

Probabilistic model for distributed generation expansion in distribution power network

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Abstract. In this paper a probabilistic model is presented to optimize the expansion of distributed generation in the electricity distribution network. The Monte Carlo technique is used to obtain probability distributions of the desired variables, such as: power flows, output power of distributed generators, costs, etc. The analysis of the results leads to optimized criteria for the expansion of distributed generation (DG) in distribution networks.

Key words

Probabilistic model, Monte-Carlo, expansion generation, renewable energy, optimization.

1 Introduction

The development of distributed generation, primarily associated with renewable energy, is related to several important topics such as the need for greater flexibility of the electrical system, new legislative and economic scenarios, price of energy and environmental impact of the generation of electricity (green house effect).

Renewable generators are intermittent due to its primary energy sources, like wind or solar radiation. But not only renewable energy sources are uncertain, also the future costs of fuels and technologies and the energy demand itself.

The presence of distributed generation has significant effects on the performance of distribution networks: reverse flows, increased contribution to short circuit currents, voltage levels and the deterioration of the protection systems and its coordination.

Electrical distribution systems are beginning to face a period of major changes which are going along with long periods of rates of return. Therefore, there is an increasing need for the development of planning tools able to efficiently address the growing uncertainty that characterizes the current situation.

Until now the main focus of the planning tools was given to deterministic methods and very few authors have considered any uncertainty in their models. This contrasts with the fact that risk management or the management of uncertainty is a very important issue for the utilities and system operators. In order to exploit the opportunities ahead, it is necessary to find an efficient way to minimize the risks and uncertainties.

Recently, several authors address the planning of generation expansion from a deterministic perspective. Kuri *et al.* [1] propose an architecture to optimize the planning of distributed generation by emphasizing the risks and uncertainties. Keane *et al.* [2] propose a methodology for determining the optimal location of distributed generators along the distribution network. Krahl *et al.* [3] present a method for assessing and minimizing the costs of distributed generation networks.

In recent years, there were proposed models based on probability distributions. In Repo *et al.* [4] the short-term planning of a distribution network is discussed, taking into account the stochastic behavior of Distributed Generation units. Marmidis *et al.* [5] show a method for the optimal location of wind turbines in a wind farm, based on Monte Carlo Simulation. Bouffard *et al.* [6] formulate a short-term electricity market-clearing problem with stochastic security, considering non-dispatchable, stochastic wind generation. It is pointed out that stochastic operation planning allows more wind power in the network, without sacrificing security. Haesen *et al.* [7] present a robust planning methodology for integration of generators in distribution networks.

In section 2 the algorithm based on the Monte Carlo technique used in the probabilistic model is exposed. In section 3, the probabilistic optimization model is described, which represents the expansion of distributed generation in an electricity distribution network. In section 4 results are presented. In section 5 the additional information given by the probabilistic model compared to

deterministic model is explained and the expansion of generation by both models is compared.

2 Monte Carlo Simulation

In Fig. 1 a flow chart of Monte Carlo simulation is shown. This simulation consists in n iterations where the input variables (consumption, wind speed, solar radiation, water flow) are changed in every simulation step following the statistical properties defined before. In every iteration wind, solar and hydro power generation profiles are estimated and introduced into the power flow model. As a result different system state variables are obtained for every iteration. Finally, if the iteration number reaches the predefined number of iterations, the loop stops. The result is a large number of deterministic results, which can be represented as histograms or approximated by distribution functions.

