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γ-Stirling engine – The effect of different working gases and pressures

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Abstract. This paper presents an investigation into the effects of changing the charge pressure as well as the working gas in a gamma type Stirling engine. An algorithm is developed to predict the effects of engine pressure and the type of working gas on the performance of a gamma-type Stirling engine. Based on the equation of state and the principle of energy and mass conservation, a set of first order differential equations are established. The fourth order Runge-Kutta method is used to solve these equations. The heat losses due to imperfect regeneration and pumping losses in the heat exchangers are included in the solution. The results show that using a working gas of lower density will cause less power losses compared to a working gas of higher density. Above 7.5bar the increase in efficiency with increased pressure becomes less significant with less than 4% increase from 7.5bar to 15bar for air. There is a linear relation between the amount of actual work and the nominal pressure charge of the engine. The results obtained from this simulation show a satisfactory working condition of the engine. The model sections and subsections were verified step by step using detailed and laborious manual calculations.

Key words

Stirling engine, simulation, working gas, pressure variations, iteration, Runge-Kutta.

1. Introduction

In 1816, Dr Robert Stirling patented a new design for a hot air engine with an "economiser" as he called his regenerator. There are two main differences to standard hot air engines, namely the closed loop of the working gas and the regenerator. There was, however, a disadvantage to his new design as the working gas, being used continuously, needed to be cooled down at the end of each cycle. The regenerator is an addition to counteract these added losses due to cooling by acting as an intermediary storage for heat. This meant that the overall efficiency of this design could reach higher values than standard steam turbines. In 1827, both Stirling brothers submitted a new patent which described the use of different gases, other than air, as the working medium. Lowering the density of the gas will lower the work required to "push" the gas around the engine [1]. Numerous individuals contributed to the development of Stirling engines. Notably here are Dr Kammerich and D. Viebach who both managed to develop a working prototype of 1kW and 0.5kW respectively without any financial contributions from major companies. A modified version of the engine designed by D. Viebach (Fig. 1) is used as the basis for this paper. Several evaluations have been presented over the years in order to predict and facilitate the design of engines. The first analysis was carried out by Schmidt almost 50 years after the actual invention of the engine. Schmidt assumed that the compression and expansion spaces are kept constant and at the same temperature throughout the cycle. His major contribution was to link the pressure in the engine to the volume which he assumed to behave in a sinusoidal fashion. An analysis of non-isothermal assumptions was first carried out by Finkelstein in 1960 [2]. He was the first to introduce the concept of conditional temperatures depending on the direction of the working gas flow, by assuming that the heat transfer within the working space takes place due to forced convection. He divided the engine into different cells (compression, cooler regenerator, heater and expansion) and represented them by control volumes using the equation of state as well as energy and mass conservation laws. This principle is further developed by adding the losses due to pumping friction in the heat exchangers by Urieli and Berchowitz [3]. The approach developed by Urieli is also used by Scollo *et al.* to design and construct a Stirling engine prototype. They use the model to estimate results and improve their design and show the usefulness of this method to get preliminary indicative information of the engine behaviour. [4]. Martini presents a good collection of simulations as well as scaling parameters with worked examples [5]. In particular he describes the various approaches and limitations of first, second and third order methods to simulate Stirling engines. Whereas first order design methods [6] are usually used to give an initial idea at the beginning of the design stage, they are limited in the way

Nomenclature:

A	free flow area	m^2
Awg	wetted area	m^2
cp	specific heat (pressure)	J/kg K
cv	specific heat (volume)	J/kg K
cwr	regenerator conductance	W/K
d	diameter	m
F	frequency	Hz
f	Reynolds friction factor	
h	heat transfer coefficient	W/m^2 K
I	enthalpy	J/kg
m	mass	kg
M	mass of gas in the engine	kg
NTU	number of transfer units	
P	pressure	Pa
Pr	Prandtl number	
Q	heat	J
R	gas constant	J/kg K
Re	Reynolds number	
St	Stanton number	
T	temperature	Kelvin
Tg	gas temperature	Kelvin
Tw	wall temperature	Kelvin
U	internal energy	J
V	volume	m^3
W	work	J

Subscripts

ac	actual
c	compression
ck	compression / cooler
e	expansion
h	heater
he	heater / expansion
in	inlet / inside
k	cooler
kr	cooler / regenerator
out	outlet / outside
r	regenerator
rh	regenerator / heater
se	swept in expansion space

