

European Association for the Development of Renewable Energies, Environment and Power Quality (EA4EPQ)

Frequency Variations of Power System Due to Switching of Renewable Energy Sources

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Abstract. Power systems mainly containing intermittently operating renewable sources require load/frequency control which is performed at the AC transmission and distribution levels. This frequency control can be achieved by employing short- and long-term storage plants buffering and complementing renewable energy sources. A representative grid consists of a natural-gas-fired plant serving as frequency leader, longterm storage plant, wind-power farm with associated short-term storage plant for energy buffering, photovoltaic farm with associated short-term storage plant interconnected by a long transmission line to two load circuits. Transient analysis is performed with Mathematica solving the differential equation system for frequency variation. Power flow through the transmission line is limited by its impedance. The long transmission line must be segmented to achieve stability and the voltage must be controlled between segments of an 800km line. The renewable plants must be operated together with the storage plants in order to minimize frequency variations by smoothing the power output of renewable plants, achieving a step-wise control of the transmission-line power.

Key words

Renewable sources, frequency variations, stability, selection of transmission line

1. Introduction

Today frequency and voltage control predominantly occurs at the AC transmission level. In distributed systems with renewable sources these control functions take place both at the transmission and distribution levels. As the intermittency of renewable sources makes frequency and voltage control more difficult, short- and long-term storage plants for electric energy must be relied on to compensate. This involves an increase of switching actions which may impair the quality [1] of energy delivered to the consumer.

The following problems which may arise due to the above-mentioned transition are detailed for Germany. Depending upon the operating levels, 26 nuclear plants supplied in 2005 about 25–42% of the energy while renewable sources provided only 14% and hydro plants supplied about 3-7% [2]. The recent decision of phasing out [3] nuclear power put tremendous pressure on government to scale up renewable sources in the transmission and distribution system. According to [4] in

2011 "green energy" accounted for more than 20% for the first time and is predicted to reach about 35% by 2020. How can the grid accommodate such a relatively high percentage of intermittently operating sources without sacrificing network stability and supply reliability?

A recent article [5] indicated that the German grid might be stressed (e.g., large frequency and voltage deviations) even at light-load conditions if the 26 nuclear plants are permanently disconnected. Distributed central stations (nuclear, coal and natural gas) ensure that the power-flow control is relatively stable because energy must not be transmitted over long (e.g., 800km) lines to reach the consumer. These interconnected grids maximize stability, efficiency and reliability while minimizing investment costs. Decommissioning nuclear plants and minimizing the operation of coal-fired plants limits generation to intermittently operating sources, natural-gasfired and a small percentage of hydro plants. While natural-gas-fired plants generate less CO₂ than coal-fired plants, they rely on methane, and the leak thereof increases global warming much more than CO₂. The National Center for Atmospheric Research (NCAR) in Boulder, Colorado, estimates that natural-gas wells leak about 10 % of their production capability [6]. In Germany, the fact that wind-power (WP) plants are predominantly located in the north (mainly offshore in the North Sea) and photovoltaic (PV) plants are mainly located in the south means that the electric energy must be transmitted over a long distance (about 800km) resulting in unacceptably large frequency and voltage perturbations [5], [7] which lead to stability problems due to large transmission-line impedances and switching actions. This is why the technical community strongly advocates [8] construction of additional 3,000km of new power lines until 2025. The construction of large short- and long-term storage plants must accompany renewable sources [7], [9], [10]. To minimize stability problems, extra-high-voltage (EHV) AC lines (e.g., 735kV) are required if generating stations and loads are very far apart [5], [11], minimizing the perunit line impedance and maximizing the transmissionline section length for which stability can be maintained. Special problems arise that require compensation equipment, e.g., synchronous capacitors, inductive reactors, static VAr compensators, shunt and series capacitors, and flexible AC transmissions (FACTS) [12] to

control voltage and assure stability of energy transmission.

