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Mathematical approach to the characterization of daily energy balance in autonomous photovoltaic solar systems

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Abstract.

Stochastic simulation methods are normally extended as the only available to assess the reliability of the PV system implies the generation, for an extended period of time, of the main state variables of the physical equations describing the energy balance of the system, that is, the energy delivered to the load and the energy stored in the batteries. Most of these methods consider the daily load as a constant over the year and control the variables indicating the reliability associated with the supply of power to the load. Furthermore, these methods rely on previous random models forgenerating solar radiation data and, since the approximations of the simulation methods are asymptotic, when more precise reliability indicators are required, the simulation period needs to be extended. This paper presents a mathematical methodology to address the daily energy balance without resorting to simulation methods.

Key words

SAPV, LLP, Aguiar matrixes, PV sizing.

1. Introduction

The Loss of Load Probability, LLP, is the reliability index given by most of the stand-alone photovoltaic system (SAPV) sizing methods [1-10]. This index represents the ratio of the global deficit to the global energy demand, both considered throughout the installation operating time.

$$LLP = \frac{\int_t Energy\ deficit}{\int_t Energy\ demand} \qquad (1)$$

On the other hand, SAPVs can be characterized by means of:

- the photovoltaic generator capacity, $C_A(\beta)$: defined as the ratio or quotient between the average daily energy produced by the generator and the average daily energy consumed by the load (equation 2).
- the accumulator capacity, C_S: defined as the maximum energy that can be extracted from the accumulator or the useful storage capacity of the accumulator, C_U, divided by the energy consumption of the load (equation 3)

$$C_A(\beta) = \frac{\eta_G A_G \overline{H}_{g\beta}}{L} \qquad (2)$$

$$C_S = \frac{C_U}{L} = \frac{PD_{\max} C_B}{L} \qquad (3)$$

In these equations A_G and η_G are the area and conversion efficiency of the photovoltaic generator respectively, $\overline{H}_{g\beta}$ is the monthly mean value of daily irradiation on the plane of the array, L is the mean daily energy load, $PD_{\max}$ is the maximum allowable discharge depth of the battery (dimensionless) and C_B is the nominal capacity of the battery [1].

Thus, when sizing SAPVs it is important to determine the pair of C_A and C_S values leading to a given value of LLP with a minimum cost.

In a previous paper, the authors presented a mathematical method to characterize the variables associated with the energy balance of a SAPV [12]. The main feature of this novel sizing technique is that it is not based on simulation so it does not have its disadvantages, such as the dependency on the number of iterations to get accurate results. To the contrary, this method is based on the estimation of the useful energy stored in the battery, B, by means of its probability density function.

Being B_i the useful energy stored in the battery before the sunrise of the day i and considering that the battery state is recorded at the beginning of the solar day, it will range from zero (discharged battery) to C_U (maximum energy storage):

$$B_i = \begin{cases} C_U & if \quad B_i^{'} \geq C_U \\ B_i^{'} & if \quad 0 < B_i^{'} < C_U \\ 0 & if \quad B_i^{'} \leq 0 \end{cases} \tag{4}$$

In equation (4) $B_i^{'}$ is an auxiliary variable which represents the net energy balance required to meet adequately the demand and it can be written as the sum of two stochastic variables: the energy stored in the battery or the state of the battery at the end of consumption on day $(i-1)$, $B_{i\text{-}1}$, and the net energy gain of the battery during the day i, E_i, that is, the difference between the incoming energy, $\eta_G A_G H_{g\beta i}$, and the energy demand on the energy demand during the day i, L_i:

$$B_i' = B_{i\text{-}1} + E_i = B_{i\text{-}1} + \left(\eta_G A_G H_{g\beta i} - L_i \right) \tag{5}$$

According to equation (4), the B probability density function is given by equation (6):

$$f(B) = p_0 \cdot \delta(0) + g(B) + p_f \cdot \delta(C_U) \tag{6}$$

where p_0 and p_f are, respectively, the probability of an empty $(B=0)$ and full $(B=C_U)$ battery and $\delta(0)$ and $\delta(C_U)$ represent the Dirac deltas centred on the possible extreme energy levels of the battery and $g(B)$ is a continuous function defined between the physical limits of the battery, 0 and C_U. In order to automatize the process and dividing the dominium of the $f(B)$ function, $[0, C_U]$, into n_B intervals of equal length, the probability of B belonging to an interval $k(k=1,2,3,...,n_B)$ is given by p_{Bk}.

