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Application to optimize the geometry of a parallel kinematics sun tracker

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Abstract. A detailed description is made of a software application that is designed to optimize a sun-tracker mechanism. It takes the main characteristics of any given installation site into account: geographical coordinates, inclination and orientation of the assembly platform. The desired result is a low-cost mechanism, which will have minimum-length construction elements, which will maximize coverage, and which will furthermore have the most energy efficient configuration. A multi-objective optimization technique (MOEAs) and numerical methods have successfully been applied to achieve these design objectives. In addition, the most computationally efficient kinematic model for the mechanism is analysed.

Keywords
Sun tracker, optimized design, parallel kinematics, Newton-Raphson, genetic algorithms, NSGA-II.

1. Introduction

The regulatory framework for photovoltaic energy production in Spain (R.D. 1578/2008 of 26th September [1] and R.D.L. 14/2010 of 23rd December [2]) makes the introduction of this technology mandatory:

a) for installations placed on roofs and building facades;

b) furthermore, the installations must have a solar tracker mechanism, preferably a dual-axis one.

However, few solar trackers exist that may be adapted to building roofs and facades. Existing designs are for installation in open terrain and their adaptation to the exteriors of buildings is no easy task [3].

Previous research [4] has described the design of two solar-tracker models that improve the distribution of the forces that they require, which makes these models more adaptable for roof-top installations. Moreover their construction elements are very simple, which means that the initial investment and subsequent maintenance needs are minimal.

This study analyzes one of them: *"Single drive solar tracker based on parallel kinematics"* patented under ES 2.331.721-B2 [5], which is shown below in fig. 1.

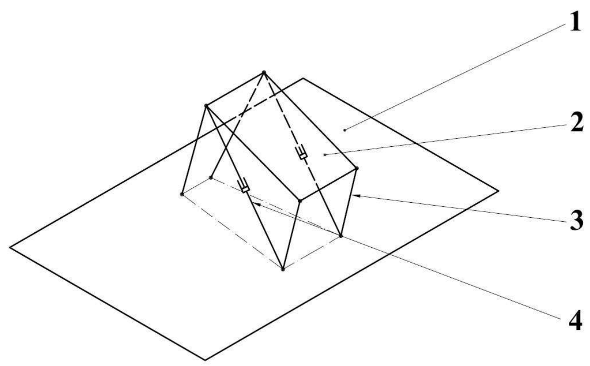

Fig. 1. Solar tracker based on parallel kinematics with a single drive. Source: patent ES 2.331.721-B2.

The mechanism consists of a base platform (1), a mobile platform (2) on which the photovoltaic panels may be installed, four bars of constant length (3), which are linked to the base platform (1) and to the mobile platform (2) by joints of two or three degrees of freedom, and two more elements of variable length (4) linked, also by joints of two or three degrees of freedom, to any previously described pair of elements, usually the mobile platform (2) and the base platform (1).

Its structure, based on parallel kinematics, requires complex study and the use of powerful mathematical tools and its optimization is complex. Hence, we developed a computer application based on Scilab (freeware numerical platform), which allows any user to optimize the geometric configuration of the mechanism according to the characteristics of the roof top. This application, which involves numerical techniques, applies the NSGA-II genetic algorithm.

2. Kinematic model

The kinematic study of the mechanism applies methods designed to obtain the mathematical expression that will define the kinematic variables -position, speed and acceleration-, according to the dimensional parameters of the mechanism under analysis and the initial kinematic variables [6].

Known as a parallel-kinematics mechanism, the mechanical structure is formed by a closed chain system, such that the final effector mechanism is connected to the base by at least two independent kinematic chains [7].

A kinematic model that describes the behavior of the mechanism was obtained. It is composed of six equations with six unknowns, two of which are the two degrees of freedom (d.o.f.) needed for solar tracking. The equations were formulated by identifying closed rings in the mechanism between a fixed point in the base platform (O) and a fixed point in the mobile platform (G), as shown in fig. 2.

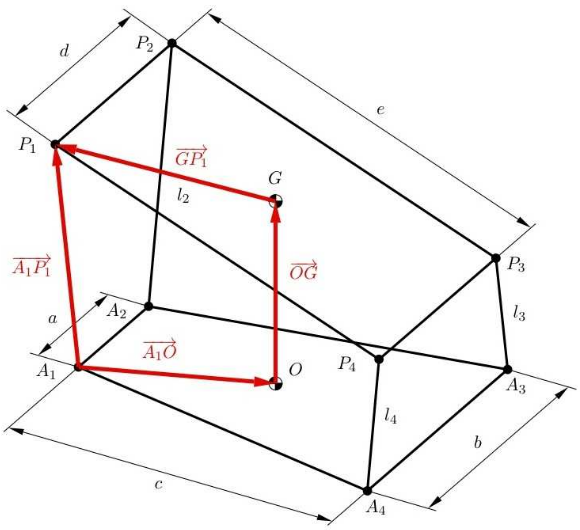

Fig. 2. Example of a closed chain in the mechanism.

