equation (9)

$$SS_T = SS_R + SS_E \qquad (9)$$

where: SS_T is the total corrected sum of squares, SS_R is the regression sum of squares and SS_E is the error sum of squares.
The value of the statistic (F_o) can be calculated using the equation (10).

$$F_o = \frac{SS_R}{SS_E/(n-2)} \qquad (10)$$

D. Correlation Coefficient

The correlation coefficient measures the possible linear relation that might exist between two random variables. Its value lies between -1 and 1. The equation (8) is used to calculate the correlation coefficient.

$$\rho(X,Y) = \frac{Cov[X,Y]}{\sqrt{\sigma_x^2 \sigma_y^2}} \qquad (11)$$

Where $Cov[X,Y]$ is the covariance between variables X and Y, and σ_x^2 and σ_y^2 are the variance of X and Y variance, respectively.

E. The Coefficient of Determination (R^2)

The coefficient of determination (also known of R^2 coefficient) is a measure of the model fitting obtained by a regression analysis. It is calculated by (12), and as its value closer to 1, better is the model fitting.

$$R^2 = \frac{SS_R}{SS_T} \qquad (12)$$

Where SS_T represent the total variation of Y around its average, and SS_R represent the variation of Y's expected value around its average.

3. Methodology

To determine the contribution of multiple harmonic-producing loads scattered in an electrical network, this paper proposes to create data-based models that describe the influence of harmonic currents produced by non-linear loads on the voltage harmonic distortion using the polynomial regression method.

4. Case Study

In this section, it is presented a study performed in a real electrical system located in the north of Brazil. In this system, it was noticed the presence of two dominant harmonic-producing loads called FH1 and FH2. This system's diagram is shown in Fig 1.
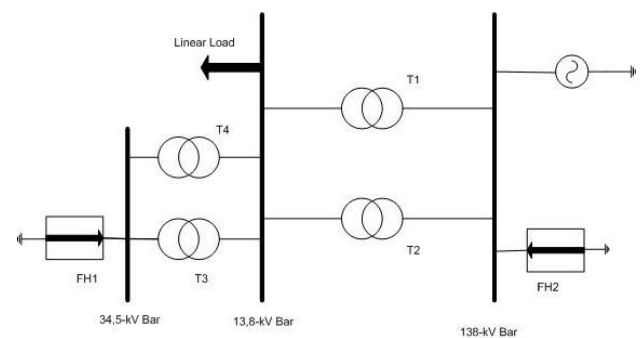

Fig. 1. Electrical diagram of the system

A measurement campaign was conducted in this system and revealed the presence of high levels of voltage and current harmonic distortion, mainly 3rd and 5th harmonics. The power-quality analyzers were installed in two sites: FH1 and FH2 in order to estimate how these harmonic sources influence the harmonic levels in bars of 34.5kV and 138kV. In Fig 2 and Fig 3 are shown the voltage-and-current trend of the 3rd and 5th harmonics measured at source FH1, respectively.

Applying the polynomial regression method to these measurement data, it was obtained the graphics shown in Fig 4 and Fig 5, that describe the voltage-current relationship for 3rd and 5th harmonics, respectively.
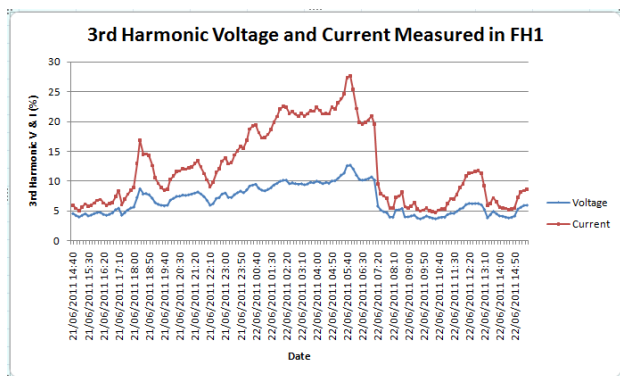