This paper calculates stress (e.g., large frequency variations, instability) in transmitting power from distant renewable sources to load centers. The development of the transmission-line infrastructure will become important in addition to the design of renewable sources and strategically located large storage plants [5], [7], [9], [10].

At light load (to fully utilize renewably generated energy) most central power stations such as coal and natural-gasfired plants will be operated at reduced levels. For example, wind energy from the north of Germany will be transmitted via 800km EHV-AC line to the south if there is not much insolation, and *vice versa*, if there is not much WP available. Note that outputs of PV and WP plants change rapidly (Fig. 5 of [9]). In this case the power system might be stressed from a frequency and reactive power point of view.

2. Grid with Renewable Sources

The objectives of operation of the grid with renewable sources and storage plants where a natural-gas-fired plant is the frequency leader are:

1) Modeling of a power system with renewable sources and storage plants and its frequency variation due to switching actions, if energy is transmitted from renewable sources to load centers 800km away with and without energy storage plants [11];

2) Determination of length of transmission line sections to ensure steady-state and dynamic stability;

3) Selection of time constants of conventional, renewable, and storage plants;

4) Operation of WP and PV plants and charging and discharging of energy storage plants must be such that the transmission line power changes only gradually. Frequency control within acceptable limits (49-51)Hz [13] can be achieved by compensating/complementing the output of PW and PV plants with those of storage plants.

A. Simplified Grid

Figure 1 illustrates a simplified grid [11] with predominantly renewable energy sources: WP farm in the north of Germany and associated short-term storage plant to compensate for the intermittent output of wind farm. Natural-gas and long-term storage plants are used for peak-power generation near the two load systems. In the south there is a PV farm with associated storage plant. Figs. 2a, and b illustrate the frequency control for either storage-plant operation or renewable-source operation. There are three modes of operation:

Mode #1: renewable sources (RS) and storage plants supply power to two load systems.

Mode #2: RS charge storage plants [9].

Mode #3: RS supply energy to two load systems.

If the transmitted power throughput across the AC transmission line is not changing slowly enough, stability problems set in due to discontinuous AC power transmission and associated frequency perturbations.

B. Transmission Line

For example, a three-phase, 735kV (line-to-neutral), 50Hz, 800km line has a reactance of $0.55\Omega/km$ [14] or $X_L = 440\Omega$ resulting at base apparent power S=1000MVA with the base impedance of $Z_{\text{base}} = (735)^2 / 1000 = 540\Omega$ in a per-unit reactance of X_{Lpu}=(440/540)=0.815pu. With a transformer reactance of X_{Tpu}=0.08pu the total line reactance is $X_{total} = X_{Lpu} + 2X_{Tpu} = (0.815 + 0.16)pu = 0.98pu$ @ 50 Hz. Transient analysis shows that this long transmission line will have to be subdivided into about five sections each having a per-unit reactance of X_{section} = X_{total}/5= 0.98/5 = 0.196 pu, and at the terminals of each section the voltage will have to be controlled to guarantee stability of transmission. The differential and algebraic equation system of Fig. 1 is listed in the Appendix, Eqs. 1-24. The transmission/tie line acts as an integrating component [9] enforcing $\Delta \omega(s) = \Delta \omega_1(s) = \Delta \omega_2(s)$. The differential and algebraic equations are solved with Mathematica software; s is the Laplace operator.

C. Limits on Stability of Transmission

Wind energy from the north to the south of Germany can be transmitted with EHV AC lines divided into five sections: one section has the per-unit reactance of X_{section}=0.196pu. Two scenarios are investigated: first (mode #1) when the WP plant, load ref₃ (s), and its associated storage plant, load ref₄(s), are supplying power via transmission line T/s to the two load systems, during which time the natural-gas-fired plant supplies load ref₁(s)=0.8pu power and the long-term storage plant supplies load ref₂(s)=0.4pu power to the two load systems defined by M_1 , M_2 , D_1 , D_2 , $\Delta P_{L1}(s)$, $\Delta P_{L2}(s)$ [11]. The time constants of the various plants (e.g., T_{CHi} for i=1 to 6) influence the stability of AC transmission. The input parameters load ref₁(s) to load ref₄(s) and [- $\Delta P_{storage4}(s)$], as defined in Fig. 1, are positive and their values depend on the power gains of the plants. Note, load $ref_1(s)$ must be nonzero because the natural-gas-fired plant is the frequency leader.