Thus, for all the links between the base platform and the mobile platform it is obtained the following vectorial equation:

$$\overrightarrow{A_iP_i} = \overrightarrow{A_iO} + \overrightarrow{OG} + R^{XYZ}_{X'Y'Z'} \cdot \overrightarrow{GP_i} \quad i \in \{1,2,3,4\} \tag{1}$$

in which, the term $R^{XYZ}_{X'Y'Z'}$ represents the *"Rotation Matrix"* between the fixed reference system of the base and the mobile reference system, which is linked to the mobile platform. The Rotation Matrix is a function of the Euler Angles: *"Pitch"* (φ), *"Yaw"* (θ) and *"Roll"* (ϕ). It is obtained by consecutively multiplying the Rotation Matrix that corresponds to each axis of the reference system.

Finally, we have to enforce the condition that expresses the norm (Euclidean distance in the space) that the vector $\overrightarrow{A_iP_i}$ is equal to the known link length:

$$\left(\overrightarrow{A_iP_i}\Big|_X\right)^2 + \left(\overrightarrow{A_iP_i}\Big|_Y\right)^2 + \left(\overrightarrow{A_iP_i}\Big|_Z\right)^2 - l_i^2 = 0 \tag{2}$$

where, $i \in \{1,2,3,4\}$. The *"government chain equations"* or *"control equations"* are formed in the same way as the previous *"structural equations"*. These equations are functions of the mechanism drives and, in this case, may be expressed as:

$$\begin{cases} \overrightarrow{A_4P_1} = \overrightarrow{A_4O} + \overrightarrow{OG} + R^{XYZ}_{X'Y'Z'} \cdot \overrightarrow{GP_1} \\ \overrightarrow{A_3P_2} = \overrightarrow{A_3O} + \overrightarrow{OG} + R^{XYZ}_{X'Y'Z'} \cdot \overrightarrow{GP_2} \end{cases} \tag{3}$$

Calculation of the Euclidean norms of both vectors allows us to obtain the following system (4):

$$\begin{cases} \left(\overrightarrow{A_4P_1}\Big|_X\right)^2 + \left(\overrightarrow{A_4P_1}\Big|_Y\right)^2 + \left(\overrightarrow{A_4P_1}\Big|_Z\right)^2 - q_1^2 = 0 \\ \left(\overrightarrow{A_3P_2}\Big|_X\right)^2 + \left(\overrightarrow{A_3P_2}\Big|_Y\right)^2 + \left(\overrightarrow{A_3P_2}\Big|_Z\right)^2 - q_2^2 = 0 \end{cases} \tag{4}$$

where, variables q_1 and q_2 are the system drive values.

Adding the structural equations system (2) to the control equations system (4), a system composed by six equations with six unknowns could be obtained that describes the kinematic behavior of the mechanism. This model can solve Direct Kinematics and Inverse Kinematics.

3. Optimization

The optimization of the mechanism is intended to achieve three design objectives:

a) To maximize the solar tracker workspace, taking account of the limits of the solar radiation vector (fig. 3). These are defined by the summer and winter solstices, respectively.

b) To center the mechanism workspace in the area of interest. Ideally, it would be desirable for the centroid of the mechanism workspace and the centroid of the solar area defined in fig. 3 to be the same.

c) To minimize the lengths of the mechanism elements, including the base and mobile platforms. In this way we may minimize the amount of material needed to build the mechanism and therefore its cost.

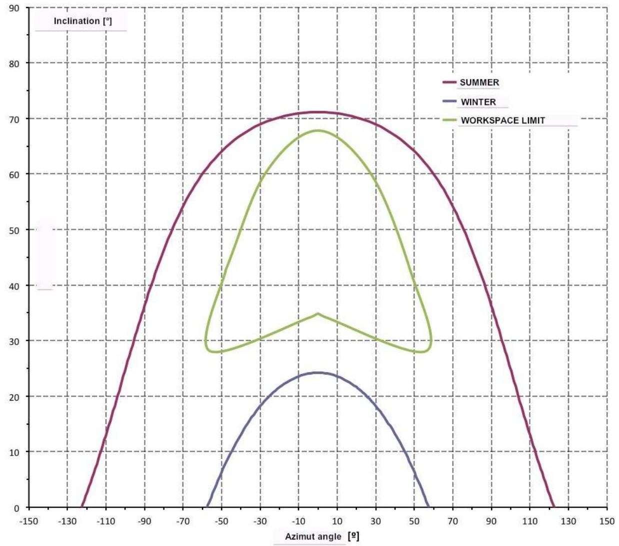

Fig. 3. Example of the mechanism workspace.