Fig. 2. Graphic of 3rd-harmonic Voltage measured in 34.5kV bar and 3rd-harmonic current measured in FH1
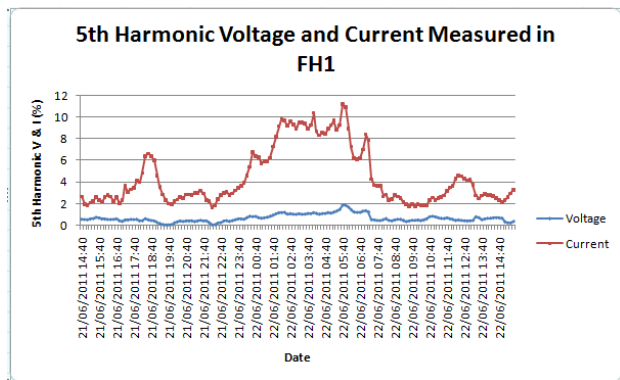

Fig. 3. Graphic of 5th-harmonic Voltage measured in 34.5kV bar and 5th-harmonic current measured in FH1

According to Fig 4, the statistical model that describes the voltage-current relationship for the 3rd harmonic is the 2nd-order polynomial equation shown in equation (13).

$$V_3 = 1.417 + 0.536I_3 - 0.005939I_3^2 \qquad (13)$$

This regression equation was selected due to its high significance since its F-statistic values were high as shown in table I. In addition, the coefficient of determination of this model was 0.97 which means that the model perfectly fitted to the data.

Table I. F-Statistics values of the 2th-order polynomial model (13)

Explanatory Variables	F Values
I_3	5653.83
I_3^2	38.71

The equation (13) suggests that the harmonic source FH1 is responsible for 80.96% of the 3rd-harmonic voltage distortion in the 34.5kV bar. And, if there wasn't 3rd-harmonic current being absorbed or injected into the system by FH1, the 3rd-harmonic voltage in the 34.5kV bar would be approximately 1.425 %.
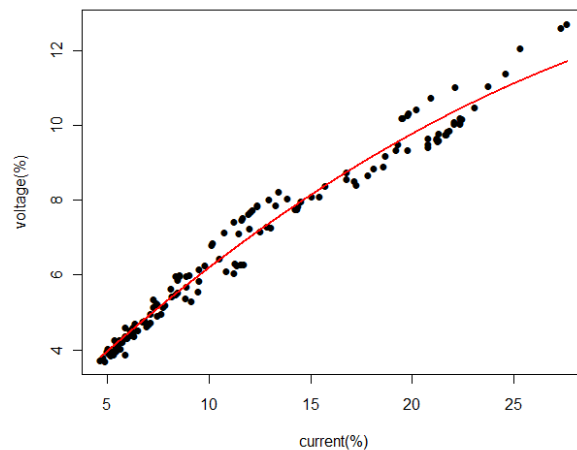

Fig 4. Correlation between the 3rd-harmonic Current FH1 and voltage of 34.5kV bar

To analyze the influence of the harmonic source FH1 on the 5th-harmonic voltage distortion of the 34.5kV bar, it was built the statistical model shown in Fig 4 that is represented by the 1^{rst}-order polynomial equation shown in equation (14).

$$V_5 = 0.1883 + 0.1091I_5 \qquad (14)$$

The regression equation (14) was the equation that presented the highest significance since its F-statistic value was 383.91. In addition, this model's coefficient of determination was 0.67 which means that the model fitted reasonably well to the measurement data.