D. Power Output Wave Shapes of Renewable Sources

Mode #1: Figures 3a-d show that at X_{section}=0.196pu at large-step outputs of WP and associated storage plant permit a stable AC energy transmission to the two load systems in the south, defined by M_1 , M_2 , D_1 , D_2 , $\Delta P_{L1}(s)$, $\Delta P_{L2}(s)$ [11]. However, the angular frequency variations due to switching are large (0.25pu). At $X_{\text{section}_{\text{max}}} = 0.33$ pu instability sets in as indicated in Fig. 4 where load ref₅(s)= $\Delta P_{\text{storage6}}(s)$ =0. Reduced time constant T_{CH1}=1.4s reduces the stability (Fig. 5). For medium-step outputs of WP and associated storage plant the frequency variation due to switching is still large (0.07pu), as indicated in Fig. 6b. Best performance with lowest angular frequency variation due to switching (0.01pu) is obtained with small-step outputs of WP and associated storage plant, as depicted in Fig. 7b; however, instability results for X_{section}=0.33pu as indicated in Fig. 8.

Mode #2: If the WP plant large-step output (load ref₃(s) $\neq 0, \Delta P_{storage4}(s)$ =load ref₃(s)) charges the associated short-

term storage plant $\Delta P_{storage4}(s)$ large (0.05pu) angular frequency variations of Fig. 9a are obtained. For smallstep charging the angular frequency variation is smaller (0.02pu, see Fig. 10a). The spikes in the $\Delta\omega(t)=\Delta\omega_1(t)=$ $\Delta\omega_2(t)$ are mitigated by smoothing the output of the WP plant based on small-step operation. The transmission of energy from the south PV farm to the two load circuits in the north is similar as discussed above. Figures 11a-d illustrates the establishment of steady-state operation at small-step reference power load $ref_5(s)=L_{r5}(t)[pu]$, fre-





Figs. 2a, b. (a) Droop characteristics when two short-term storage plants (associated with PV and PW plants) are operating; (b) Droop characteristics when two renewable (PV and PW) plants are operating

quency variation with small-step outputs, and the transmission power $\Delta P_{tie}(s)$ when the PV farm and its associated short-term storage $\Delta P_{storage6}(s)$ supply power to the two load circuits. The functions load ref₁(s), load ref₂(s), $\Delta P_{storage6}(s)$ are positive, and load ref₅(s), load ref₆(s), (as defined in Fig. 1) are negative; their values depend on the power gains of the plants. Note, load ref₁(s) must be nonzero and positive because the natural-gas-fired plant is the frequency leader.



Fig. 3a. Angular frequency variation $\Delta\omega_1(t)$ [pu] at large-step WP plant output and $\Delta P_{storage4}(s) = -2$ {load ref₃(s)} with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0, X_{section}= 0.196pu, T_{CH1}=3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s) = 0.8pu$, and $\Delta P_{L2}(s)=0.6pu$ from 0-200s to establish steady state, mode #1



Figs. 3b, c. (b) Large-step reference power load ref₃(t)=L_{r3}(t) [pu] for WP plant output and $\Delta P_{storage4}(s) = -2 \{\text{load ref}_3(s)\}$ with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}(s)=\text{load ref}_5(s)=\text{load ref}_6(s)=0$; (c) angular frequency variation $\Delta \omega_1(t)$ [pu] at large-step WP output, $X_{section} = 0.196$ pu, $T_{CH1} = 3$ s, load ref₁(s)=0.8pu, load ref₂(s) =0.4pu, $\Delta P_{L1}(s)=0.8$ pu, and $\Delta P_{L2}(s)=0.6$ pu, transient operation from 200- 6000s, mode #1



