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Space Vector Fundamental Frequency Modulation Compared with the Selective Harmonic Elimination Method

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Abstract. A new modulation strategy for three-phase multilevel-converters, called Space Vector Fundamental Frequency Modulation will be presented. Derived from conventional Space Vector PWM, it was developed to overcome Selective Harmonic Elimination Method in mathematical complexity. Besides the harmonic optimization, the method offers a variable modulation index and is applicable for real-time applications. But in contrast to Selective Harmonic Elimination Method it is an approximation procedure. In this paper, both methods are compared, concerning several criteria such as voltage quality and mathematical complexity. Also, a classification with reference to EN 50160 is presented.

Key words

Multilevel-converter, harmonics, fundamental frequency, voltage quality, EN 50160

1. Introduction

Multilevel-converters synthesize a staircase voltage waveform. This leads to low switching frequencies and a high voltage quality. The distribution of voltage stress over several active elements allows the use of IGBTs instead of thyristors for high voltage applications, such as HVDC or FACTS. Switching every active element only once a load cycle (fundamental switching frequency), switching losses can be minimized. But, in case of a low number of voltage levels, fundamental frequency switching will cause significant low order harmonics.

Nonetheless, to achieve a high quality of the spectral characteristic the Selective Harmonic Elimination Method (SHE) was developed. Here, the switching angles are chosen in such way that the most disturbing harmonics are suppressed. A set of transcendental equations is derived, out of which the switching angles are calculated by the use of iterative and numerical methods. Due to its complexity the SHE is an offline-method and only suitable for a low number of levels.

To overcome these disadvantages the space vector fundamental frequency modulation (SVFFM) was derived from conventional space vector modulation [1]. Due to its simplicity it is suitable for a high number of levels and real-time applications. But as a disadvantage it is not possible to eliminate certain harmonics and the fundamental signal slightly differs from the reference signal.

Subsequent, both methods are explained and compared.

2. Selective Harmonic Elimination Method

SHE can work both, in PWM and fundamental switching frequency. Against the background of low switching losses the PWM strategy will not be outlined.

For each voltage level of a staircase voltage waveform two degrees of freedom are given: the switching angle and the amplitude. These degrees of freedom are used to eliminate single harmonics and to adapt the fundamental voltage to a reference voltage [2].

In fundamental frequency the low order harmonics dominate. Because of the 6-pulse character of the voltage waveform it is reasonable to eliminate for example the 5^{th} and 7^{th} harmonics. The switching angles and amplitudes of the voltage levels are calculated with iterative and numerical methods, e.g. Newton Raphson.

Usually multilevel-converters operate with voltage levels of the same amplitude to achieve voltage redundancies. These redundancies are used for voltage balancing. But as a result, one degree of freedom is relinquished.

Fig. 1 illustrates the output voltage waveform of a seven level converter in fundamental frequency. At defined angles θ the voltage levels are added. The Fourier series of a staircase waveform is given with (1).

$$u_n(\omega t) = \underbrace{\frac{4U_{dc}}{\pi \cdot n} \sum_n [\cos(n\theta_1) + ... + \cos(n\theta_s)]}_{b_n} \cdot \cos(n\omega t) \quad (1)$$

In (1) n is the ordinal number of the harmonic and s is coincident with the degrees of freedom, here three.

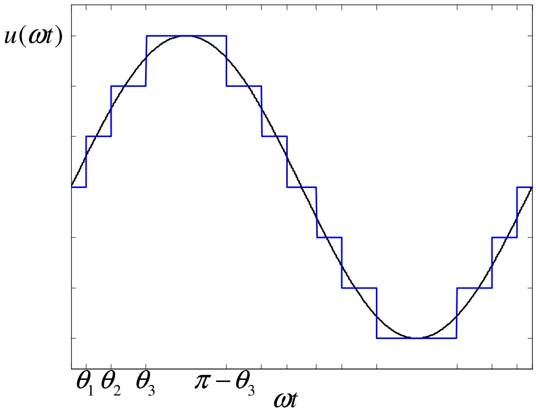

Fig. 1. Staircase voltage waveform

One degree of freedom is used to adapt the fundamental voltage to the sinusoidal reference voltage

$$u_1(\omega t) = \hat{U}^* \cdot \cos(\omega t) = m_a \cdot s \cdot U_{DC} \cdot \cos(\omega t)\,. \quad (2)$$

