Sensor optimum location algorithm for estimating harmonic sources injection in electrical networks

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Abstract. Today, the presence of disturbing loads in low, medium and also directly in high voltage networks is producing a growing presence of harmonics and unbalance in transmission levels. Economic reasons difficult the installation of harmonic measurement devices in all the buses of the system. Instead, it would be possible to install harmonic measurement equipment in selected buses, in a fraction of the buses of the network. With them, estimation of harmonic current injection can be made.

In this paper, a method to estimate the best buses to install harmonic voltage measurement equipment is presented. Based on the configuration of the network supplied by the TSO (by means of its impedance matrix), the method allows to determine which are the best buses.

The method is being implemented for the Spanish TSO REE, to allow the watching of the harmonic and unbalanced presence in the Spanish transmission network, with the actual equipment and to determine the future equipment installation buses. It is developed by the UPM Electrical and Mathematics Departments under an investigation project (GENINF) with REE funding.

Key words

Harmonic Distortion; Harmonic Source Location; Multiple Harmonic Sources; Power Quality.

1. Introduction

Traditionally, the harmonic sources considered in transmission networks studies are the ones which are directly connected to this network, or great power loads in distribution networks very close to the transmission network. This scenery is changing with the growing presence of a great number of low power distorting loads in the distribution network, and with the presence of distorting generators in the same network.

The injection of harmonics and unbalances in the transmission networks is so becoming unlocalized, and more busses of the transmission network are possible sources of harmonic currents. Transmission system operators (TSOs) are due to keep theses buses under vigilance, in order to monitor the compliance with the emission limits for the sake of a good power quality in their networks. From this, the necessity of identify the network buses in which distorted currents are connected is becoming a must for the company.

The optimal solution should be to install measurement equipment in all the suspicious buses. But from an economic point of view, this is unviable [1]. When a big number of buses are measured, technics of state estimation can be used to identify harmonic sources [2]. However, this is not practical in a real power system. Only partial measurements are usually made, and even these in buses in which no current source is present. In an early paper [3], neural network methods were suggested to identify and give value to the harmonic current sources.

With this, the normal situation is to obtain an underdetermined system. In the case of some TSOs, the number of measurement devices is only a small part of the number of busses of the network, resulting in a very underdetermined system. Besides, in most cases only voltage harmonics are measured, with the measurement of currents being limited to the point of connection of large loads.

In this way, the estimation of harmonic sources in a power network must necessarily be made with a limited number of measures taken on specific buses and lines of the network. Also, measurement errors adds to the limitation on the number of measurement, and this makes harmonic detection even more problematic. One of the first references to the solution of this problem is [2], where in imitation of the procedure used in the

fundamental state estimation, it is proposed to use a least squares technique. In **[1]**, this procedure is systematized.

If it is possible to restrict to certain buses of the network the suspicion that are buses where harmonics are generated, there is naturally the question of where to place the sensors to obtain the best possible detection. To deal with this problem, has been proposed variants of this technique of least squares **[4] [5]**. In **[6]**, the problem is solved by using a procedure of combinatorial optimization which seems computationally expensive for implementation in an actual network with a large number of nodes.

2. Problem and solution

The proposed problem is the problem of optimally selecting the nodes of a network wherein voltage should be measured, in order to estimate in the best possible way the harmonic currents injected into certain nodes of the network. Following paragraphs are a mathematical formulation of this electric scenery.

The scenery is one in which a network of n buses is considered. In it, a subset of $q < n$ buses are the nodes where it is supposed to exist an injection of harmonic current (for example, buses in which a non-linear load is directly connected, or connected to an industrial load). In the rest of the nodes is admitted that the injection of harmonic currents is zero.

As is well known, the relationship between nodal voltages and currents is of the form $\mathbf{V} = \mathbf{ZI}$, where $\mathbf{Z}$ is the sub-matrix of the impedance matrix of dimension $q \times n$ with columns corresponding to nodes in which there are harmonic current sources.

If there are $p < n$ harmonic voltage meters installed, it is possible to select from $\mathbf{Z}$ and $\mathbf{V}$ the rows corresponding to these measured voltages are obtained. With this, a system of p equations with q unknowns is obtained, of the form

$$\mathbf{Z}_m \ \mathbf{I} = \mathbf{V}_m$$

If p<q, i.e. there are less meters than suspicious buses, this system of equations is indeterminate, admitting multiple solutions.