As explained in the previous paper, this probability depends on:

- The relation between $H_{g\beta}$ and this variable in a previous day, $H_{g\beta(i-1)}$: this relation can be estimated following the method proposed by Aguiar [11] for the relation between H_{gi} and $H_{g(i-1)}$ but recalculating the limits and lengths of the $H_{g\beta}$ class intervals as a function of the tilt angle, β [2,3].
- The geometrical area correction factors, $m(j,k,q)$, that represents the probability of $B_i^{'}$ being included in its j interval when B_{i-1} and E_i belongs to the k and q intervals respectively. This correction factors are based on equation (5) and the geometrical representation of B_{i-1} and E_i.
- The E probability conditioned to the B occurrence in the previous day, $p(q/k)$: Due to the fact that the state of the battery, B, depends on the net energy gain, E, both variables are probabilistic interdependent and the E probability conditioned to the B occurrence in the previous day is needed.From Bayes theorem, this conditioned probability, $p(q/k)$, is given by equation (7)

$$p^i(r/j) = \frac{\sum_{k,q} p_{Bk}^{i-1} m(j,k,q)\, p^{i-1}(q/k)\, p_A(q \rightarrow r)}{\sum_{k,q} p_{Bk}^{i-1} m(j,k,q)\, p^{i-1}(q/k)} \tag{7}$$

where $p_A(q \rightarrow r)$ is identified with the a_{qr} element of the Markov transition matrix, given by Aguiar [11], and represents the probability of the solar radiation changing from a q state in a particular day to a state r in the following day.

Taking into account all these dependences, equation (8) provides the probability of B' belonging to a j interval in a day i:

$$p_{Bj}^i = \sum_{k,q} p_{Bk}^{i-1} m(j,k,q) p^{i-1}(q/k) \tag{8}$$

From these probabilities and taking into account that the $j=0$ interval of B' represents a situation of empty batteries, that is, a deficit, $\overline{d}^i$, the LLP can be calculated as the ratio of the sum of the expected deficits, $\overline{d}^i$,for a number of days to the sum of demands, L^i:

$$LLP = \frac{\sum_{i=1}^{365} \overline{d}^i}{\sum_{i=1}^{365} L^i} \tag{9}$$

In this paper, this novel method proposed is applied to an example of SAPV sizing problem in order to describe how to apply the method and to point out its main features and advantages.

2. Mathematical method to a study application of the case

The methodology described is applied to the sizing example proposed by Posadillo and López [3] for a photovoltaic installation with monthly variable demands and panel inclinations. Specifically, it is a 12 V photovoltaic installation located in Cordoba (Spain) (ϕ=37.85ºN; Lon=4.48º W), with a three annual position solar traking. It must satisfy the daily demands listed in Table 1. This demand profile matches a constant monthly demand but a variable seasonal load which could be the representative profile of a Mediterranean farm.

Table 1. Monthly loads and collector inclinations for the sizing method application example mean clearness indexes for Cordoba (Spain).