Greek symbols

γ	gas gamma ratio	
δ	derivative	
ε	effectiveness	
μ	dynamic viscosity	kg/m s

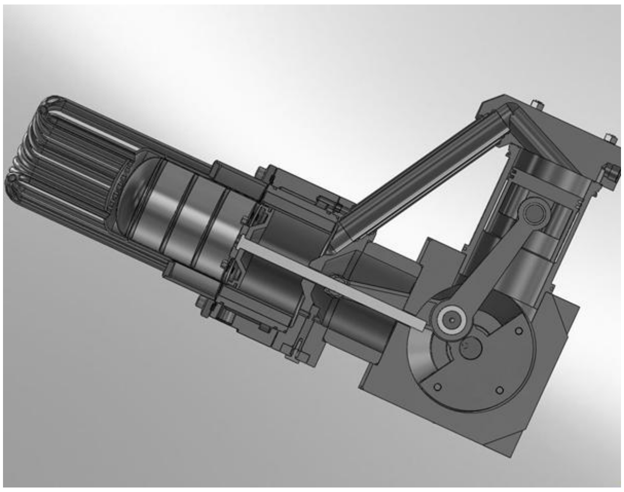

Fig. 1 Viebach gamma type Stirling engine

that neither the working gas properties nor the engine layout is used in the calculations. Second order design methods are relatively easy computational procedures that are particular useful for optimizing the design. Notably here is the Philips second order design method, described by Martini. The simulation used in this paper is of the third order form. The third order is an amelioration of the second order by means of modelling the interaction between the different processes, assumed to be going on simultaneously and independently. The advantage of these methods is the capability to compute flows and temperatures inside the engine which cannot be measured in practice [7-9].

This paper presents an investigation into the effects of changing the charge pressure as well as the working gas in a gamma type Stirling engine. A developed third order method for alpha engines is modified to account for the engine type and parameters. It is estimated that a lighter gas (than air) will generate a higher efficiency as less work is lost due to pumping friction losses.

2. Thermodynamic Analysis of a γ-type Stirling engine

The model discussed before had to be reconfigured prior to being applied to the configuration of the engine used in this setup. Additionally, the heater, regenerator and cooler configurations had to be remodelled. Fig. 2 shows a layout of the engine divided in five cells with four walls separating them. For simplicity, it is assumed that at the beginning of the cycle, the working gas flows from the compression space to the expansion space via the cooler, the regenerator and then the heater.

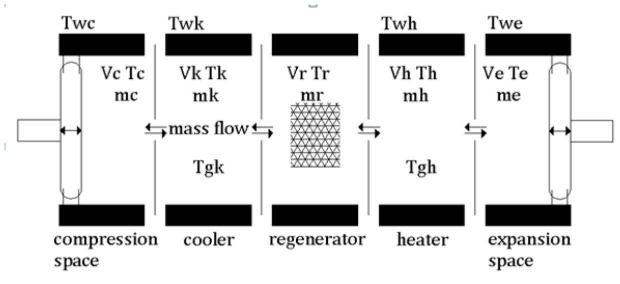

Fig. 2 Five engine cells with separating walls

These volumes are seen as open thermodynamic systems with a variable mass of working gas. The main equations for the mathematical model are the energy conservation equations, the equations of mass conservation which are written for each control volume, and the van der Waal's gas state equation, which is written for the whole internal volume for the engine, as follows:

$$\begin{cases} \boldsymbol{\delta m_i = \delta m_{i(in)} - \delta m_{i(out)}} \\ \boldsymbol{\delta Q_i = \delta U_i + \delta W_{i(in)} + I_{i(in)}\delta m_{i(in)} - I_{i(out)}\delta m_{i(out)}} \\ \boldsymbol{i = 1, 2, \ldots, 5} \end{cases} \quad (1)$$

$$\sum_{i=1}^{5} dm_i = 0 \quad (2)$$

$$\left(P + \frac{a}{V^2}\right)(V - b) = R\,T \quad (3)$$

where, a and b are substance specific constants of the Van der Waal's equation of state.