The equation (14) suggests that the harmonic source FH1 is responsible for 65.64% of the 5th-harmonic voltage distortion in the 34.5kV bar. And, if there wasn't 5th-harmonic current being absorbed or injected by FH1 into the system, the 5th-harmonic voltage in the 34.5kV bar would be approximately 0.1883 %.
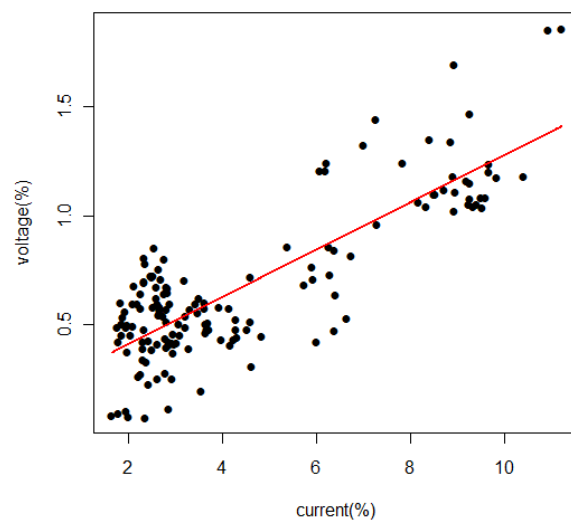

Fig 5. Correlation between the 5th-harmonic Current FH1 and voltage of 34.5kV bar

In Fig 6 and Fig 7 are shown the voltage-and-current trend of the 3rd and 5th harmonics measured at source FH2, respectively.

Applying the polynomial regression method to these measurements, it was obtained the graphics shown in Fig 8 and Fig 9, that describe the voltage-current relationship for 3rd and 5th harmonics at FH2, respectively.
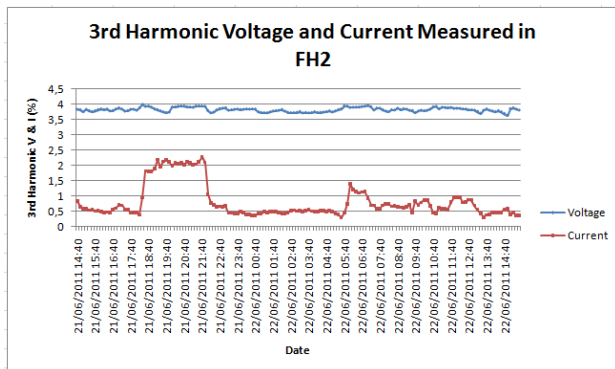

Fig. 6. Graphic of 3rd Harmonic Voltage and Current Measured in FH2
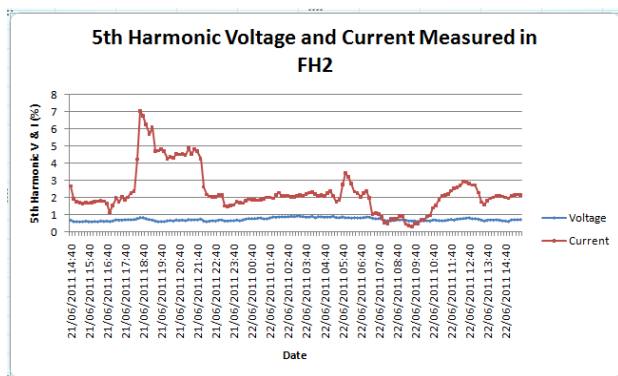

Fig. 7. Graphic of 5th Harmonic Voltage and Current Measured in FH2

To analyze the influence of the harmonic source FH2 on the 3rd-harmonic voltage distortion of the 138kV bar, it was built the statistical model shown in Fig 8 that is represented by the 4th-order polynomial equation shown in equation (15).

$$V_3 = 4.15 - 1.64I_3 + 2.43I_3^2 - 1.3I_3^3 + 0.23I_3^4 \qquad (15)$$

The regression equation (15) presented higher significance than the other equations since its F-statistic values shown in table II were higher than the other equation's values. In addition, the coefficient of determination of this model was 0.25 which means that the model poorly fit to the data.