In (2) m_a is the modulation index.
The remaining two degrees of freedom are used to eliminate two harmonics, e.g. the 5th and 7th. With (1) and (2) the following set of equations results.

$$\begin{aligned} &\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) = 0 \\ &\cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) = 0 \\ &\frac{4U_{dc}}{\pi} \cdot \left(\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3)\right) = \hat{U}^* \end{aligned} \quad (3)$$

With additional voltage levels the set of equations can be enlarged to eliminate more and more harmonics. In general, depending on the number of levels L

$$\begin{aligned} n &= \frac{L-3}{2} \text{ für } L = 2k+1,\ k \in \mathbb{N} \\ n &= \frac{L-4}{2} \text{ für } L = 2k+2,\ k \in \mathbb{N} \end{aligned} \quad (4)$$

harmonics can be eliminated [2].

If the amplitudes of the levels differ from each other (1) is converted to (5).

$$u_n(\omega t) = \underbrace{\frac{4}{\pi \cdot n} \sum_n [U_{DC1} \cos(n\theta_1) + ... + U_{DCn} \cos(n\theta_s)]}_{b_n} \cdot \cos(n\omega t) \quad (5)$$

As a result (4) can be enlarged and

$$n = L - 2 \quad (6)$$

harmonics can be eliminated [2]. It must be pointed out, that the amplitudes of the voltage levels cannot be changed under operation. As a result these degrees of freedom get lost by variation of the modulation index.
The maximum modulation index $m_a = 1$ occurs, if all switching angles are set to zero. It is

$$m_a = \frac{\hat{U}^*}{\hat{U}_{max}} = \frac{\pi \hat{U}^*}{4 \cdot \sum_{i=1}^{s} U_{DCi}} \quad (7)$$

in the case of different voltage levels and

$$m_a = \frac{\pi \hat{U}^*}{4 s U_{DC}} \quad (8)$$

for equal voltage levels.

Despite approaches to reduce the mathematical complexity, e.g. [3], the set of equations is usually solved numerically. More than about five harmonics cannot be canceled by standard algorithms like Newton Raphson. Practically, precalculated switching angles for different modulation indices have to be stored in a look-up table.

3. Space Vector Fundamental Frequency Modulation

SVFFM bases on traditional SVPWM. In contrast to SVPWM it works strictly in fundamental frequency. By the use of several physical considerations, the spectral behavior is optimized. In contrast to SHE it is only applicable for three-phase multilevel converts.

Fig. 2 shows the space vector diagram of a 5-level converter. As the amplitudes of the voltage levels are equal, the triangles are equilateral. Here, the length of each side is set to one.

Each corner of a triangle equates to a certain three-phase output voltage, which is defined by a dedicated vector

$$\vec{\underline{U}} = U_R + \underline{a} \cdot U_S + \underline{a}^2 \cdot U_T \text{ with } \underline{a} = e^{j\frac{2\pi}{3}} \quad (9)$$

For example, the switching state (2, -1, 0) means that phase R is connected to $2U_{DC}$, phase S to $-U_{DC}$ and phase T to 0 V respectively ground. The appendant vector is given with

$$\vec{\underline{U}}_1 = 2 \cdot U_{DC} - \underline{a} \cdot U_{DC} = (\frac{5}{2} - \frac{\sqrt{3}}{2} j) \cdot U_{DC} \quad (10)$$

At this numerous redundancies are useable. So, the vector $\vec{\underline{U}}_1$ is also defined by the triplet (1, -2, -1).