The total mass of gas in the engine is determined using the Schmidt analysis described in numerous papers and textbooks (Martini, Urieli, Organ).

$$m = P\, V_{se}\, B \times \frac{\sqrt{(1+s^2)}}{2\, R\, T_k} \tag{4}$$

where, B and s are constants depending on the layout of the engine.
The conditional temperature conditions introduced by Finkelstein can be expressed mathematically as follows:

$$\text{If } m_{ck} > 0 \text{ then } T_{ck} = T_c \text{ else } T_{ck} = T_k \tag{5}$$
$$\text{If } m_{he} > 0 \text{ then } T_{he} = T_h \text{ else } T_{he} = T_e \tag{6}$$

Applying the steady flow energy equation, Eq. (1), for the working gas to a generalised cell of the engine can be expressed mathematically as follows:

$$\delta Q + (cp\, T_{in}\, m_{in} - cp\, T_{out}\, m_{out}) = \delta W + cv\, \delta(mT) \tag{7}$$

It is generally assumed that the working gas in Stirling engines is behaving like an ideal gas and that the total mass of gas in the engine is constant The pressure inside the engine can be determined, according to Eq. (3) as follows:

$$P = \frac{M\, R}{\frac{V_c}{T_c}+\frac{V_k}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}+\frac{V_e}{T_e}} \tag{8}$$

The value of for the regenerator temperature is based on the logarithmic mean temperature difference:

$$T_r = \frac{T_e - T_c}{\ln\frac{T_e}{T_c}} \tag{9}$$

If Eq. (7) is applied to the compression / expansion space and considering continuity, then the mass derivative in the compression / expansion space can be determined as follows:

$$\delta m_c = \left(P\, \delta V_c + \frac{V_c\, \delta P}{\gamma}\right) / (R\, T_{ck}) \tag{10}$$
$$\delta m_e = \left(P\, \delta V_e + \frac{V_e\, \delta P}{\gamma}\right) / (R\, T_{he}) \tag{11}$$

where: $\gamma = {}^{cp}/_{cv}$
The equation of state for a generalised cell of the engine is represented in both its standard and differential form as follows:

$$P\, V = M\, R\, T \tag{12}$$
$$\frac{\delta P}{P} + \frac{\delta V}{V} = \frac{\delta M}{M} + \frac{\delta T}{T} \tag{13}$$

Differentiating the equation for the total mass of gas in the engine gives:

$$\delta m_c + \delta m_k + \delta m_r + \delta m_h + \delta m_e = 0 \tag{14}$$

Since the respective volumes and temperatures in each cell of the heat exchangers are constant, the differential form of the equation of state, equation (13), thus reduces to:

$$\frac{\delta m}{m} = \frac{\delta P}{P} = \frac{\delta P\, V}{R\, T} \tag{15}$$

Substituting equations (10) and (11) as well as equation (15) for each heat exchanger into the differential equation for the total mass of gas, equation (14), the following equation can be obtained:

$$\delta P = \frac{-\gamma P\left({}^{\delta V_c}/_{T_c} + {}^{\delta V_e}/_{T_h}\right)}{{}^{V_c}/_{T_c} + \gamma\left({}^{V_k}/_{T_k} + {}^{V_r}/_{T_r} + {}^{V_h}/_{T_h}\right) + {}^{V_e}/_{T_e}} \tag{16}$$

Applying the equation of state to the working spaces and differentiating, gives the temperature derivatives for the expansion and compression spaces, respectively:

$$\delta T_c = T_c\left({}^{\delta P}/_{P} + {}^{\delta V_c}/_{V_c} - {}^{\delta m_c}/_{m_c}\right) \tag{17}$$
$$\delta T_e = T_e\left({}^{\delta P}/_{P} + {}^{\delta V_e}/_{V_e} - {}^{\delta m_e}/_{m_e}\right) \tag{18}$$

In order to determine the heat in the different heat exchangers of the engine, the energy equation, Eq. (1), can be applied whilst substituting for the equation of state for a heat exchanger:

$$\delta Q = \frac{V\, \delta P\, cv}{R} - cp\, (T_{in}\, m_{in} - T_{out}\, m_{out}) \tag{19}$$