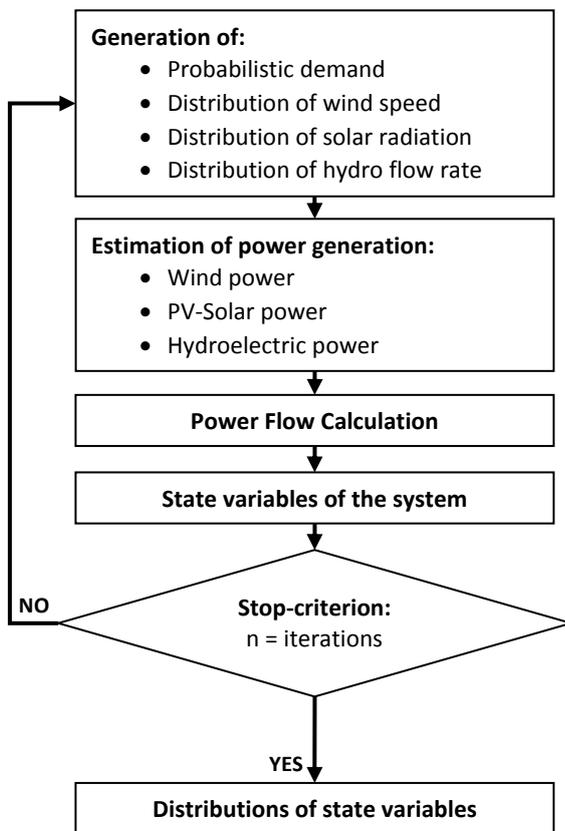


Fig. 1. Flowchart of Monte Carlo simulation.

The main variables with randomness in the presented model: electricity demand, wind generation, photovoltaic generation and hydro generation.

The variability of the demand is usually modelled with the normal distribution, combined with a daily profile.

In general, the statistics of the wind speed is described with the Weibull distribution. From here, taking into account the power curve of the wind turbine, the probability density function of the generated power generated can be estimated.

The solar resource is modelled with the Beta distribution. Different parameters are obtained for every hour of the day, from long-term measurements.

The water flow rate of the hydro power resource is modeled with the Generalized Extreme Value

Distribution. Parameters were obtained from a time series of daily mean values from 17 years. The power, generated with that flow rate is obtained with the power curve of a standard Francis Turbine.

These statistical characteristics are introduced in the Monte Carlo model. More details on the probabilistic models can be found in [8].

3 Mathematical formulation

A. The objective function

In the model for optimal expansion planning of distributed generation, the objective function $F(x)$ is minimized being subject to a set of constraints $h(x)$ and $g(x)$:

$$F(x) = \min[f(x)] \quad (1)$$

$$h(x) = 0 \quad (2)$$

$$g(x) \leq 0 \quad (3)$$

In the optimization problem presented here, the objective function is the total system cost and restrictions are: balance of power in nodes (Kirchhoff's current law), balance of generation, balance of energy in the storage units (state of charge), power capacity limits of substations, generators and storage units, capacity limits of power lines, voltage limits in nodes.

The probabilistic model of the system provides a large number of deterministic results, representing different expansion scenarios. These results are expressed as histograms expanded in time. The stochastic input variables are: electricity demand, wind generation, photovoltaic generation and hydro generation.

The objective function to be minimized is the total cost C_{total} over the planning horizon, which in this case is 20 years. To do this, annual costs are calculated for the expansion for every year. Project costs are derived by adding the fixed costs and variable costs per day multiplied by 365 and by 20. Finally, the costs are annualized with the so-called "Capital Recovery Factor". As a result, the objective function can be formulated as follows:

$$C_{total} = \min \left[\sum_a \sum_k \left\{ f_{re} CF_{a,k} + CRF \sum_{j=1}^y \left(\frac{1+g_k}{1-r} \right)^j \left(CVA_{a,k} + 365 \sum_t CV_{a,k,t} \right) \right\} \right] \quad (4)$$

with

$$CRF = \frac{r(1+r)^y}{(1+r)^y - 1} \quad (5)$$

Where C is the total cost of the system, CRF is the capital recovery factor, k are the network elements, t are the periods, CF_k is the fixed cost of element k , f_{re} is the reduction factor of installation costs of element k , CVA_k is the annual variable cost of element k , $CV_{k,t}$ is the variable cost of element k as a function of time t , g_k is the rate of increase of variable cost of element k , r is the market interest rate, and y is the project planning horizon in years.

In the first year, the whole distribution system is set up, under the condition that no distributed generation and no storage are present. As a result, lines are sized according to the load flows and the associated annualized costs are

calculated. From the second year onwards, the installation of distributed generation and storage is allowed, but no additional lines can be added. The installation cost of the new capacity is annualized and added to the costs originated from the first year, and so on. For every year, the sum of these annualized costs is minimized. If this annual cost is divided by the annual demand of energy, the cost of energy can be calculated for every year of the planning horizon as shown in (6). This way, the evolution of the minimized **COE** is obtained in €/kWh.

$$COE_a = \frac{C_a}{E_a} \quad (6)$$

Where C_a is the cumulated annualized cost after a years, E_a is the energy demand in year a .