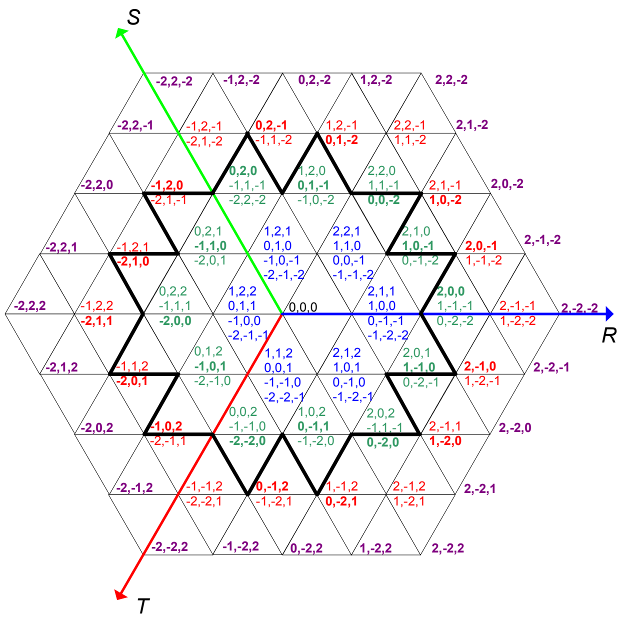

Fig. 2. Space vector diagram of a 5-level converter

An effective approach to drive a three phase multilevel-converter in fundamental frequency is making use of two concentric hexagons. Following a particular zig-zag line between two hexagons, as shown in Fig. 2, every phase voltage level is still only employed once a load cycle. At this, only selected switching states – the bold triplets – are permitted. Analogue to traditional multilevel space vector modulation, this fact allows generating arbitrary vectors by averaging the vectors of three adjacent switching states. To ensure fundamental frequency operation, some specialties have to be considered. In the first place, every triangle of adjacent states may only contain one reference point, meaning it may only be used once in each cycle. Otherwise, the same switching state would be used repeatedly as in high-frequency modulation. Furthermore, the last state of the previous vector must be the first state of the next vector. Fig. 3 illustrates the procedure for the generation of the two reference points *A* and *B*. In contrast to classical SVPWM, it is also possible to combine two small triangles to a large one to ensure fundamental frequency for high modulation indices (c.f. Fig. 4).

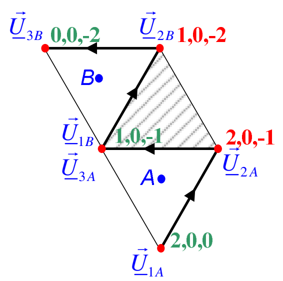

Fig. 3. Moving through the space vector diagram

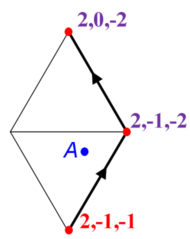

Fig. 4. Combination of two triangles to a single on to ensure fundamental frequency switching

Fig. 5 shows the phase to ground voltages of the example given in Fig. 2. The voltage amplitudes correspond with the bold triplets of the space vector diagram. To illustrate the coherence between the space vector diagram and the phase voltages the first triplet is marked in the figure. As can be seen, every single voltage level of the phases *R*, *S*, and *T* appears only once a space vector revolution (load period) and is increasing or decreasing by a single step. So, fundamental frequency switching is apparent.

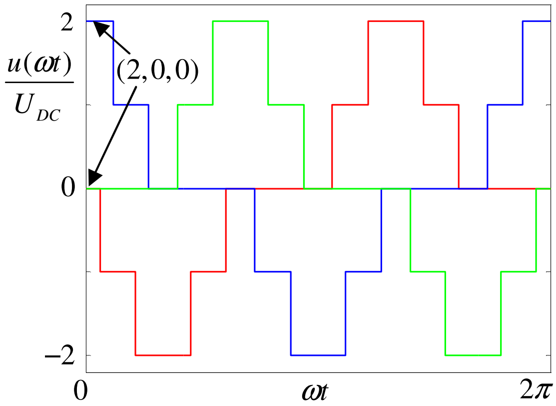

Fig. 5. Phase to ground voltages of the example given in Fig. 2

The individual duty cycle *T* of each employed state can be calculated as in high-frequency space vector modulation, e.g. as released in [4], [5].

Overall, this method allows the fundamental frequency generation of

$$n = (L-1)\cdot 3 \qquad (11)$$

reference points anywhere within the diagram. Analog to the generation of highly pulsating space phasors the spectrum is optimized by a symmetrical placement of the reference points [1],[6],[7].

The way through the space vector diagram is calculated successively with a user-friendly algorithm, which was released in [1]. Only the radius and the initial angle of the first reference point must be chosen. The modulation index of the SVFFM is defined by

$$m_a = \frac{r}{r_{\max}} \qquad (12)$$

and is nearly identical to (8).

SVFFM is an online modulation strategy for unlimited numbers of levels. But as it is not an exact method, no harmonics are eliminated. In the next chapter SVFFM and SHE are compared, which allows a classification of SVFFM.

4. Comparison of SHE and SVFFM

In the following a classification of both methods regarding EN 50160 will be presented. On basis of this analysis SHE and SVFFM will be compared with respect to several criteria such as voltage quality, variation of the modulation index and mathematical limitations.

Also an examination of performance versus benefits will be part of this chapter.