The work done in the compression and expansion cells is given by

$$\delta W = P\, V_c + P\, V_e \tag{20}$$

Since the regenerator is not ideal, some of the heat, when the gas flows from the heater to the cooler, stored in the regenerator matrix, will not be transferred back to the gas when the gas flows back from the cooler to the heater. This results in heat needing to be added in the heater at each passage of the gas. It is useful to quantify this heat loss in the regenerator in order to predict a more "real" behaviour of the engine. Therefore, the actual heat transferred (Qr) is dependent on the effectiveness of the regenerator. Based on Kays & London, the Stanton number and the Reynolds friction factor for a mesh type heat exchanger can be determined as follows [10]:

$$St = 0.46\, \frac{Re^{-0.4}}{Pr} \tag{21}$$
$$f = 54 + 1.43\, Re^{0.78} \tag{22}$$

As NTU is a function of the type of heat exchanger as well as its physical size, it can be expressed as a function of wetted area as well as the actual mass flow through the heat exchanger. Kays & London linked the NTU number to the Stanton number as follows, which relates the NTU more the fluid properties.

$$NTU = \frac{St\, Awg}{2\, A} \tag{23}$$

The total heat loss in the regenerator over one cycle can then be obtained through the following equation:

$$Q_{rloss} = (1-\varepsilon)\left(Q_{r,max} - Q_{r,min}\right) \tag{24}$$

Where: $\varepsilon = \frac{NTU}{1+NTU}$
So far in the analysis it is assumed that the gas temperatures in the compression/expansion spaces are

equal to its respective wall temperature. However, they are dependent on the heat transfer coefficients of the heat exchanger walls. Hence, the actual gas temperature can be expressed as follows:

$$T_g = T_w - \left[(Q_h + Q_{rloss}) \times \frac{F}{h\ Awg}\right] \quad (25)$$

In order to determine the heat transfer coefficient h, it is assumed that the flow is always turbulent; hence the Blasius relation can be used for the average Reynolds number in each heat exchanger:

$$f = 0.0791\ Re^{0.75} \quad (26)$$
$$h = \frac{f\ \mu\ cp}{2\ d\ \ Pr} \quad (27)$$

The thermal efficiency of the engine can be expressed as follows:

$$\varepsilon = \frac{W_{ac}}{Q_{h,ac}} \quad (28)$$

Since the actual power output of the engine can be expressed as the sum of the total power output minus the losses due to friction/pumping, the value for work in Eq. (28) can be determined by subtracting the sum of all individual pumping losses from each heat exchanger. The actual heat input to the engine, in Eq. (28), can be expressed mathematically as follows:

$$W_{ac} = W - \sum pumping\ losses \quad (29a)$$
$$Q_{h,ac} = Q_h + Q_{rloss} + Q_{wleak} \quad (29b)$$

Here, Q_{wleak} represents the heat leakage across the regenerator wall:

$$\mathrm{Q_{wleak} = h\ A_{wg}\ (T_{wh} - T_{wk})} \quad (30)$$

3. Computer model

The engine used in this study was originally developed by Dieter Viebach in 1992. The drawings of the engine are available under the Creative-commons licensing rights. The computer model generated is divided in two parts. The first part uses the Schmidt analysis to determine the total mass of gas in the heat exchanger. In the first part of the model, the user input is read by the macro and stored in variables and the initial conditions for temperatures are set as well as the gas properties depending on the type of gas used as working fluid. In the second part, the free flow area, void volume, wetted area and hydraulic diameter are determined for the heater, cooler and regenerator, respectively. These are dependent on the geometric shape of the heat exchangers and are programmed to account for the type used in the engine of this study; tubular for the heater, finned for the cooler and mesh wire for the regenerator. In the third part a Schmidt analysis is carried out for this gamma type engine. Eq. (4) is used to determine the total mass of the working gas in the engine. The second part of the computer model is slightly more complicated. Firstly, the heater and cooler temperatures specified by the user are not equal to the temperatures of the respective working spaces due to the impossibility of 100% heat transfer in the heater and cooler. Hence, an iterative method is used, which can be compared to a warm-up period of the engine, until the temperatures are balanced. This means that the temperatures at the end of the cycle will be more or less equal to the temperatures at the beginning of the cycle. This temperature difference is set to 1 degree and is limited by a maximum number of iterations of 10 (on average it took 4-5 iterations to converge the results). Secondly, the equations above are partially dependent on derivative forms of their corresponding equations. Hence, a priori, it is unknown what happens to the different values for each crank angle increment of the engine. Analysing the above equations shows a set of seven ordinary differential equations (Te, Tc, Qk, Qr, Qh, We and Wc). In order to solve this set of equations, the system is regarded as an initial value problem, where the equations are integrated from the known initial state. The classical fourth order Runge-Kutta method is used here. In the fourth part, the initial conditions are set as well as the resetting of the values for Q and W to zero at the beginning of each cycle. In the fifth part, the analytical variables and derivatives for pressure, masses and volumes are computed and in the sixth part, the seven ordinary differential equations are solved with a step increment of one degree crank angle (0.0174 rad) over an entire cycle. In seventh part, the sum of the absolute temperature difference between beginning and end of the cycle is evaluated. If the sum is higher than 1, then the temperatures at the beginning and the end of the cycle are not yet balanced and require a new cycle run with the initial temperatures for the heater and cooler now being the temperatures determined for the end of the cycle. When the temperature difference computed becomes less than 1, then two options are presented in the last part. The heat loss and resulting gas temperatures have or have not yet been computed. From Eq. (25) it can be seen that the actual gas temperature will be less than the determined wall temperature. This will generate a more realistic gas temperature. Once they are computed, the condition of ΔT is re-checked and if still true the results are presented. If the condition is not satisfied the iteration is run once more.