One possible way to estimate the currents injected into the nodes is by the solution of minimum norm, which (on the assumption that the rank of the matrix Z_m matches the number of rows p) is given by the expression

$$\mathbf{I}_{est} = \mathbf{Z}_m^{'}[\mathbf{Z}_m\mathbf{Z}_m^{'}]^{-1}\mathbf{V}_m$$

The general solution can be represented in the form

$$\mathbf{I} = \mathbf{I}_{est} + \mathbf{Nh}$$

where h is a vector of $q - p$ freely varying parameters and N is a matrix of dimensions $q \times (q - p)$ and full rank $q - p$ which comply with

$$\mathbf{Z}_m\mathbf{N} = \mathbf{O}$$

Where $\mathbf{O}$ is the null matrix of dimensions $p \times (q - p)$. Eventually, it is shown in large scale networks that some of the rows of matrix $\mathbf{N}$ are all zero elements, which

means that the corresponding currents are uniquely determined by the topology of the network. These rows are characterized by the index of the rows of a submatrix $\mathbf{U}$ of the identity matrix of order q such that

$$\mathbf{UN} = \mathbf{O}$$

Equivalently, can be expressed that those rows are determined because they are necessarily linear combinations of the rows of the matrix $\mathbf{Z}_m$.

A subset of nodes where it is interesting to detect the existence of harmonic injections can be chosen. Once these nodes have been predetermined, the optimum selection of the voltages to measure can be stated as finding the rows of the matrix $\mathbf{Z}$ that best approximate as a whole the rows of the corresponding matrix $\mathbf{U}$.

Being $\mathbf{S}$ a submatrix of the identity matrix of order n which selects p rows of the impedance $\mathbf{Z}$, the best approach in Frobenius norm of the matrix $\mathbf{U}$ by linear combinations of the rows of the $\mathbf{SZ}$ matrix is given by the expression

$$\mathbf{U_S} = \mathbf{U}[\mathbf{Z}^{'}\mathbf{S}^{'}\mathbf{SZ}]^{-1}\mathbf{Z}^{'}\mathbf{S}^{'}$$

where $\mathbf{S}^{'}$ and $\mathbf{Z}^{'}$ are the transpose and conjugate transpose matrices of matrices $\mathbf{S}$ and $\mathbf{Z}$ respectively. Ideally the optimum solution involves determining a matrix $\mathbf{S}$ that solves the following minimization problem:

$$\min_{\mathbf{S}}(\text{trace}[\mathbf{U}^{'}\mathbf{U_S}])$$

Obviously this problem has a combinatorial complication that makes it intractable. However it is possible to obtain a suboptimal solution by extending to the matrix case the procedures known as matching pursuit. With then it is possible to iteratively determine the subsystem with a determined number of vectors extracted of a predetermined set of vectors, which comes closest to a given vector linearly. In the next paragraphs these methods are reviewed and extended.

3. Greedy subset selection algorithms

Suppose fixed a set $\{\mathbf{a}_1,\dots,\mathbf{a}_\nu\}$ of vectors in $\mathbf{C}^n$ that, for convenience, we assume have unit norm. Given $p < n$ and a vector $\mathbf{b}$, the problem is to select p vectors in the above set such that the orthogonal projection $\mathbf{b}$ of the vector $\mathbf{b}$ on the subspace generated by these vectors give the best approximation of this vector, in the sense that the euclidean norm $\|\mathbf{b} - \mathbf{b}\|$ will be minimum.

To determine the optimal solution, we would have to check over the $\binom{\nu}{p}$ possible subsets and the computation quickly becomes infeasible as the numbers ν and p increases. Therefore several suboptimal methods of reasonable complexity have been developed. In the greedy algorithms, also known as forward selection, the subset is sequentially arranged, building and incorporating one vector in each iteration. The idea is to start by finding the vector $\mathbf{a}_i$ closest to $\mathbf{b}$ and then add vectors one by one until p vectors have been

selected, adding each time the vector that gives the largest decrement of the least squares residual.

The so-called orthogonal matching pursuit is briefly described now. The index of the first selected vector is obtained as

$$i_1 = \text{argmax}_{1 \le i \le \nu} \left| \mathbf{a}_i' \mathbf{b} \right|$$

Having selected the first k vectors $\mathbf{a}_{i_1}, \ldots, \mathbf{a}_{i_k}$ the subspace linearly generated by these vectors is the range subspace of the matrix

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{a}_{i_1} & \cdots & \mathbf{a}_{i_k} \end{bmatrix}$$

whose columns are these vectors. The orthogonal projection of the vector $\mathbf{b}$ on this subspace is given by

$$\mathbf{b}_k = \mathbf{A}_k \left(\mathbf{A}_k' \mathbf{A}_k \right)^{-1} \mathbf{A}_k' \mathbf{b}$$

and the corresponding orthogonal residual is the vector $\mathbf{b}_k^\perp = \mathbf{b} - \mathbf{b}_k$. Having obtained this vector, the index of the next vector is given by

$$i_{k+1} = \text{argmax}_{i \in J_k} \left| \mathbf{a}_i' \mathbf{b}_k^\perp \right|$$

where J_k is the set of indexes of the of the $\nu - k$ remaining vectors after excluding of the initial set the k previously selected vectors.