A. *Classification concerning EN 50160*

The European standard EN 50160 is the definite ruling concerning voltage quality. Purpose of the standard is the definition and characterization of the supply voltage with respect to several criteria, such as frequency, amplitude, waveform and symmetry. As standard voltage the agreed supply voltage is consulted – here the line to line voltage. To evaluate the voltage quality of both modulation strategies the voltage waveform has to be analyzed. The absolute maximum ratings of the harmonic components for the low and medium voltage level are given in Table I. For the high voltage level, the ratings are similar but in part still in consultation. As voltage qualities figure of merit *THD* is defined in the standard as

$$THD = \frac{\sqrt{\sum_{n=2}^{40} \hat{U}_n^2}}{\hat{U}_1} \cdot 100\% . \qquad (13)$$

Its upper limit in EN 50160 is 8.0 % [8].

Table I. – Absolute maximum ratings of the harmonic components with reference to EN 50160

odd harmonics				even harmonics	
multiple of 3		non multiple of 3			
n	$\frac{\hat{U}_n}{\hat{U}_1}$	n	$\frac{\hat{U}_n}{\hat{U}_1}$	n	$\frac{\hat{U}_n}{\hat{U}_1}$
5	6.0 %	3	5.0 %	2	2.0 %
7	5.0 %	9	1.5 %	4	1.0 %
11	3.5 %	15	0.5 %	6 - 24	0.5 %
13	3.0 %	21	0.5 %		
17	2.0 %				
19	1.5 %				
23	1.5 %				
25	1.5 %				

The use of the standard allows a practical benchmark of both modulation strategies. Also it becomes apparent which minimum requirements have to be fulfilled for a filterless converter layout.

The examinations have shown that for both modulation strategies at least eleven levels are necessary to meet EN 50160.

At first the results for SVFFM will be outlined. Fig. 6 illustrates the *THD* in dependence of the modulation index, for $m_a > 0.5$. It meets for the whole range the limit of EN 50160. The highest voltage quality with a *THD* of 2.53 % is given for $m_a = 0.830$.
Next the harmonics around $m_a = 0.830$ are illustrated in Fig. 7. As they also meet the standard a reasonable operating point is given with $m_a = 0.830$.
Fig. 8 shows the dedicated space phasor. Also the reference points are illustrated in the picture. It can be seen, that one reference point is always enclosed by one triangle (c.f. Fig. 3 and Fig. 4).

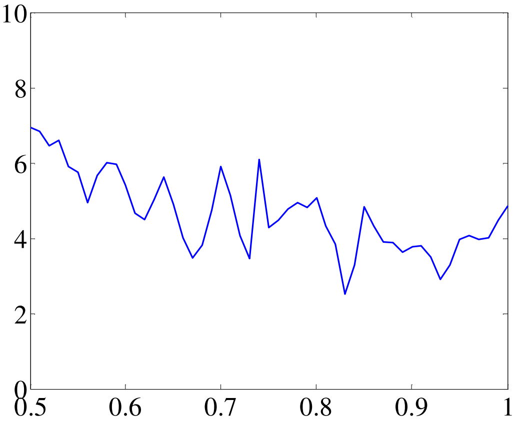

Fig. 6. *THD* in dependence of m_a (SVFFM)

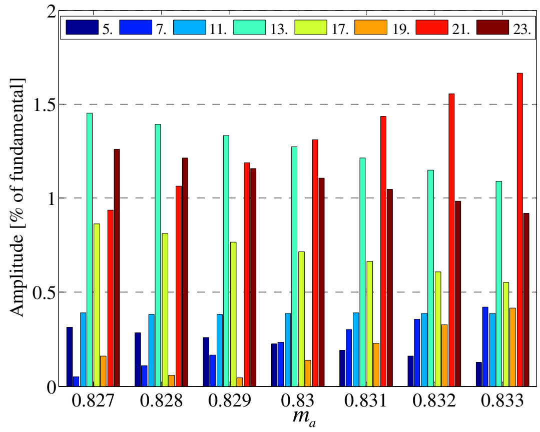

Fig. 7. Harmonics in dependence of m_a (SVFFM)