Table II. F-Statistics values of the 4th-order polynomial model (15)

Explanatory Variables	F-statistic Values
I_3	36.13
I_3^2	4.44
I_3^3	7.73
I_3^4	5.87

The equation (15) suggests that the harmonic source FH2 is responsible for -7.25 % of the 3rd-harmonic voltage

distortion in the 138kV bar. And, if there wasn't 3rd-harmonic current being absorbed or injected into the system by FH2, the 3rd-harmonic voltage in the 138kV bar would be approximately 4.15 %.

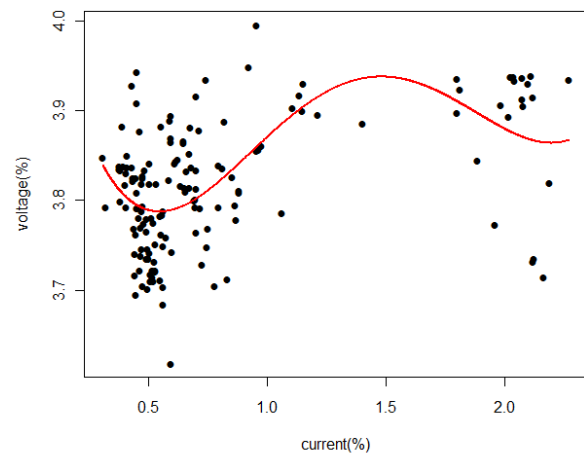Fig 8. Correlation between the 3rd-harmonic Current FH2 and voltage of 138kV bar

To analyze the influence of the harmonic source FH2 on the 5th-harmonic voltage distortion of the 138kV bar, it was built the statistical model shown in Fig 9 that is represented by the 5th-order polynomial equation shown in equation (16).

$$V_5 = 0.858 - 0.566I_5 + 0.52I_5^2 - 0.174I_5^3 + 0.024I_5^4 - 0.001I_5^5 \quad (16)$$

The regression equation (16) presented higher significance than the other equations since its F-statistic values shown in table III were higher than the other equation's values. In addition, the coefficient of determination of this model was 0.22 which means that the model poorly fitted to the data.

Table III. F-Statistics values of the 5th-order polynomial model (16)

Explanatory Variables	F-statistic Values
I_5	2.889
I_5^2	8.82
I_5^3	17.71
I_5^4	12.24
I_5^5	5.03

The equation (16) suggests that the harmonic source FH2 is responsible for 9.55% of the 5th-harmonic voltage distortion in the 138kV bar. And, if there wasn't 5th-harmonic current being absorbed or injected by FH2 into the system, the 5th-harmonic voltage in the 138kV bar would be approximately 0.858 %.
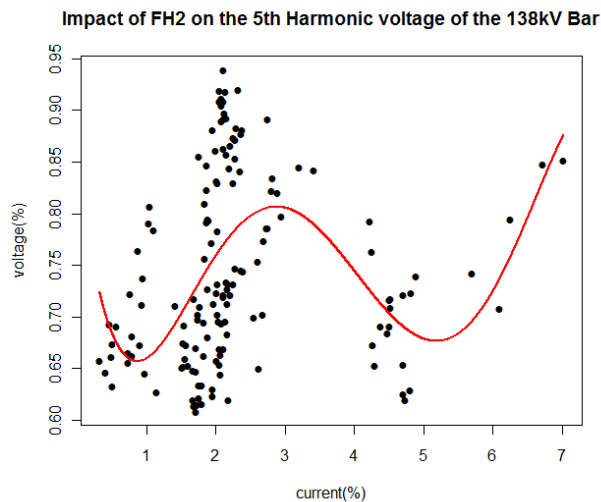
Impact of FH2 on the 5th Harmonic voltage of the 138kV Bar

Fig 9. Correlation between the 5th-harmonic Current of FH2 and voltage of 138kV bar

Impact of FH1 on the 3rd Harmonic voltage of the 138kV Bar

Fig 10. Correlation between the 3rd-harmonic Current of FH1 and voltage of 138kV bar

Impact of FH1 on the 5th Harmonic voltage of the 138kV Bar

Fig 10. Correlation between the 5th-harmonic current of FH1 and voltage of 138kV bar