B. Fixed costs

In the following equation, the fixed costs of the network elements are formulated.

$$C_L = \sum_{ij} c_{f,ij} P_{ijmax} \quad (7)$$

$$C_W = \sum_i \sum_w c_{f,w} P_{wmax,i} \quad (8)$$

$$C_{PV} = \sum_i \sum_w c_{f,pv} P_{pvmax,i} \quad (8)$$

$$C_H = \sum_i \sum_w c_{f,mgh} P_{mghmax,i} \quad (9)$$

$$C_S = \sum_i \sum_{st} \{c_{fp,st} P_{stmax,i} + c_{fe,st} E_{stmax,i}\} \quad (10)$$

Where C_L , C_W , C_{PV} , C_H and C_S are the fixed costs of distribution lines ij , wind generators w , solar generators pv , mini-hydro generators mgh and storage units st , with corresponding fixed cost coefficients c_f in €/kW and for storage $c_{fp,st}$ in €/kW and $c_{fe,st}$ in €/kWh, P_{ijmax} , $P_{wmax,i}$, $P_{pvmax,i}$, $P_{mghmax,i}$, $P_{stmax,i}$ are the installed power of lines and generators and $P_{stmax,i}$ and $E_{stmax,i}$ are de power and energy capacity of the storage unit st at node i .

C. Variable costs

Next, variable costs are described. The cost related to energy obtained from the grid $C_{E,t}$ in every time period t is given in (11).

$$C_{E,t} = f_{pe} \cdot PE_t \sum_i \sum_q E_{q,t} \quad (11)$$

Where f_{pe} is the escalation factor of the electricity Price, PE_t at time t , given in €/kWh, $E_{q,t}$ is the energy in kWh imported from the grid at time t through substation q at node i .

In this model, the variable costs are referred as operational costs, which are supposed to be proportional to the investment cost. In case of storage, the cost of energy losses is included too.

One exception is the variable cost of distribution lines, which is assumed to be exclusively the energy loss. If line losses are linearized, the variable cost of the lines at time t $C_{LO,t}$ can thus be written as

$$C_{LO,t} = \sum_{ij} c_{v,ij,t} \cdot P_{ij,t} \cdot P_{ijmax} \quad (12)$$

With

$$c_{v,ij,t} = PE_t \cdot 0.75 \cdot FP_{ij} \cdot \frac{R_{ij}}{U_{n,ij}^2} \cdot p_t \quad (13)$$

Where $c_{v,ij,t}$ is the loss cost coefficient of line ij in period t in $\text{€}/(\text{kW})^2$, $P_{ij,t}$ is the power in kW transported by line ij in period t , P_{ijmax} is the maximum capacity in kW of line ij , PE_t is the electricity Price in period t in €/kWh, FP_{ij} is the loss factor, R_{ij} the phase resistance in Ω , and $U_{n,ij}$ the nominal voltage in V of line ij , p_t is the duration of period t in hours.

The annual operational costs of the renewable generators are calculated as a percentage of the investment cost with a coefficient of variable costs c_v as can be seen in the following equations.

$$C_{WO} = \sum_i \sum_w c_{v,w,t} \cdot C_{w,i} \quad (14)$$

$$C_{PVO} = \sum_i \sum_{pv} c_{v,pv,i} \cdot C_{pv,i} \quad (15)$$

$$C_{HO} = \sum_i \sum_{mgh} c_{v,mgh,i} \cdot C_{mgh,i} \quad (16)$$

Where c_v are the annual O&M cost coefficients in p.u. and C are the investment costs with the indices w , pv y mgh for wind, solar pv and mini-hydro generation and i is the node.

In the case of storage, the energy loss has to be added to the variable costs.

$$C_{SO,t} = \sum_i \sum_{st} \{c_{v,st,i} \cdot C_{S,st,i} + 365 \cdot C_{p,st,i}\} \quad (17)$$

with

$$C_{p,st,i} = PE_t \cdot p_t \cdot (P_{c,st,i,t}(1 - \eta_{c,st,i,t}) + P_{d,st,i,t}(1 - \eta_{d,st,i,t})) \quad (18)$$

Where $c_{v,st,i}$ the annual O&M cost coefficient per unit of investment cost $C_{S,st,i}$ and $C_{p,st,i}$ is the daily cost of energy losses for storage unit st at node i , $P_{c,st,i,t}$ and $P_{d,st,i,t}$ are the power of charge and discharge in kW during period t , $\eta_{c,st,i,t}$ and $\eta_{d,st,i,t}$ are the efficiencies of charge and discharge.