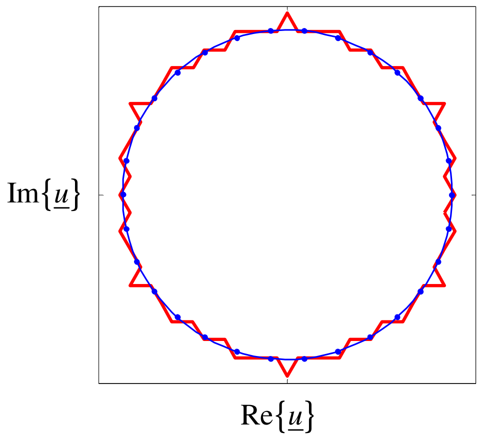

Fig. 8. Space phasor for m_a=0.830

Next, the same analysis is done for the SHE. With eleven levels it is possible to eliminate four harmonics. It is expediant to eliminate the lowest order harmonics, which are the 5th, 7th, 11th and 13th.
The *THD* in dependence of the modulation index is given in Fig. 9. By the use of a standard algorithm, here Newton Raphson, it is not possible to find solutions for the whole modulation range. One reason is under-modulation which results in a decrease of switching angles. Another one is that Newton Raphson does not converge for each modulation index. By the use of special optimization algorithms, such as [3], the permitted range of the modulatin index increases.

The highest voltage quality, where also the limits for the harmonics are met is around $m_a = 0.646$.

Fig. 10 shows the dedicated harmonics. It is an interesting fact, that with the modulation index one additional degree of freedom is given which allows the elimination of one more harmonic, as here the 17th at $m_a = 0.647$. At this point the *THD* (3.22 %) is significant higher compared to SVFFM. The reason is the increase of higher order harmonics.

In Fig. 11 and Fig. 12 the spectra of both modulation strategies are given. Whereas SVFFM has a positive impact on the entire spectrum, the effect of SHE is limited on certain harmonics. In fact the spectral behavior of SHE is not foreseeable.

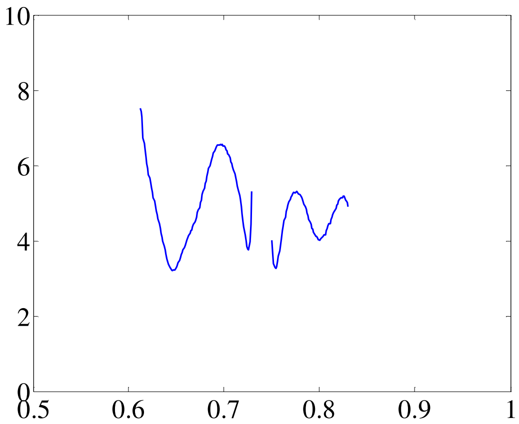

Fig. 9. *THD* by variation of m_a (SHE)

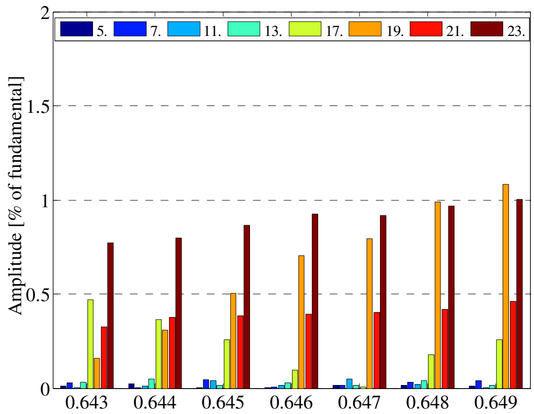

Fig. 10. Harmonics by variation of m_a (SHE)