To analyze the influence of the harmonic source FH1 on the 3rd-harmonic and 5th-harmonic voltage distortion of the 138kV bar, it were built the statistical models shown in Fig 10 and Fig 11, respectively, that were represented by the 3rd-order and 2nd-order polynomial equations shown in equation (17) and (18), respectively.

$$V_3 = 3.47 + 0.086I_3 - 0.006I_3^2 + 0.0001I_3^3 \qquad (17)$$

$$V_5 = 0.53 + 0.064I_5 - 0.0028I_5^2 \qquad (18)$$

The regression equations (17) and (18) presented higher significance than the other equations since their F-statistic values shown in tables V and VI, respectively, and were higher than the other equation's values. In addition, the coefficient of determination of the 3rd-order and 2nd-order polynomial models were 0.16 and 0.88, respectively, which means that the 3rd-order model poorly fitted to the data and 2rd-order model fitted well to the data.

Table V. F-Statistics values of the 3rd-order polynomial model (17)

Explanatory Variables	F-statistic Values
I_3	0.02
I_3^2	7.49
I_3^3	24.76

Table VI. F-Statistics values of the 2nd-order polynomial model (18)

Explanatory Variables	F-statistic Values
I_5	0.02
I_5^2	7.49

The equation (17) and (18) suggest that the harmonic source FH1 is responsible for 4.0% and 38.87%, respectively, of the 3rd-order and 5th-harmonic voltage distortion in the 138kV bar. And, if there wasn't 3rd-order and 5th-harmonic currents being absorbed or injected into the system by FH1, the 3rd-harmonic and 5th-harmonic voltage in the 138kV bar would be approximately 3.47% and 0.53%, respectively.
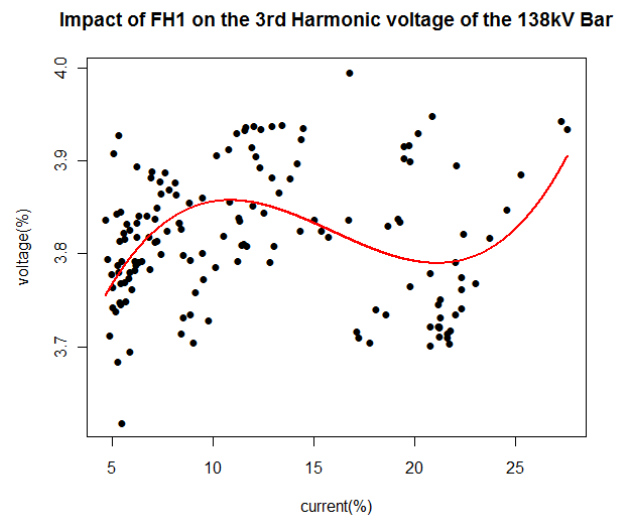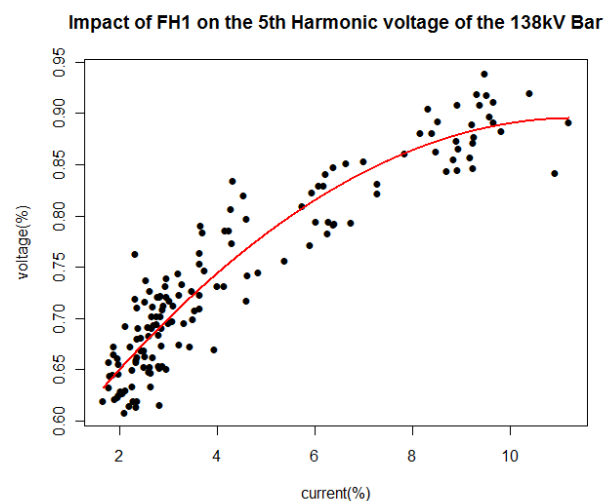
To analyze the influence of the harmonic source FH2 on the 3rd-harmonic and 5th-harmonic voltage distortion of the 34.5kV bar, it were built the statistical models shown in Fig 12 and Fig 13, respectively, that were represented by the 3th-order and 2nd-order polynomial equations shown in equation (19) and (20), respectively.