Note that for simplification, in the variable costs for storage there are included several quite different costs, such as normal O&M costs and the replacement of lager parts of the storage system, due to a shorter lifetime (inverters, battery packs, etc.)

D. Modeling of charge and discharge cycles of the storage

Charge and discharge cycles of the storage units are introduced into the model assuming that they are completed in 24 h, which leads to the condition that SOC is zero at the beginning and at the end of the day. Charge and discharge efficiency is defined separately, including wiring and the inverter. To adequately model the storage it is necessary to introduce a model of the state of charge SOC which provides the energy stored in any moment. This is necessary to model the charge limits. The depth of discharge (DoD) is assumed to be 100%. Possible limitations of the DoD depend on the storage technology

and can be taken into account when sizing the storage unit to be installed. Below see the model of SOC:

$$SOC_{st,i,t} = SOC_{st,i,(t-1)} + E_{c,st,i,t} - E_{d,st,i,t} \quad (20)$$

with

$$E_{c,st,i,t} = P_{c,st,i,t} \cdot \eta_{c,st,i,t} \cdot P_t \quad (21)$$

$$E_{d,st,i,t} = \frac{P_{d,st,i,t}}{\eta_{d,st,i,t}} \cdot P_t \quad (22)$$

E. Restrictions

The restrictions imposed here on the optimization problem for distributed generation planning are: balance of power in nodes (Kirchhoff's Current Law), capacity limits of power flow in lines, maximum generator power capacity, power and energy limits in storage units, substation power limit and voltage limit in nodes.

In (19) the balance in every node is shown. Note that distribution losses $P_{p,ij,t}$ and storage losses $P_{p,st,i,t}$ are included.

$$D_{i,t} = \left\{ \begin{array}{l} \sum_{ij} [(P_{ij,t} - P_{p,ij,t}) - (P_{ji,t} - P_{p,ji,t})] + \sum_q P_{q,i,t} \\ + \sum_w P_{w,i,t} + \sum_{pv} P_{pv,i,t} + \sum_{mgh} P_{mgh,i,t} \\ + \sum_{st} [P_{d,st,i,t} - P_{c,st,i,t} - P_{p,st,i,t}] \end{array} \right. \quad (23)$$

Where $D_{i,t}$ is the demand in node i in time period t , $P_{p,ij,t}$ and $P_{p,ji,t}$ are los line losses in directions ij and ji , $P_{p,st,i,t}$ are the storage losses.

Finally, several limits are imposed on the different system elements. Power lines are limited by their thermal limit $P_{ij,max}$ and as power flows in opposite directions are modeled separately ($P_{ij,t}$ and $P_{ji,t}$), no negative power flows are permitted. In (24) the set of restrictions of the distribution power lines is shown.

$$\begin{aligned} 0 &\leq P_{ij,t} \leq P_{ij,max} \\ 0 &\leq P_{ji,t} \leq P_{ij,max} \end{aligned} \quad (24)$$

The intermittence of renewable generators is modeled superposing a generation profile over the nominal power of the generator as shown in the following equations.

$$0 \leq P_{w,i,t} \leq P_{w,max,i} \cdot P_{perf,w,t} \quad (25)$$

$$0 \leq P_{pv,i,t} \leq P_{pv,max,i} \cdot P_{perf,pv,t} \quad (26)$$

$$0 \leq P_{mgh,i,t} \leq P_{mgh,max,i} \cdot P_{perf,mgh,t} \quad (27)$$

As only one day of the year is simulated, worst-case situations are introduced to the probabilistic profile, as shown in Fig. 2.

The stochastic behavior is introduced separately by the probabilistic model of each generator.

Finally, the restrictions of the storage are defined. Power limits for charging and discharging are separated and SOC is limited between zero and the maximum storage capacity $E_{st,max}$.

$$P_{c,st,i,t} \leq P_{c,st,max,i} \quad (28)$$

$$P_{d,st,i,t} \leq P_{d,st,max,i} \quad (29)$$

$$0 \leq SOC_{st,i,t} \leq E_{st,max,i} \quad (30)$$

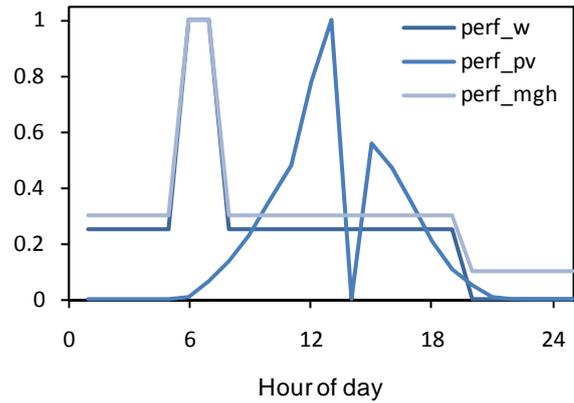


Fig. 2. Generation profiles for wind (perf_w), solar (perf_pv) and mini-hydro (perf_mgh) generators.