The above procedure can be extended step by step to the subset selection for several vectors. Let these vectors be the columns of the matrix

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}_1 & \cdots & \mathbf{b}_m \end{bmatrix}$$

and as before consider for each subset of p vectors the matrix

$$\hat{\mathbf{B}} = \begin{bmatrix} \hat{\mathbf{b}}_1 & \cdots & \hat{\mathbf{b}}_m \end{bmatrix}$$

whose columns are the orthogonal projections of the columns of $\mathbf{B}$ on the subspace linearly generated by the vectors of this subset.

Now the problem is to obtain the subset such that the Frobenius norm of the matrix difference $\mathbf{B} - \hat{\mathbf{B}}$, which is given by

$$\left\| \mathbf{B} - \hat{\mathbf{B}} \right\|_F^2 = \text{trace} \left(\mathbf{B} - \hat{\mathbf{B}} \right)' \left(\mathbf{B} - \hat{\mathbf{B}} \right) = \left\| \mathbf{B} \right\|_F^2 - \left\| \hat{\mathbf{B}} \right\|_F^2$$

will be minimal. In the k iteration, the matrix

$$\mathbf{B}_k^\perp := \mathbf{B} - \mathbf{A}_k \left(\mathbf{A}_k' \mathbf{A}_k \right)^{-1} \mathbf{A}_k \mathbf{B}$$

is obtained and the index of the next vector will be given by

$$i_{k+1} = \text{argmax}_{i \in J_k} \left\| \mathbf{a}_i' \mathbf{B}_k^\perp \right\|_1$$

4. Backward subset selection algorithms

All the forward selection algorithms have a limitation. If a vector is selected in one iteration, it remains forever in the selected set. Then if, for example, the first vector is erroneously selected, it is clear that the correct set of vectors cannot be selected. For this reason, sequential backward elimination algorithms have been proposed for the subset selection problem as an alternative to forward selection. Now the idea is to start with all vectors present and remove one vector each time until only p vector remains.

As before, a set $\{\mathbf{a}_1, \ldots, \mathbf{a}_\nu\}$ of ν vectors in $\mathbf{C}^n$ is supposed to exist but now, to simplify the exposition, we also suppose that they are linearly independent, for which $\nu \le n$ necessarily. By considering the matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \cdots & \mathbf{a}_\nu \end{bmatrix}$$

the best approximation of the vector $\mathbf{b}$ using all the columns of this matrix is given by

$$\hat{\mathbf{b}} = \mathbf{A} \left[\mathbf{A}' \mathbf{A} \right]^{-1} \mathbf{A}' \mathbf{b}$$

To describe the first step of the elimination process, let us consider the matrices

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{a}_1 & \cdots & \mathbf{a}_{k-1} & \mathbf{a}_{k+1} & \cdots & \mathbf{a}_\nu \end{bmatrix}, \qquad k = 1, \ldots, \nu$$

Each matrix is obtained by elimination the k column in $\mathbf{A}$. For each of these matrices the best approximation is given by

$$\hat{\mathbf{b}}_k = \mathbf{A}_k \left[\mathbf{A}_k' \mathbf{A}_k \right]^{-1} \mathbf{A}_k' \mathbf{b}$$

Clearly for all k we have

$$\left\| \hat{\mathbf{b}} \right\| - \left\| \hat{\mathbf{b}}_k \right\| \ge 0$$

Then the column that gives the least of these differences is eliminated from the matrix $\mathbf{A}$.