Month	L (kWh/day)	β(degree)	KT
January	3	45	0.46
February	4	45	0.51
March	4	45	0.50
Aril	5	30	0.49
May	8	20	0.48
June	9	20	0.59
July	9	20	0.64
August	9	20	0.64
September	5	30	0.58
October	5	30	0.49
November	5	45	0.46
December	3	45	0.42

As a starting point, a photovoltaic installation consisted of 20 modules of 80 Wp with a maximum power point traking and 30 Pb-acid accumulator cell batteries system of 600 Ah, 2V and a discharge depth of 50% is analysed. Accordingly, the effective area of the installation, $A_G\eta_G$, will be given by equation (9):

$$A_G\eta_G = 20\cdot\frac{80\,Wp}{1000\,W/m^2} = 1.6m^2 \tag{9}$$

and the battery useful storage capacity will be given by equation (10):

$$C_U = 0.5\cdot 30\cdot 2V\cdot 600\,Ah\cdot 3600\,(s/h) = 64\,800\,000\,J = 18\,kWh \tag{10}$$

Once C_U and $A_G\eta_G$ have been calculated, it is necessary to determine the extreme values of the subintervals considered for the variables involved in the study, that is, B, B' y E.

The method presented considered that the variation of E can be described as a Markov process equal to the one of Aguiar [11]. Thus, the extreme values of the E variable, that according to equation (5) depend on the extreme values of $H_{g\beta}$, can be estimated from Aguiar matrixes that depend, in turn, on the typical clearness indexes of the emplacement. In that way, for the example analysed, the Aguiar matrix corresponding to each month will be determined by the typical clearness indexes of Cordoba listed in table 1

From these clearness indexes and the suitable Aguiar matrix, it is possible to determine, for each month, the minimum and maximum values of the dominium of the variable E, that is, $E_{\min}$ and $E_{\max}$ respectively (table 2). In addition, in order to use the probability transition values of Aguiar [11], it is necessary to divide the dominium of E into 10 intervals. According to that, table 3 shows, together with $E_{\min}$ and $E_{\max}$, the nine intermediate values, denoted by the subindex q, that divide the dominium of E into intervals.

Table 2. Frontier values, in kWh, of the intervals considered for the variable E.

M	$E_{\min}$	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	$E_{\max}$
Jan	-2.67	**-2.22**	**-1.72**	-1.12	-0.42	0.48	1.62	3.01	4.60	6.21	7.55
Feb	-3.55	**-2.95**	**-2.30**	-1.58	-0.76	0.23	1.43	2.84	4.42	6.02	7.39
Mar	-3.42	-2.64	-1.83	-0.96	-0.02	1.03	2.22	3.57	5.03	6.50	7.82
Apr	-4.25	-3.21	-2.17	-1.10	-0.01	1.12	2.30	3.54	4.81	6.09	7.32
May	-7.13	-5.92	-4.72	-3.52	-2.31	-1.10	0.11	1.33	2.55	3.77	4.98
Jun	-8.50	-7.10	-5.71	-4.32	-2.94	-1.58	-0.24	1.09	2.40	3.72	5.05
Jul	-8.07	-6.66	-5.26	-3.85	-2.46	-1.07	0.30	1.67	3.04	4.42	5.79
Aug	-8.15	-6.87	-5.58	-4.27	-2.95	-1.60	-0.22	1.19	2.60	3.97	5.35
Sep	-4.63	-3.62	-2.59	-1.52	-0.39	0.82	2.14	3.56	5.05	6.48	7.87
Oct	-4.48	-3.76	-3.02	-2.23	-1.36	-0.39	0.72	1.98	3.36	4.74	5.98
Nov	-4.64	-4.14	-3.60	-2.97	-2.23	-1.30	-0.15	1.25	2.83	4.44	5.79
Dec	-2.66	-2.28	-1.87	-1.38	-0.80	-0.07	0.84	1.96	3.29	4.74	6.16

On the other hand, table 3 shows the intervals of the variables B and B', represented by the indexes k and j respectively. These intervals do not depend on the month and remain constant along the year.

To start the recursive method, as Aguiar proposes in his method [11], the probability density function for the first day is considered known. Specifically, the batteries are supposed to be full in the first day. According to that, the probability of the $k=11$ interval of B is equal to the unit whereas the probability of the rest of the intervals is considered null. On the other hand, since information of the previous days is not available, the matrix C, consisted of the elements $C_{kq} = p\left(q/k\right)$ that represent the probability of E conditioned to B (equation 7), is initialized for the day zero as a uniform probability distribution $C_{kq} = p\left(q/k\right) = 1/10$.