The following optimization functions should be defined, in order to satisfy the aims of the design:

a) Mechanism workspace area.

b) Euclidean distance between the centroid of the mechanism workspace and the centroid of the solar area.

c) Lengths of all the elements of the mechanism.

A multi-objective optimization technique is needed, as there are three optimization objectives to satisfy. Among the many multi-objective methods, some of the simplest and most efficient are based on Multi-Objective Evolutionary Algorithms (MOEAs). These algorithms are able to find multiple Pareto optimal solutions in only one simulation step. One of the most powerful algorithms in this field is NSGA-II, developed by Srnivas and Deb in 2002 [8]. This genetic algorithm is not-dominated and implements elite groups and a crowd operator. Briefly, its operation could be described as follows [9]:

a) The descendant population Q_t of size N is firstly created, using the parent population P_t (also of size N).

b) Both populations are combined to form R_t of size $2N$.

c) With a non-dominated arrangement, the R_t population is classified in different Pareto fronts.

d) A new population of size N is generated according to the previous non-dominated fronts.

e) A *"fitness function"* is assigned to each solution according to its level of non-dominance. This function should decrease during the optimization process.

f) A new descendant population Q_0 of size N is created with tournament selection, crossover and mutation.

g) The process is repeated iteratively.

Fig. 4 shows the Genetic Algorithm process graphically.

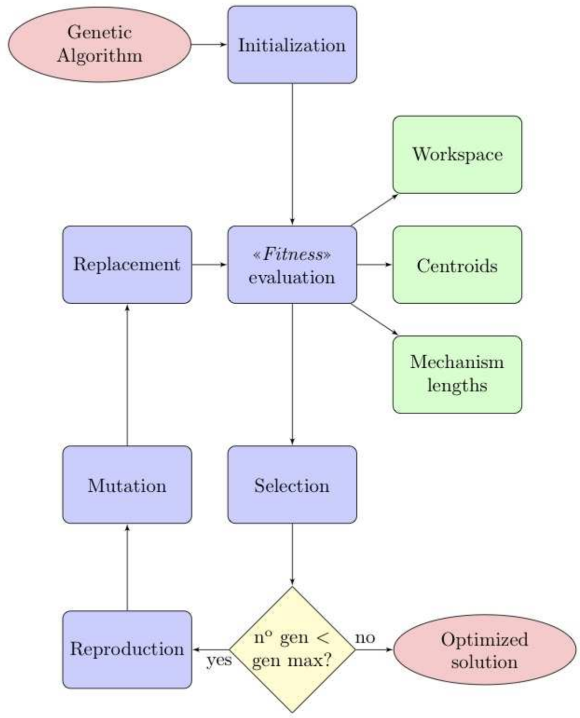

Fig. 4. Diagram of the Genetic Algorithm.

The NSGA-II algorithm is implemented in Scilab with the *"optim_nsga2"* function, the main input parameters that have to be taken into account are:

- **Population size:** size of the population of each stage. In this case its value was set at 100.

- **Generations:** number of generations to compute (equivalent to the number of iterations in the classic numerical optimization process). Its value was set at 10.

- **Mutation probability:** set at 0.1 as recommended in [10].

- **Crossover probability:** set at an optimal value of 0.7.

The values of these parameters appear to be the best suited for the case study at present. They provide good results with acceptable computation lapses.

The initial population of individuals is generated at random, taking into account physical restrictions and singularities.

The selection mechanism in this case is the tournament selection. With this operator the worst individuals are selected with lower probabilities than the best ones [9].

The crossover operator implemented in the Scilab function performs a convex combination between the parent individuals, taking into account both minimum and maximum bounds for each variable.

The mutation process is occasional, which means that each gene (bit) of the chromosome and its mutation process is independent from the rest. This is the most common mutation process used in general applications.

In the replacement stage a new generation is created from the junction of the previous one and the one created by the selection, crossover and mutation operators. Different fronts are extracted from this junction, selected according to the number of solutions that dominate them. The first one is the Pareto front. The new population is created including the different fronts, from the best to the worst, until the maximum population size is reached.

Finally it is important to understand that the NSGA-II algorithm implements the crowd operator. This special mechanism selects the most dispersed solutions from the last front in the new population. The larger the crowd distance from one solution to the rest of the front, the better, as it means that the concentration in that area is lower.