4. Results

Fig. 3 shows the variation of the compression, expansion and total volume as a function of the crank angle. The compression volume reaches its maximum value when the cycle has not begun yet, because the flow direction of the working gas has been arbitrarily set to flow from the compression space to the expansion space. From the compression space the working gas is pumped through the cooler, the regenerator and the heater to the expansion space. After the gas has fully expanded in the expansion space, the gas flows back to the compression space.

Fig. 4 shows the variation of the working gas temperature in the expansion, compression space and regenerator as well as the heater and cooler temperature as a function of the crank angle. The temperature within the compression and expansion space increases due to the decreasing total volume before the volume starts to increase again. As the volume increases, the cooling occurs due to the expansion of the gas and the temperature in the compression and expansion spaces decrease. The pressure for the individual cells of the engine shows very little differences, which reflects the assumptions of Eq. (8).

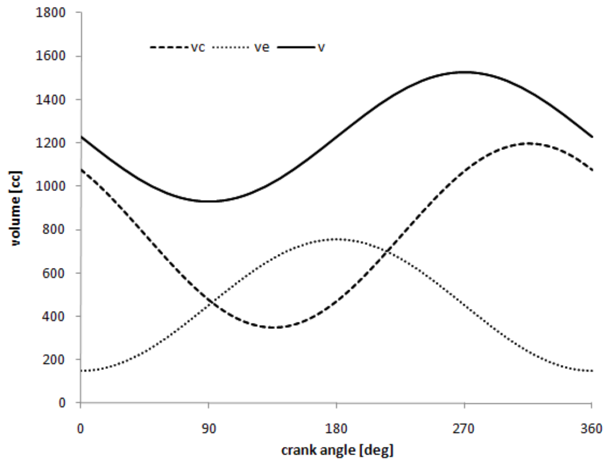

Fig. 3 Volume variation as a function of crank angle

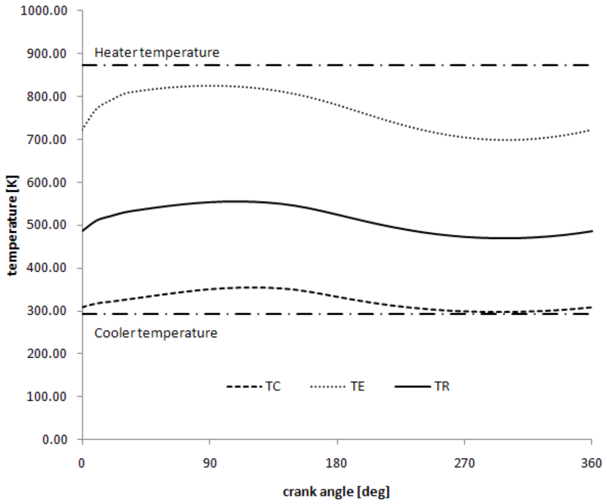

Fig. 4 Temperature variation as a function of crank angle

Fig. 5 shows the p-V diagram for the expansion and compression spaces for air, Hydrogen and Helium at 10bar charge pressure. As the pressures are identical across the engine (for each cycle step), the actual pressure in the expansion space is equal to the sum of pressure in the compression space and the pressure losses due to pumping friction. The shape of the graphs are very similar for the three different working gases and since the work output is the mathematical integral of this graphical shape, there should be little difference between the work output for the different gases at equal charge pressure. Fig. 6 shows the pressure drop for the three heat exchangers for air and Hydrogen at 10bar nominal pressure charge as a function of crank angle. The pressure drops are shown with positive values for comparison, although the numerical values are negative for crank angles ranging from 140 to 330 degrees. The negative or positive sign of the value of the pressure drops are relating to the direction of flow of the working gas. From the graph it can be observed that the pressure drop for air is of a higher magnitude than the one for Hydrogen.