This elimination procedure is reiterated until only p vectors remains.
Obviously the complexity of this algorithm is larger than the complexity of the forward algorithm, but frequently the subset selected is better than the obtained in that case. Using the results obtained in [8], the computational burden can be reduced with the following considerations. Introducing the matrix

$$\mathbf{G} = \begin{bmatrix} \mathbf{A}' \mathbf{A} \end{bmatrix}^{-1}$$

let $\gamma_k = \mathbf{G}(k,k)$ and $\mathbf{g}_k$ the $n-1$ vector obtained after removing in the k column of $\mathbf{G}$ the k component. Then it can be proved that by taking the vector

$$\mathbf{h}_k = \mathbf{A}_k \mathbf{g}_k + \gamma_k \mathbf{a}_k$$

We have the following relation:

$$\left\| \hat{\mathbf{b}} \right\|^2 - \left\| \hat{\mathbf{b}}_k \right\|^2 = \frac{1}{\gamma_k} \left| \mathbf{h}_k' \mathbf{a}_k \right|^2$$

Hence, to minimize the difference $\left\| \hat{\mathbf{b}} \right\| - \left\| \hat{\mathbf{b}}_k \right\|$ it is only necessary to search for the

$$\text{argmin}_h \ \frac{1}{\gamma_k} \left| \mathbf{h}_k' \mathbf{a}_k \right|^2$$

The initial matrix $\mathbf{G}$ can be efficiently obtained from a qr factorization of the matrix $\mathbf{A}$. In the remaining steps, it is possible to use a recursive formula to reflect the removal of columns. If $\mathbf{G}_k$ is the matrix resulting from the elimination of the k row and column of matrix $\mathbf{G}$, it can be proved that

$$\begin{bmatrix} \mathbf{A}_k' \mathbf{A}_k \end{bmatrix}^{-1} = \mathbf{G}_k - \frac{1}{h_k} \mathbf{g}_k \mathbf{g}_k'$$

This forward selection algorithm can be extended to the case of subset selection for several vectors in the obvious way, with the only difference that in this case we have to check the relations

$$\left\|\mathbf{B}\right\|_F^2 - \left\|\mathbf{B}\right\|_F^2 = \frac{1}{\gamma_k}\left\|\mathbf{h}_k'\mathbf{A}_k\right\|^2$$

to determine the column to be removed.

5. Application to node selection in electrical networks

To examine the validity of the proposed algorithm, a network of the Spanish TSO REE has been used. Using data obtained for a determined demand situation of the Spanish power system, the nodes with nominal voltages of 220 and 400 kV have been selected, together with their lines and transformers. With this data, the admittance matrix of the network for each harmonic is calculated by means of a harmonic power flow program, INTAR, commissioned to the Electrical Department of the Polytechnic University of Madrid by REE within the framework of previous R&D projects. The following results have been obtained for a 5th harmonic, but the method can be applied to any other harmonic.

The network has n = 846 nodes in total. A superficial analysis of the possible loads has allowed to conclude that there exists q = 347 nodes where injection of harmonics will eventually be presented.
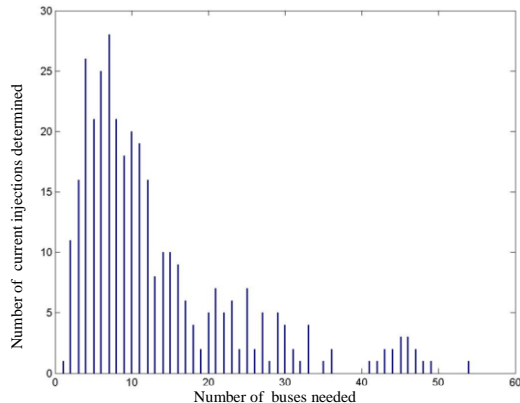

Fig. 1: Distribution of voltage measurement needed to exactly estimate a current source.

In a first analysis of the network, for each of these nodes the localization of the corresponding minimal subset of the nodes where the measure of voltages allows to exactly determine the harmonic injection in the node has been determined by using the forward greedy algorithm for only one vector. The histogram of figure 1 gives the distribution of the different number of voltages measures for each injection node.

By putting together all the nodes obtained, it is observed that there are 590 nodes in which the measure of voltage is needed to exactly determine the possible injection of harmonics in all the 347 suspect nodes. It is obvious that the installation of such a number of equipment is not an option.

In a second analysis, a subset of 40 nodes of higher interest has been selected among the 347 suspect nodes. To determine the suboptimal selection of nodes to place the

voltage sensors to characterize these nodes, a mixed forward-backward subset selection algorithm have been employed. First an exhaustive greedy forward search is implemented to obtain a set of nodes where the measure of voltage exactly determines the harmonic injection in the selected nodes. Then a backward iterative process is initiated, where nodes to be removed are selected one by one, trying to minimize in each step the least squares errors increments in the numerical solutions, until the specified numbers of voltage sensors is obtained.
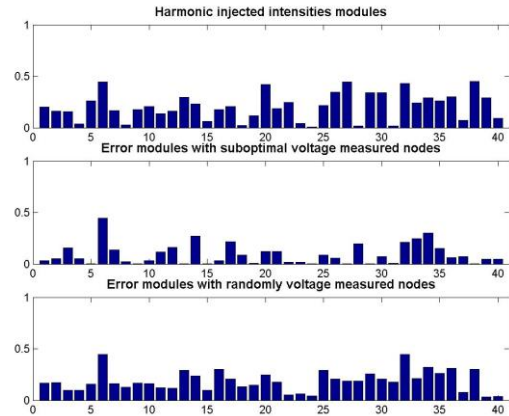