4 Results

In this section results obtained with the deterministic and probabilistic model are presented. Different cases are analyzed and solutions from the deterministic and probabilistic method are compared.

The case study considers a 15-node distribution network, connected to the transmission grid by a substation with 30 MVA, 115/10 kV. The feeders of the lines are 3X1X400Al, using a double circuit for trunk lines and simple circuit for branch lines. The topology of this distribution network is shown in Fig. 3.

Possible placements of wind generators are restricted to nodes 5, 7, and 11 and of mini-hydro generators to nodes 7 and 15. Solar photovoltaic and storage can be places in any node (2-15).

The model is implemented in GAMS (General Algebraic Modeling System) and solved using its solver XPRESS.

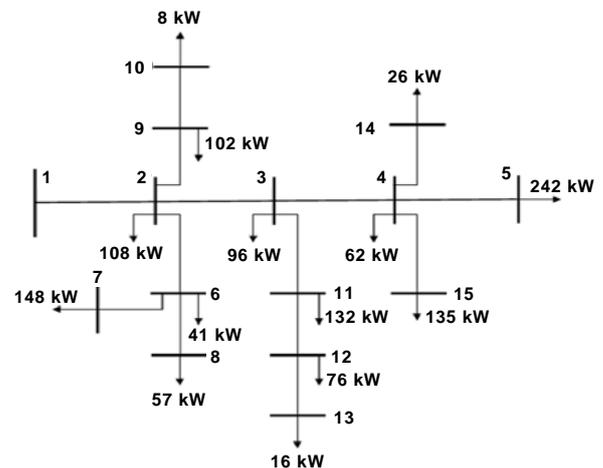


Fig. 3. Topology of the distribution network of 15 nodes.

The results are shown in three scenarios in the deterministic model: No DG, DG and DG + storage. For the probabilistic model results are only shown for the third case.

Figures 4 and 5 show the evolution of energy costs expressed as COE over the planning horizon of 20 years. In Fig. 4 results from the deterministic model are shown. The base case with no distributed generation ("conventional") shows an exponential growth of energy costs, because it was assumed that the electricity price

will rise in this horizon exponentially. With the introduction of DG in the network COE still grows but much slower, reaching after 20 years 0.15 €/kWh, starting from 0.13 €/kWh. In the third case, when storage is considered, from year 7 onwards COE decreases down to a value of 0.095 €/kWh. The sudden decrease is due to the fact that storage starts to be implemented at that moment, because it has become profitable to install it. It shall be mentioned, that a 20% annual decrease of installation costs has been considered here, and after 7 years costs have fallen enough to make storage profitable.

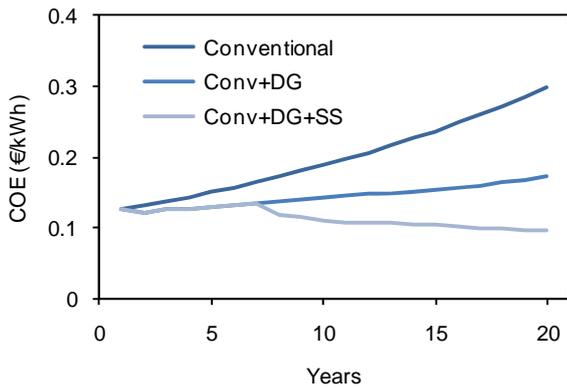


Fig. 4. Evolution of the cost of energy (deterministic model).

In Fig. 5 the result of COE from the probabilistic model is represented. The same tendency can be observed when storage starts to be installed from year 8 onwards. The additional information which can be drawn from the probabilistic model is that the expected COE may rather lie in a corridor of 0.10 – 0.13 €/kWh with the highest probability around 0.12 €/kWh. In general, expected costs are slightly higher than those obtained from the deterministic model, which can be explained by the better simulation of the stochastic behavior of renewable sources.