The calculations showed that the pressure drop for Helium was slightly higher than the one for Hydrogen and would overcrowd the graph. This shows that there is more work required to push the air around the engine than the other two working gases. For each of the three working gases the highest pressure drop is noticed in the regenerator with the lowest pressure drop in the cooler as the pressure drop is directly proportional to the Reynolds friction factor.

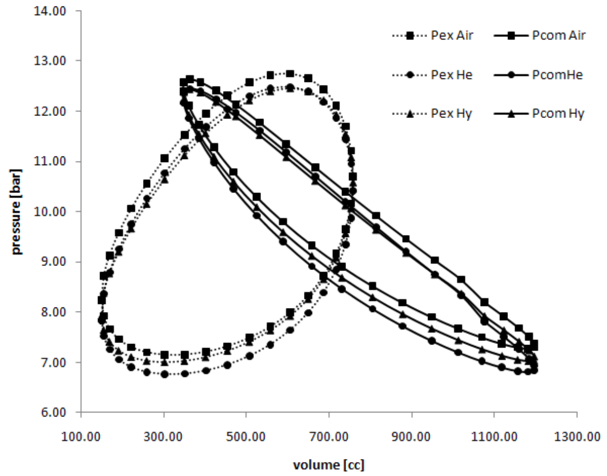

Fig. 5 p-V diagram at 10bar charge pressure

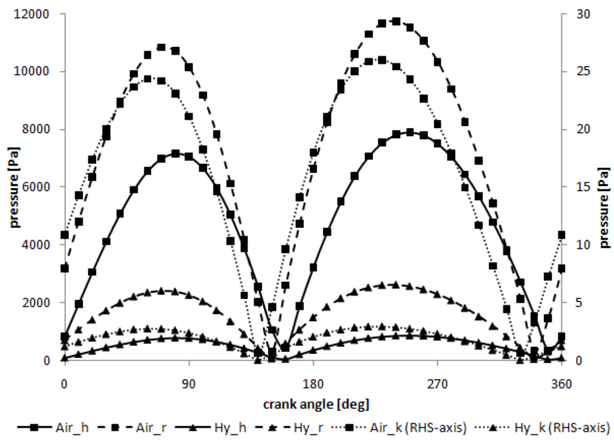

Fig. 6 Pressure drop in the heat exchangers at 10bar

Fig. 7 shows the variation of the total mass of working gas as a function of the nominal charge pressure for all three working gases. Considerable differences can be noted with a total mass of air of 16.05g (10bar) in comparison to 2.22g and 1.11g for Helium and Hydrogen respectively. This is reflected on the amount of work required to push the gas around the engine, resulting in increased pressure drops for air compared to the other two gases. These differences in pressure drop for the different gases will have an impact on the actual work output and the efficiency of the engine. Fig. 8 shows the variation of compression, expansion and total actual work output for air as a function of crank angle for 10bar. The results show that there is an increase in work for increased pressure as expected. As the compression volume is decreasing, the work done by the compression space is of increasing negative magnitude as work is required to compress the gas. Similarly, as the expansion volume is increasing, the work done by the expansion space on the displacer is of increasing positive magnitude. At the end of the cycle, the values for W_e and W_c are equal to the values of heat transferred in the heater and cooler respectively.