Table 3. Frontier values, in kWh, of the intervals considered for the variables B and B'.

B' Intervals			B Intervals		
j Index	Lower limit	Upper limit	k Index	Lower limit	Upper limit
0	-∞	0.0	0	0.0	0.0
1	0.0	1.8	1	0.0	1.8
2	1.8	3.6	2	1.8	3.6
3	3.6	5.4	3	3.6	5.4
4	5.4	7.2	4	5.4	7.2
5	7.2	9.0	5	7.2	9.0
6	9.0	10.8	6	9.0	10.8
7	10.8	12.6	7	10.8	12.6
8	12.6	14.4	8	12.6	14.4
9	14.4	16.2	9	14.4	16.2
10	16.2	18.0	10	16.2	18.0
11	18.0	∞	11	18.0	18.0

Figure 1 explains the basis of the geometrical estimation of the variable B' as the sum of the two stochastic variables B and E (equation 5). In this figure, the occurrence probabilities of both addends for the first day, that is, B_0 and E_1, are represented. The intersection between their interval frontiers sets up the rectangular clusters the plane $\left(B_0, E_1\right)$ is initially divided into. All this clusters are rectangular except for the ones corresponding to $B_0 = 0$ and $B_0 = C_U$ that are lineal segments. However, in order to generalize and systematize the process, they can be considered as rectangles of null height. As hypothesis, it is assumed that occurrence probability in a rectangular cluster is uniformly distributed inside it.

Once the plane (B_0, E_1) has been defined, it is possible to obtain B' as the geometrical sum of B and E. In that way, the probability of each sub-interval j of the variable B' will be given by the sum of the probabilities of all the polygons between the two oblique lines that demarcate the sub-interval. These polygons depend on $m(j,k,q)$ that represents the part of points of the whole rectangular cluster (with indexes k and q) whose coordinate sum is included in the interval j of B' (figure 2).