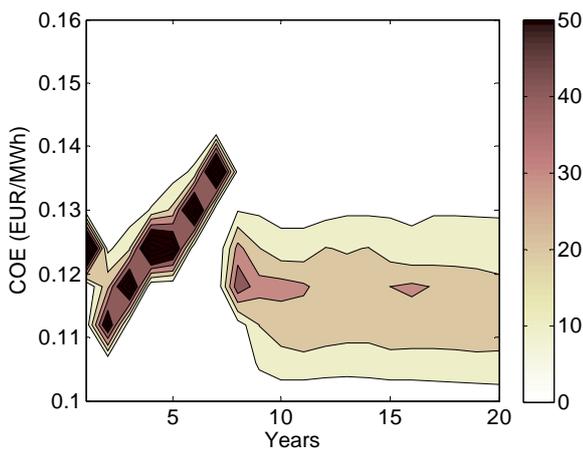


Fig. 5. Evolution of the cost of energy (probabilistic model).

In Fig. 6 the share of energy generated annually by each generation technology is depicted. It can clearly be observed how the energy imported from the grid is reduced gradually unit, after 11 years the distributed generation takes over the entire supply. Hydro power reaches quickly its limit and maintains its production. Wind power enters one year later and its share grows up

to year 14, when solar photovoltaic generation starts to enter the market. It may be mentioned here, that a rather pessimistic initial investment cost of 4000 €/kW and a yearly decrease rate of installations costs of 10% was assumed for solar power. Recent developments on the world market of solar pv suggests, that less than 3000 €/kW and a somewhat higher cost reduction rate would be more realistic. Germany, which installed in 2010 almost 50% of the world's capacity plans to reduce the feed-in tariff by 24% during 2011 in order to adjust the unexpected advances in cost reduction in the solar industry.

Interesting is to see how energy cycled through storage grows through the years 7 – 11 and then just increase at the same rate as global demand grows. In the year 20, an amount of approximately 50% of the total demand is cycled through the storage units. In Fig. 7 it is shown that the losses due to such a massive storage are far less than excess energy produced by renewable generators. This excess energy would reach in year 20 about 24% of total consumption, while storage losses only reach 6%. Another result seen in Fig. 7 is that at the moment when storage is introduced to the network, excess energy disappears completely. This is the reason for the cost reduction, achieved by storage.

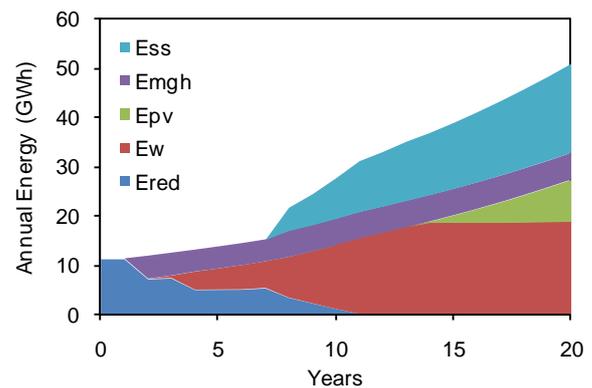


Fig. 6. Evolution of total annual generated energy (deterministic model).

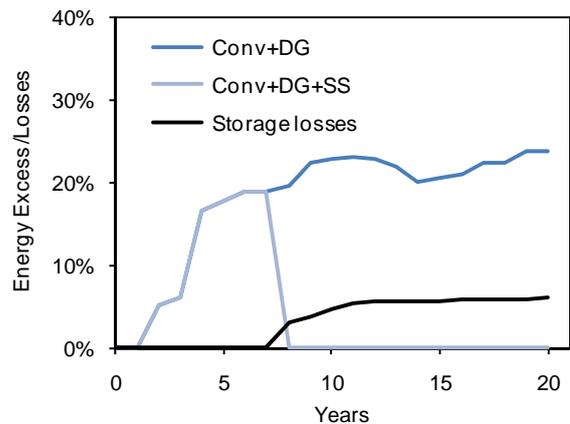


Fig. 7. Evolution of excess energy and storage losses (deterministic model) as a percentage of total demand.

In Fig. 8 the modeled growth in energy consumption is shown. It can be seen, how the uncertainty about the expected consumption grows every year. This is a direct result of the variation of the demand growth rate, introduced to the probabilistic model.