Fig. 3d. Transmission-line power ΔP_{tie} [pu] at large-step WP plant output ref₃(t)=L_{r3}(t) [pu] and $\Delta P_{storage4}(s)$ = -2{load ref₃(s)} with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₅(s)=load

ref₆(s)=0, $X_{section}$ = 0.196pu, T_{CH1} =3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ =0.6pu, transient operation from 200-6000s, mode #1



Fig. 4. Angular frequency variation $\Delta\omega_1(t)$ [pu] at large-step WP plant output ref₃(t)=L_{r3}(t) [pu] and $\Delta P_{storage4}(s)$ = -load ref₃(s) with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₅(s)=load ref₆(s)=0, $X_{section}$ = 0.33pu, T_{CH1}=3s, load ref₁(s)= 0.8pu, load ref₂(s)= 0.4 pu, $\Delta P_{L1}(s)$ = 0.8pu, and $\Delta P_{L2}(s)$ =0.6pu, transient operation from 200-6000s, mode #1



Fig. 5. Angular frequency variation $\Delta\omega_1(t)$ [pu] at large-step WP plant output ref₃(t)=L_{r3}(t) [pu] and $\Delta P_{storage4}(s)$ = -2 {load ref₃(s)} with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0, X_{section}= 0.196pu, T_{CH1}=1.4s, load ref₁(s) = 0.8pu, load ref₂(s)= 0.4pu, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ =0.6pu from 0-200s to establish steady state, mode #1



Figs. 6a,b. (a) Medium-step reference power load ref₃(t)=L_{r3}(t) [pu] for WP plant output and $\Delta P_{storage4}(s)$ = -load ref₃(s) with delay of 0.5s, load ref₄(s)=0, and $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0; (b) angular frequency variation $\Delta \omega_1(t)$ [pu] at medium-step WP output, $\Delta P_{storage4}(s)$ = -load ref₃(s), load ref₄(s) = 0, $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0, X_{section}=0.196pu, T_{CH1} = 3s, load ref₁(s)=0.8pu, load ref₂(s)= 0.4pu, $\Delta P_{L2}(s)$ =0.6pu, transient operation from 200-6000s, mode #1



Fig. 6c. Transmission-line power ΔP_{tie} [pu] at medium-step WP plant output ref₃(t)=L_{r3}(t) [pu] and $\Delta P_{storage4}$ (s)= -load ref₃(s) with delay of 0.5s, load ref₄(s)=0, $\Delta P_{storage6}$ (s)=load ref₆(s)=load ref₅(s)=0, $X_{section}$ = 0.196pu, T_{CH1} =3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, ΔP_{L1} (s) = 0.8pu, and ΔP_{L2} (s)=0.6pu, transient operation from 200-6000s, mode #1



Figs. 7a,b. (a) Small-step reference power load ref₃(t)=L_{r3}(t) [pu] for WP plant and $\Delta P_{storage 4}(s) \approx$ -load ref₃(s) with no delay, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0; (b) angular frequency variation $\Delta \omega_1(t)$ [pu] at small-step WP output and $\Delta P_{storage 4}(s) \approx$ -load ref₃(s), load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₅(s)=0, $X_{section} = 0.196$ pu, $T_{CH1}=3$ s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s)=0.8$ pu, and $\Delta P_{L2}(s)=0.6$ pu, transient operation from 200-7000s, mode #1



Fig. 7c. Transmission-line power ΔP_{tie} [pu] at small-step WP plant output and $\Delta P_{storage 4}(s)\approx$ -load ref₃(s) with no delay, load ref₄(s)=0, $\Delta P_{storage6}(s)=$ load ref₆(s)=load ref₅(s)=0, X_{section}=0.196 pu, T_{CH1}=3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s)=0.8$ pu, and $\Delta P_{L2}(s)=0.6$ pu, transient operation from 200-7000s, mode #1