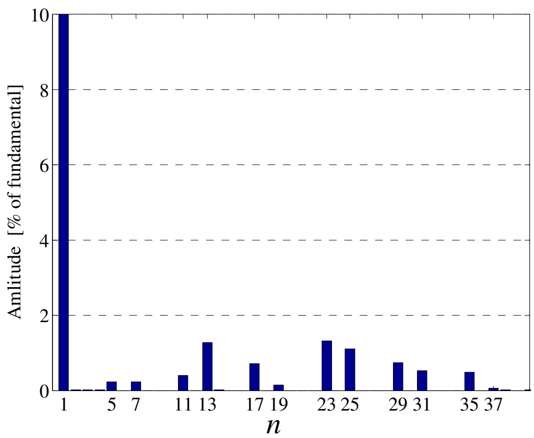

Fig. 11. Spectrum of the SVFFM for m_a=0.830

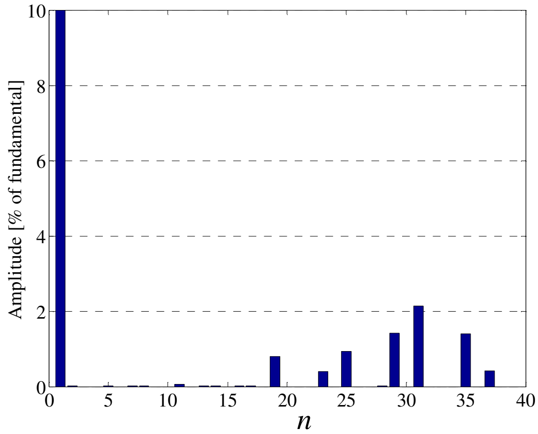

Fig. 12. Spectrum of the SHE for m_a=0.647

B. *Performance versus benefits*

Both methods allow meeting the standard EN 50160. Furthermore in this chapter the performance, benefits and application range of the methods shall be discussed.

The main disadvantage of SHE compared to other modulation strategies is its performance.

For real-time applications like drives precalculated switching angles have to be stored in a look-up table. Furthermore not the whole range of the modulation index is useable.

Especially for higher number of levels no solutions for the switching angles can be found. FACTs operate already with more than 20 levels and HVDC solutions are planned up to about 300 levels [9]. As result the application range of the SHE is mainly limited to static low level applications, e.g. photovoltaic.

To improve the voltage quality basic physical considerations can be used instead of complicated on- and offline methods. As outlined in chapter 4A a numerical enhancement of voltage quality is not necessary and often misleads. Analytical methods, like SVFFM, are useable for high numbers of levels and real-time applications.

The advantage of SHE is the elimination of certain harmonics. This could be helpful to avoid resonances between the converter and the voltage grid or to suppress abnormally disturbing harmonics.

A concluding comparison of both methods concerning several criteria is given in Table II.

Table II. – Comparison of SHE and SVFFM

	SHE	SVFFM
Analytical method	-	+
Suitable for real-time applications	x	+
Improvement of the voltage quality	x	+
Elimination of harmonics	+	-
Suitable for high number of levels	-	+
Suitable for single-phase	+	-
Suitable for three-phase	+	+

- no, + yes, x limited

5. Conclusion

Numeric modulation strategies for multilevel-converters, such as SHE, were developed to improve the spectral behavior. In fact, although it is possible to eliminate selected harmonics, voltage quality remains mainly unaffected. Because of the complex computation the scope for real-time applications is limited. In contrast, good enough switching angles can be figured out by basic physical and mathematical considerations. As a result no complicated on- / offline optimazion methods are necessary. As a real analytical method SVPWM was introduced. The method allows to improve the spectral behaviour. Contrary to SHE it is not possible to eliminate single harmonics. The user-friendly algorithm allows the application of this method for high levels and real-time applications. This makes SVPWM suitable for drive applications, FACTs and renewable energies.

References

[1] Mehlmann G., Zimmermann v. M., Piepenbreier B., Herold G., "Harmonic Optimization for Multilevel Converters Using Space Vector Fundamental Frequency Modulation", 14th European Conference on Power Electronics and Application (EPE), 2011

[2] Holmes Grahame D. and Lipo Thomas A., Pulse Width Modulation For Power Converters, IEEE Press, 2003

[3] Chiasson J. N., Tolbert L., McKenzie K. J., and Zhong D. "A New Approach to Solving the Harmonic Elimination Equations for a Multilevel Converter", IEEE Industry Applications Society Annual meeting, 2003

[4] Alonso O., Marroyo L., and Sanchis P., "A Generalized Methodology to Calculate Switching Times and Regions in SVPWM Modulation of Multilevel Converters", 9th European Conference on Power Electronics and Applications, 2001

[5] Khomfoi S. and Tolbert L., Multilevel Power Converters, Power Electronics Handbook, ch. 17 pp. 451-482, 2007

[6] Mehlmann G. and Herold G., "The generation of symmetrical space phasors compared with the selective harmonic elimination method of multilevel converters", Electric Power Quality and Supply Reliability (PQ2010), 2010

[7] Mehlmann G., Svensson F., Herold G., "18-pulse Cascaded-Multilevel-Converter" Electrical Power Quality and Utilization (IEEE EPQU2011), 2011

[8] European Committee for Electrotechnical Standardization CENELEC, "Voltage characteristics of electricity supplied by public electricity networks", European Standard EN 50160, June 2010

[9] Retzmann D., "Power Electronics with HVDC and FACTS – Solutions and Applications" (Leistungselektronik mit HGÜ und FACTS – Lösungen und Anwendungen), in EEI Koloquium, Erlangen, 12.01.2012 (in german)