Figure 9 shows the share of different technologies in the total cost (including distribution losses). The cost of losses tends to decrease in both cases. A first difference can be observed in the share of storage costs. With the probabilistic model (b) storage reaches rapidly a larger proportion than with the deterministic model. This is a sign that the probabilistic model represents better the intermittence of renewable sources.

Participation of hydropower in the early years is higher than in the past 6 years. Wind enters the scene in two years later in the probabilistic case, due to higher costs caused by better modeling of its intermittency. In solar energy no significant differences are observed. The cost associated to energy imports from the grid is lowering more slowly in the probabilistic case.

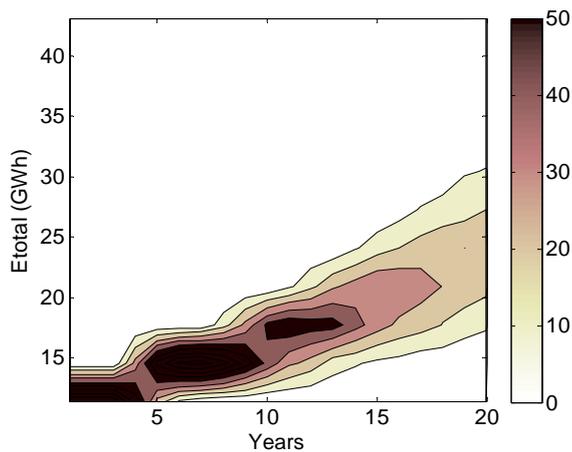


Fig. 8. Evolution of total energy consumption (probabilistic model).

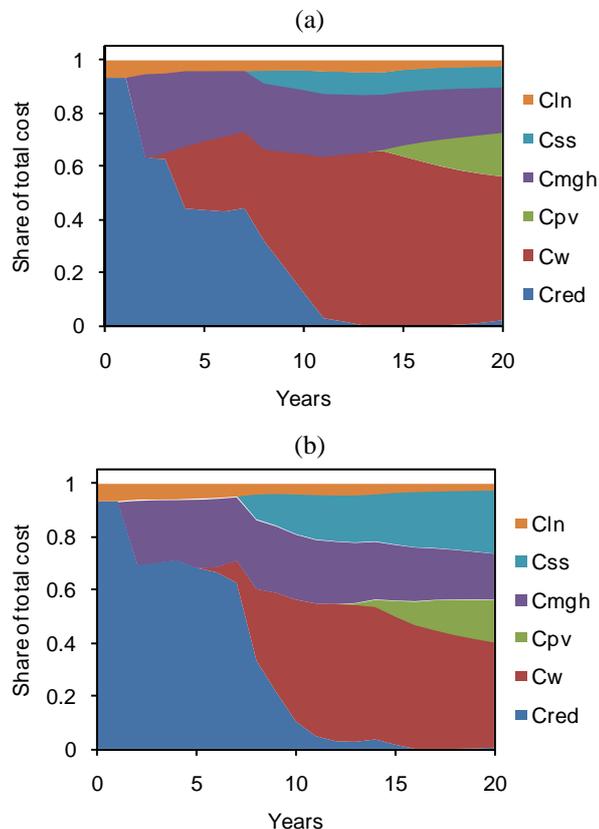


Fig. 9. Share of total system cost of different elements from (a) deterministic model and (b) probabilistic model.

5 Conclusions

The probabilistic model developed in this paper, allows modeling the uncertainty of renewable energy, to determine the scenarios that are more favorable to the expansion of distributed generation and its influence. The histogram representation of the output variables gives an idea of the uncertainty of the solutions obtained, the feasibility of installing a specific technology and risk (cost) involved. Therefore, the first advantage of using probabilistic models is that more information is available. In the deterministic model the evolution of a solution to a particular scenario is obtained and in the probabilistic model solutions for many scenarios are obtained in parallel.

The trend is observed, that the cost of energy is lower with the installation of distributed generation compared to that obtained with conventional energy supply. With the introduction of storage a further reduction is achieved. Results from the probabilistic model indicate, that in the deterministic model the cost of renewable energies are underestimated. This is due to the fact that the intermittence is not well simulated, while this is done much better in the probabilistic approach. This can also be seen in the implementation of storage in the network. In the probabilistic model, more storage is proposed, because the randomness of the distributed generation is taken into account and storage is the key to deal with that. The probabilistic model accounts also for the growing uncertainty in total demand. As a consequence, planning results show a broader distribution for years which lie more in the future.

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