Fig. 8. Angular frequency variation $\Delta \omega_1(t)$ [pu] at small-step WP plant output and $\Delta P_{storage 4}(s) \approx$ -load ref₃(s) with no delay, load

ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₆(s)= load ref₅(s)=0, $X_{section} = 0.33$ pu,T_{CH1}=3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s) = 0.8$ pu, and $\Delta P_{L2}(s)=0.6$ pu, transient operation from 200 – 7000s, mode #1



Figs. 9a,b. (a) Angular frequency variation $\Delta\omega_1(t)$ [pu]; (b) transmission-line power ΔP_{tie} [pu] at large-step WP plant output charges the associated short-term storage plant with $\Delta P_{storage4}(s)$ =load ref₃(s) with 0.5s delay, load ref₄(s)=0, $\Delta P_{storage6}(s)$ = load ref₆(s)=load ref₅(s)=0, X_{section}=0.196pu, T_{CH1}=3s, load ref₁(s) = 0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ = 0.6pu, transient operation from 200-7000s, mode #2



Figs. 10a,b. (a) Angular frequency $\Delta\omega_1(t)$ [pu] variation; (b) transmission-line power ΔP_{tie} [pu] at small-step WP plant output charges the associated short-term storage plant with $\Delta P_{storage4}(s)$ =load ref₃(s) with 1s delay, load ref₄(s)=0, $\Delta P_{storage6}(s)$ =load ref₆(s)=load ref₅(s)=0, $X_{section}$ =0.196pu, T_{CH1} =3s, load ref₁(s)=0.8pu, load ref₂(s)=0.4pu, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ =0.6pu, transient operation from 200-7000s, mode #2



Fig. 11a. Angular frequency variation $\Delta \omega_1(t)$ [pu] at small-step PV plant output ref₅(t)=L_{r5}(t) [pu] and $\Delta P_{storage6}(s)$ = -load ref₅(s), load ref₆(s)=0, $\Delta P_{storage4}(s)$ =load ref₄(s)=load ref₃(s)=0,

 $X_{section}{=}0.196pu,~T_{CH1}{=}3s,~load~ref_1(s){=}0.2pu,~load~ref_2(s){=}0,~\Delta P_{L1}(s){=}0.8pu,~and~\Delta P_{L2}(s){=}0.6pu$ from 0-200s to establish steady state, mode #1



Figs. 11b,c. (b) Small-step reference power load ref₅(t)=L_{r5}(t) [pu] for PV plant and $\Delta P_{storage6}(s)$ = -load ref₅(s), load ref₆(s)=0, $\Delta P_{storage4}(s)$ =load ref₄(s)=load ref₃(s)=0; (c) angular frequency variation $\Delta \omega_1(t)$ [pu] at small-step PV output, $\Delta P_{storage6}(s)$ = -load ref₅(s), load ref₆(s)=0, $\Delta P_{storage4}(s)$ =load ref₄(s)=load ref₃(s)=0, $X_{section}$ =0.196pu, T_{CH1} =3s, load ref₁(s)=0.2pu, load ref₂(s)=0, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ = 0.6pu, transient operation from 200-7000s, mode #1



Fig. 11d. Transmission-line power ΔP_{tie} [pu] at small-step PV plant and $\Delta P_{storage 6}(s)$ = -load ref₅(s), load ref₆(s)=0, $\Delta P_{storage4}(s)$ = load ref₄(s)=load ref₃(s)=0, X_{section}=0.196pu, T_{CH1}=3s, load ref₁(s) =0.2pu, load ref₂(s)=0, $\Delta P_{L1}(s)$ =0.8pu, and $\Delta P_{L2}(s)$ =0.6pu, transient operation from 200-7000s, mode #1

According to the newspaper [8] "Financial Times Deutschland", DC transmission lines are planned to avoid stability problems associated with long AC lines and result in smaller line voltage drops (no reactive drop). DC lines could be mounted on existing towers/pylons presently used for AC transmission lines to reduce costs and avoid protests by citizens. Although DC lines solve the stability problem associated with AC lines, DC lines still require short-term and long-term storage to control the frequency at the AC load side by enforcing power balance. DC lines are known for their voltage distortions, thus harmonic filters would also be required.