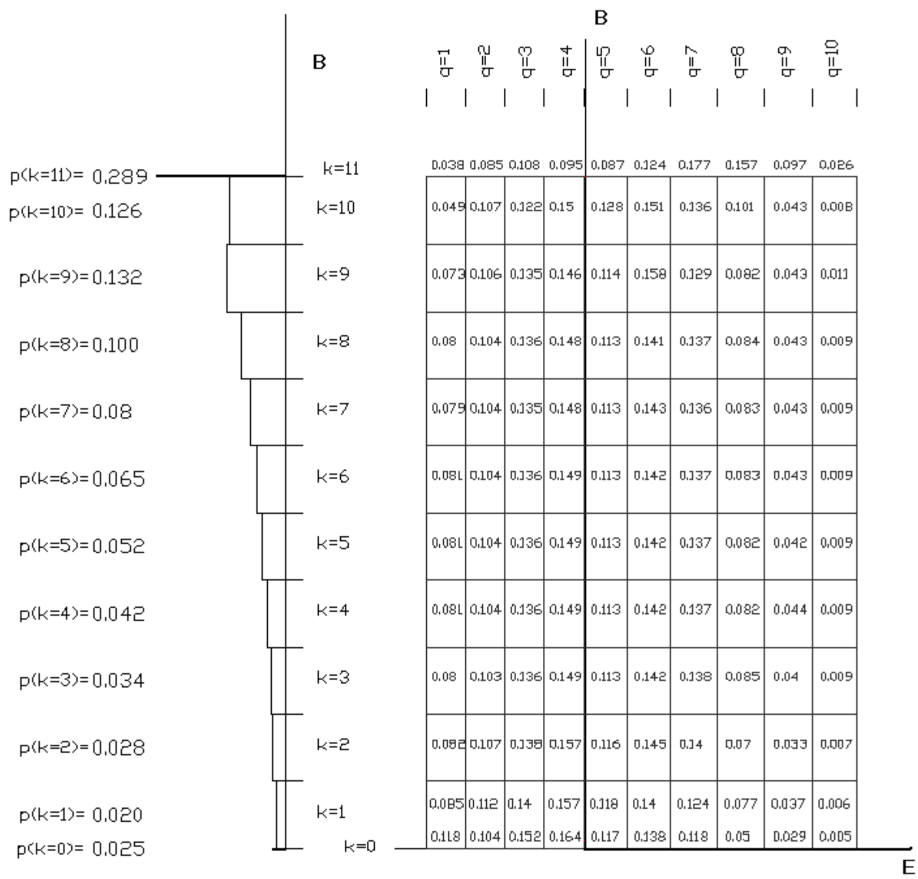

Fig. 1. Probability distributions for the variables B and E in the first day of the installation It can be seen the dependence of the probabililities of E on interval k

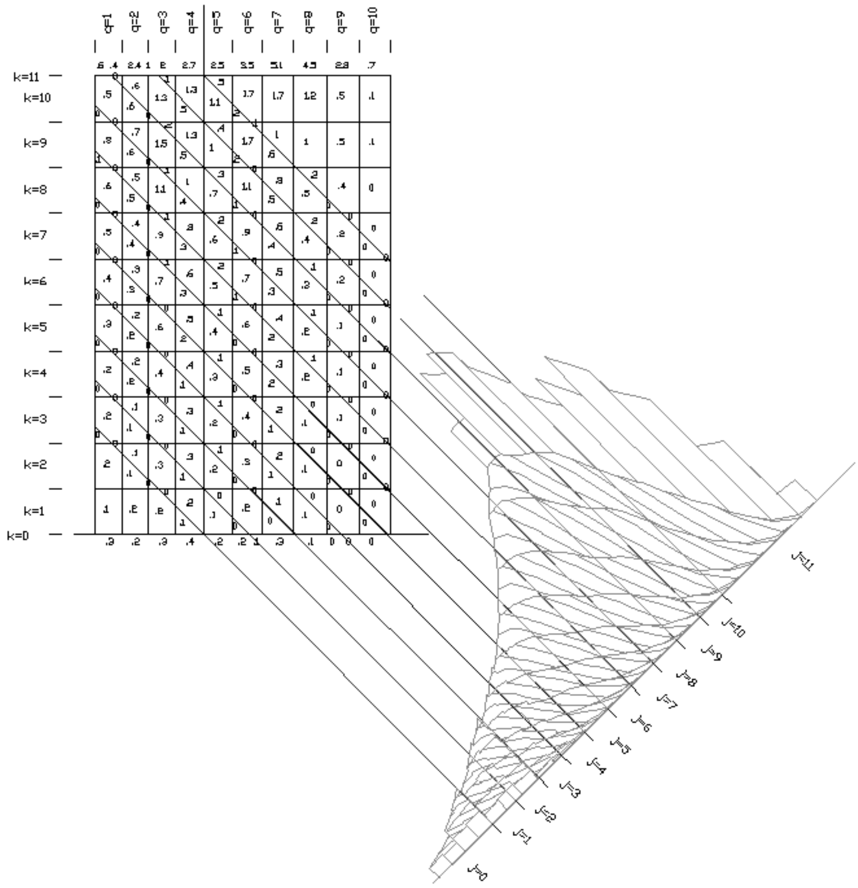

Fig. 2. Geometrical Estimation of the Probability distributions for the variables B' as the sum of theProbability distributions for the variables B and E in a generic day of the installation (probabilities on the rectangle has been multiplied by 100 in order to a better representation).

On the other hand, figure 2 shows the geometrical interpretation of equation 8 for the estimation of the probability corresponding to each interval j of the variable B'. This probability is obtained as the accumulative recount of the probabilistic mass of all the polygons included between the oblique lines that delimit the interval j of the variable B'.