$$V_3 = 14.84 - 27.94I_3 + 27.41I_3^2 - 7.54I_3^3 \qquad (19)$$

$$V_5 = 0.42 + 0.24I_5 - 0.045I_5^2 \qquad (20)$$

The regression equations (19) and (20) presented higher significance than the other equations since their F-statistic values shown in tables VII and VIII, respectively, and were higher than the other equation's values. In addition, the coefficient of determination of the 3th-order and 2nd-order polynomial models were 0.01 and 0.11, respectively, which means that both 3th-order 2rd-order models poorly fitted to the data.

Table VII. F-Statistics values of the 3th-order polynomial model (19)

Explanatory Variables	F-statistic Values
I_3	1.0074
I_3^2	0.78
I_3^3	14.22

Table VIII. F-Statistics values of the 2nd-order polynomial model (20)

Explanatory Variables	F-statistic Values
I_5	5.09
I_5^2	15.44

The equation (19) and (20) suggest that the harmonic source FH2 is responsible for -82.26% and 17.94%, respectively, of the 3rd-order and 5th-harmonic voltage distortion in the 34.5kV bar. And, if there wasn't 3rd-order and 5th-harmonic currents being absorbed or injected into the system by FH1, the 3rd-harmonic and 5th-harmonic voltage in the 138kV bar would be approximately 14.84% and 0.42%.
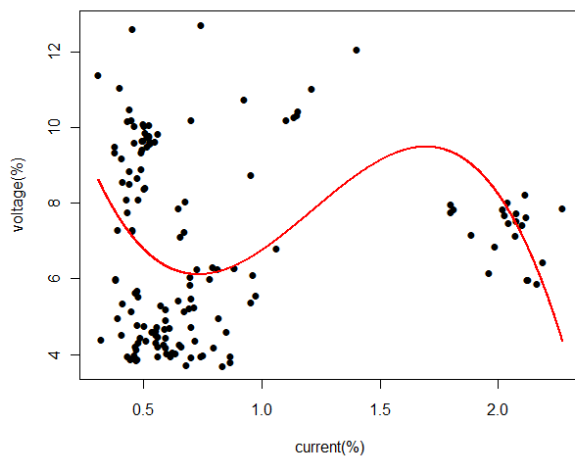
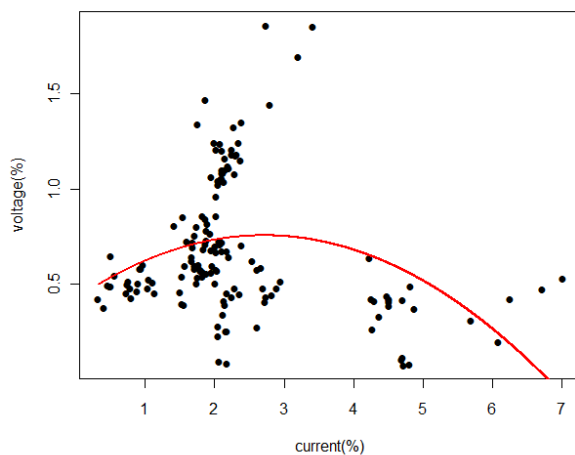Fig 11. Correlation between the 3rd-harmonic Current of FH2 and voltage of 34.5kV bar

Fig 12. Correlation between the 5th-harmonic Current of FH2 and voltage of 34.5kV bar

5. Conclusion

This paper developed statistical models using the polynomial regression method that described the relationship between the harmonic voltage and current of an electrical system located in the north of Brazil.

Some of the models presented a satisfactory performance and significance and were able to describe the influence of some harmonic sources in the harmonic voltage level of this system.

In this study, the harmonic source FH1 presented a high influence on the levels of voltage harmonic distortion in the studied sites of the system.

Acknowledgement

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