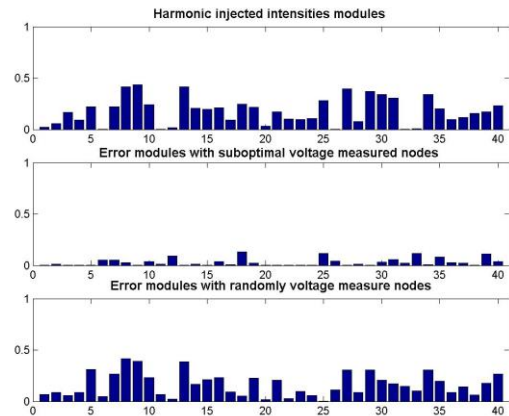Figure 2: Errors in the determination of harmonic current injection (50 voltage measurements).

Figure 3: Errors in the determination of harmonic current injection (100 voltage measurements).

This algorithm has been implemented in a MATLAB program. Numerical simulations were performed with this program by assuming 40 nodes of interest which are located in randomly chosen nodes selected from the 347 potential sources.

The 5th harmonic injections are done in the 347 potential sources, with random modules of a mean value of 0.2 p.u. and a variation of 0.5, with the phases selected randomly. In Fig. 2, a typical result is shown. The 5th harmonic current sources are shown. Also, the differences in absolute values of the obtained modules in p.u. of the estimated harmonic sources with the actual values are represented. In the first case, 50 voltage sensors sub-optimally and randomly selected are used. If 100 voltage sensors are used, the results improve, as shown in Fig. 3.
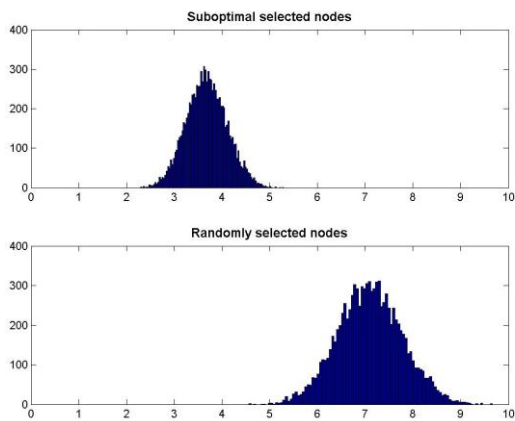
Figure 4: Histograms of the results for a 10000 executions of the algorithm with 50 measures.
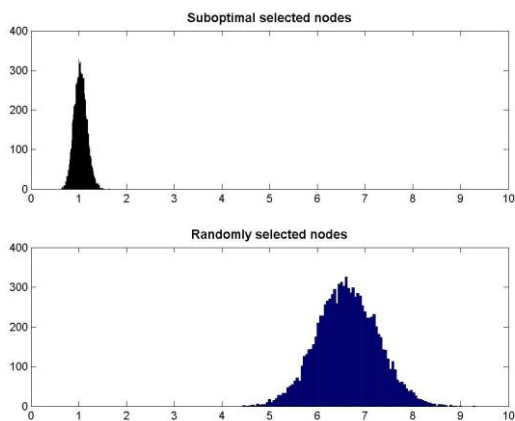

Figure 5: Histograms of the results for a 10000 executions of the algorithm with 100 measures.

In Fig. 4 and Fig. 5, the results of a big number of executions with random sources in fixed buses, with the measurements in a fixed set of random buses, and in a fixed set of sub-optimal buses are shown. The global errors in the estimated harmonic injection in the 40 buses have been represented, for a set of 50 and 100 voltage measurement devices. They are expressed as the addition of the absolute difference between estimated and actual values in the 40 buses.

3. Conclusion

A variation of the problem of obtaining the best location of sensors to improve the location of harmonic voltage measurement devices is shown. An extension to matrix cases of standard greedy forward-backward algorithms for subset basis selection is presented. It can be applied to real networks, and results for an actual transmission grid are presented.
Even when the necessity of a substantial number of measurement systems cannot be avoid, a sub-optimal selection of the buses to install can reduce the number of devices required.

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