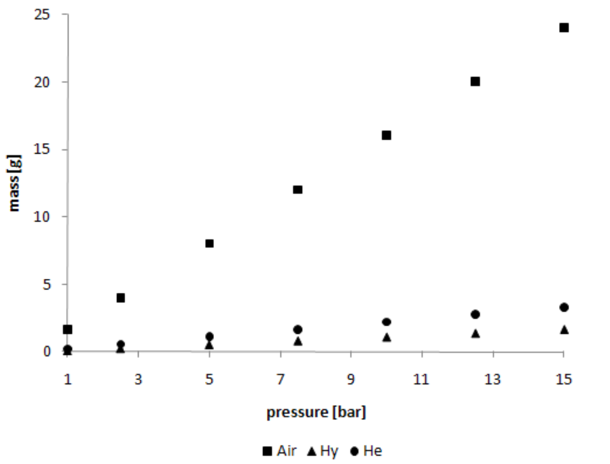

Fig. 7 Total mass of working gas in the engine

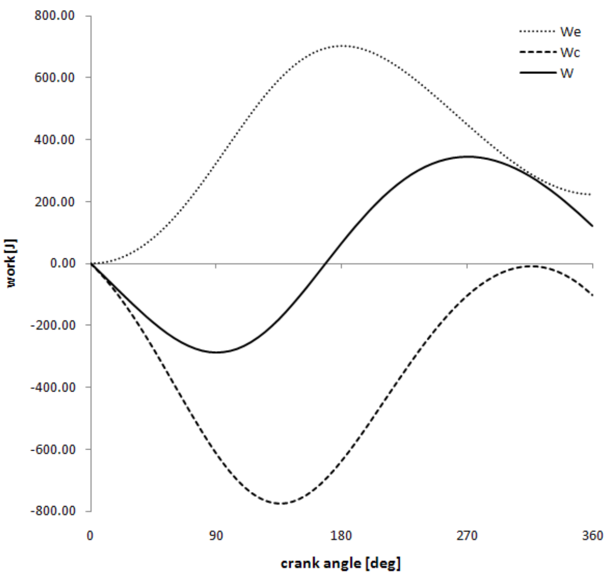

Fig. 8 Actual work output for air at 10bar

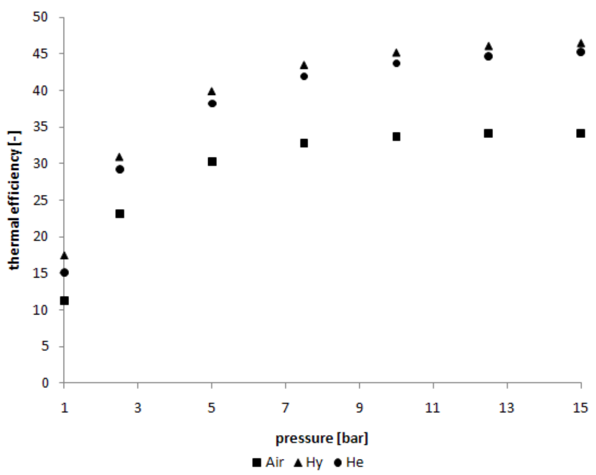

Fig. 9 Comparison of thermal efficiency for all gases

Figure 9 shows the variation of thermal efficiency for all three working gases as a function of the nominal pressure charge. As expected, from the previous discussion points, there is a notable increase of thermal efficiency for increased nominal pressure charge. However, this difference becomes less significant above 7.5bar. The difference in efficiency for air at 7.5bar and 15bar is less than 4% whereas the actual work output for the same pressure conditions is about 100%. This obviously reflects a higher heat input at 15bar than at 7.5bar. As a matter of fact the heat required is almost doubled. The engine was originally designed for a 10bar nominal charge, which obviously shows a good work output and efficiency. Increasing the pressure to 15bar will increase the work output, however, it is not certain if the engine will actually be able to withstand those pressures from a stress related point of view.

5. Conclusions

This paper presents an investigation into the effects of changing the charge pressure as well as the working gas in a gamma type Stirling engine. A third order simulation based on the works of Urieli and Berchowitz was modified to investigate the effects of changing nominal pressure charge as well as the working gas in a particular gamma type Stirling engine. This simulation, although well used is not compared to experimental findings with this particular engine. The results presented in this paper will be investigated in laboratory experiments once the engine is build. As expected, the use of a gas lighter than air will result in a higher thermal efficiency of the engine as less work is required to overcome the flow friction. This flow friction is due to a heavier gas being pushed around the engine. The amount of heat required to drive the engine becomes more important with higher pressures.

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