3. Conclusions

Frequency control can be achieved by employing shortand long-term storage plants complementing output of renewable sources by small step-wise control of the AC transmission-line power. Stable AC transmission results by appropriate power management: wave shaping of power outputs, appropriate transmission line segmentation, and selection of plant time constants.

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Appendix: Equations for Fig. 1		
System #1:	ε_{11} =load ref ₁ - $\Delta \omega_1/R_1$,	(1)
	$\Delta P_{valve1} + T_{G1} d(\Delta P_{valve1})/dt = \varepsilon_{11}$	(2)
	$\Delta P_{mechl} + T_{CHl} d(\Delta P_{mechl})/dt = \Delta P_{valvel}$	(3)
$\epsilon_{12} = \Delta P_m$	$ech_1 - \Delta P_{L1} - \Delta P_{tie} + \Delta P_{mech_3} + \Delta P_{mech_4} - \Delta P_{storage_4}$, (4)
	$\Delta \omega_1 D_1 + M_1 d(\Delta \omega_1)/dt = \varepsilon_{12}.$	(5)
Coupling (tie, i	<i>transmission) network:</i> $(1/T)d(\Delta P_{tie})/dt = \varepsilon_3$,	(6)
	$\varepsilon_3 = \Delta \omega_1 - \Delta \omega_2.$	(7)
System #2:	ε_{22} =load ref ₂ - $\Delta\omega_2/R_2$,	(8)
	$\Delta P_{valve2} + T_{G2} d(\Delta P_{valve2})/dt = \epsilon_{22},$	(9)
	$\Delta P_{mech2} + T_{CH2} d(\Delta P_{mech2})/dt = \Delta P_{valve2},$	(10)
$\varepsilon_{21} = \Delta P_{mech2} - \Delta P_{L2} + \Delta P_{tie} + \Delta P_{mech5} + \Delta P_{mech6} - \Delta P_{storage 6}$, (, (11)
	$\Delta \omega_2 D_2 + M_2 d(\Delta \omega_2)/dt = \varepsilon_{21}.$	(12)
System #3:	ε_{33} =load ref ₃ - $\Delta\omega_1/R_3$,	(13)
	$\Delta P_{valve3} + T_{G3} d(\Delta P_{valve3})/dt = \varepsilon_{33},$	(14)
	$\Delta P_{\text{mech3}} + T_{\text{CH3}} d(\Delta P_{\text{mech3}})/dt = \Delta P_{\text{valve3}}.$	(15)
System #4:	ε_{44} =load ref ₄ - $\Delta\omega_1/R_4$,	(16)
	$\Delta P_{valve4} + T_{G4} d(\Delta P_{valve4})/dt = \varepsilon_{44},$	(17)
	$\Delta P_{mech4} + T_{CH4} d(\Delta P_{mech4})/dt = \Delta P_{valve4}.$	(18)
System #5:	ε_{55} =load ref ₅ - $\Delta\omega_2/R_5$,	(19)
	$\Delta P_{valve5} + T_{G5} d(\Delta P_{valve5})/dt = \varepsilon_{55},$	(20)
	$\Delta P_{\text{mech5}} + T_{\text{CH5}} d(\Delta P_{\text{mech5}})/dt = \Delta P_{\text{valve5}}.$	(21)
System #6:	ε_{66} =load ref ₆ - $\Delta\omega_2/R_6$,	(22)
	$\Delta P_{valve6} + T_{G6} d(\Delta P_{valve6})/dt = \varepsilon_{66},$	(23)
	$\Delta P_{\text{mech6}} + T_{\text{CH6}} d(\Delta P_{\text{mech6}})/dt = \Delta P_{\text{valve6}}.$	(24)