Once the probabilities of the variable B' have been computed for the first day of the installation, the second day is simulated. For that purpose, the probabilities of B' on the previous day become the probabilities of the variable B for this second day (equation 4). As far as the probabilities of each sub-interval of the variable E are concerned, as explained before, they will be conditioned to the values of this variable in the previous day. Once more, from the probabilities of each sub-interval of the variables B and E, it is possible to estimate the probabilities of the different sub-intervals of the variable B' for the second day, following the same procedure that for the first one.

In a similar and recursive way, for any day i, it is possible to determine the probabilities of the different sub-intervals considered for the variable B' related to the battery state. Figure 4 shows the sum of the variables B and E for an intermediate day of the simulation process. Values of B' corresponding to the sub-interval $j=0$, that is, negative values of this variable, represent possible deficits and, consequently, the probability of a system failure in the day i. The probability associated to the sub-interval $j=11$ of the variable B' represents the probability of full batteries in the day i.

Thus, repeating this recursive method, it is possible to obtain not only the annual loss of load probability of the system, LLP, or the number of failures, f, but also the distribution of these values along the year. Figures 3 and 4 show respectively the LLP and f monthly distributions for the example previously described. These distributions computed by means of the method proposed in this paper are compared with the ones obtained by classical simulation methods [1] applied for a 20 year period of time [3]. Thus, from the information of figures 3 and 4, any user would be aware of the monthly restrictions of his energy demand.

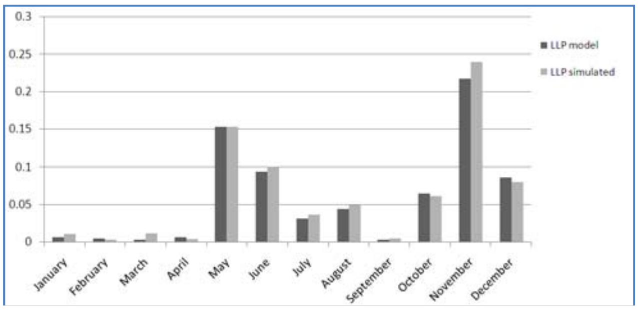

Fig.3.LLP monthly distributions for the 20 module (of 80 Wp) and 30 vase solution. The distributions calculated by the method presented in the paper are compared with the ones obtained by simulation.

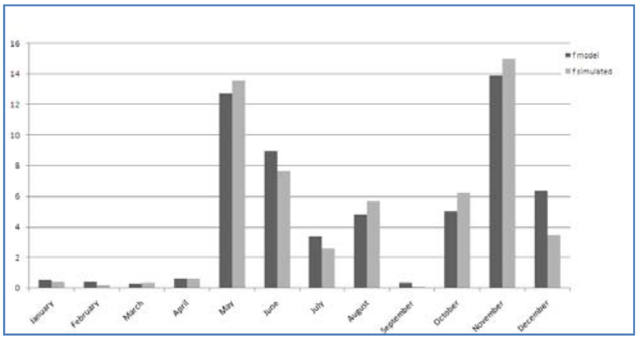

Fig.4.f monthly distributions for the 20 module (of 80 Wp) and 30 vase solution. The distributions calculated by the method presented in the paper are compared with the ones obtained by simulation.

3. Conclusions

In this paper, by means of an application example, the usefulness of the method presented by the authors in [12]. According to the results, it is possible to conclude that this method estimate the LLP value and the number of failures, f, for a stand-alone photovoltaic installation quickly and in an analytic and precise way. Furthermore, since it is a method based on the estimation of frequency distributions without resorting to simulation techniques, its results do not entail the uncertainty characteristic of these techniques. For these reasons, the method could be promising for the analytic study of the effects of different determining factors on the LLP value and the number of failures, f, not studied till this moment due to their complexity, such as: the daily demand changes, solar tracking or solar power concentration.

On the other hand, the method proposed is directly based on the descriptive models of Aguiar [11] for the solar radiation prediction. Thus, these method improvements will imply the proposed method improvement. In that sense, future works will be aimed towards the analytic study of the hourly balance.

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