4. Numeric implementation

The problem that is presented involves the resolution of several systems of non-linear equations. In general, for these kinds of problems, we try to approximate the non-linear system to a linear system, but in this case that technique is not possible and we have to face the problem directly.

The most useful method for solving non-linear equations systems is the Newton-Raphson Method for several variables [11], a technique that generally entails quadratic convergence. As is known, the functional iteration procedure goes from the selection of $x^{(0)}$ to generate, for $k \geq 1$,

$$x^{(k)} = G\left(x^{(k-1)}\right) = x^{(k-1)} - J\left(x^{(k-1)}\right)^{-1} F\left(x^{(k-1)}\right) \qquad (5)$$

where, J is the Jacobian Matrix of the system. However, the Newton-Raphson Method for equation systems is very hard and can converge to local solutions of little interest or, on the contrary, can quickly increase the distance to the desired solution. Hence, the Maximum Slope Method [11] is recommended, also known as the Steepest Descent Method, to obtain more reliable initial approximations $x^{(0)}$.

5. Description of the application

The optimization process requires complex calculus, performed on a Scilab-based computer application (freeware, similar to Matlab) [10]. The program, shown in fig. 5, was developed as a modular one, in such a way that each module controls a specific function. We briefly describe the main modules below:

- **Data request module:** function that asks the user to input the location data (latitude, longitude and time zone GMT) and the characteristics of the installation surface (inclination and orientation).
- **Solar area module:** according to the location data, this module calculates the solar workspace delimitated by the summer and winter solstices (fig. 3).
- **Optimization module:** executes the NSGA-II algorithm, which optimizes the geometric configuration of the mechanism. The algorithm has to calculate the mechanism workspace limits, at each iteration, for every possible configuration. The mechanism workspace is calculated according to the previously explained kinematic model.

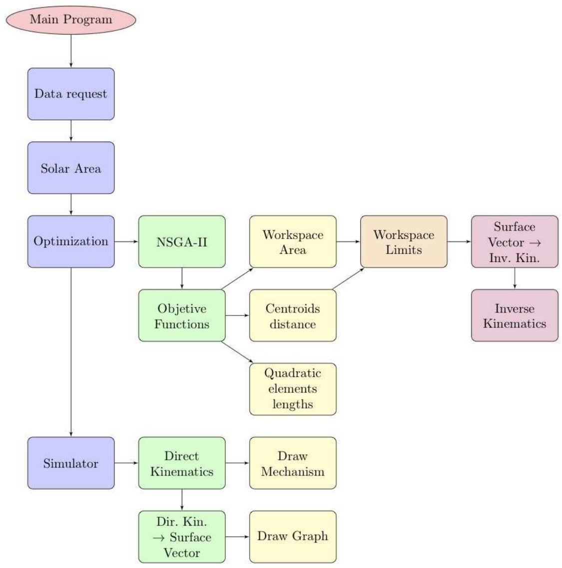

Fig. 5. Diagram of the application structure.

- **Simulation module:** once the optimal solution has been reached, the user is able to simulate that configuration, as shown in fig. 6. In that simulation, the movement of all the mechanism components in a 3D representation may be appreciated, as well as the traced trajectory of the mobile platform for the orientation and inclination coordinates.

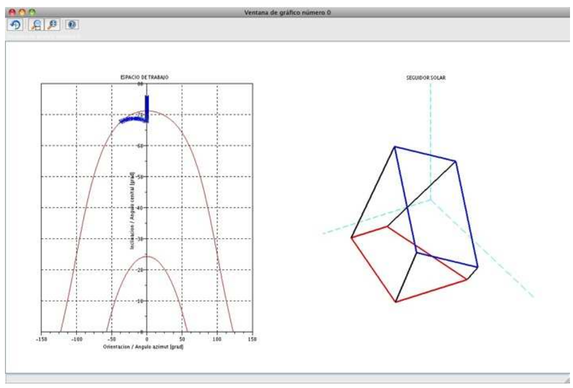

Fig. 6. Screenshot of the simulation window.

6. Conclusions

The legislation that regulates the production of photovoltaic energy in Spain makes the installation of new technology mandatory, in order to increase the technical, financial and environmental efficiency of such systems. This research has sought to optimize a design found in previous works, in such a way as to achieve those aims. Hence, in the first place, we have constructed a complete, computationally efficient, kinematic model using numerical techniques. On that basis, we were able to implement the NSGA-II optimization algorithm, currently one of the most useful methods at solving multi-objective optimization problems. The entire process has been integrated into computer software, which allows quick and efficient optimization of possible installations by any user.

Needless to say that the kinematic model that is implemented and the resolution process with numerical techniques can be adapted to any other mechanism based on parallel kinematics.

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