International Conference on Renewable Energies and Power Quality (ICREPQ'16)
Madrid (Spain), 4th to 6th May, 2016
Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, No.14 May 2016

Stochastic Modelling Applied to Prediction of Electricity Saving by using Solar Water Heating Systems for Low-Income Families

Stochastic Modeling Applied to Prediction of Electricity Saving by using Solar Water Heating Systems in Low-Income Families

B. G. Menita[1], J. L. Domingos[1], E. G. Domingues[1], A. J. Alves[1], and W. P. Calixto[1]

[1] Graduate Program in Technology of Sustainable Processes
Federal Institute of Education, Science and Technology of Goiás (IFG)
Goiânia Campus – Goiás (Brazil)
e-mail: **brunomenita@gmail.com, jose.domingos@ifg.edu.br, prof.eldergd@gmail.com, aylton.alves@ifg.edu.br, wpcalixto@gmail.com**

Abstract. Solar water heating systems for low-income families as Energy Efficiency Action bring energetic benefits for the consumers and the Brazilian Electrical System and also contribute for the reduction of the environmental impacts associated with generation, transmission and distribution of electricity. This paper presents the stochastic modelling for the generation of future scenarios of electricity saving of Energy Efficiency Projects that involves solar water heating systems for low-income families. The model is developed by using the Geometric Brownian Motion Stochastic Process with Mean Reversion (GBM-MR) associated with the Monte Carlo simulation technique. As a result it is possible to obtain the time series and the probability distribution function of the energy saving for each year of the simulation period. Once there is no historical data available for obtaining the standard deviation and the mean reversion speed of the stochastic process, it is presented a sensitivity analysis in order to verify how these parameters influence on the results.

Keywords

Solar water heating, energy efficiency, Geometric Brownian Motion, Monte Carlo simulation, sensitivity analysis.

1. Introduction

The use of solar energy in residential water heating has growing acceptance as an alternative or supplementary way to the heating provided by electric showers. Recently, Brazilian government programs have promoted the use of solar water heaters in homes of low-income families, such as Energy Efficiency Projects (EEP) of electricity distribution companies, as it is shown in Figure 1. These

Fig. 1. Solar water heating system in home of low-income family

EEP are part of Energy Efficiency Program of Brazilian Electricity Regulatory Agency - ANEEL [1].

The benefits of the Energy Efficiency Action (EEA) of an EEP can be evaluated from some points of view: i) the consumer, that save money with reduction of electricity consumption; ii) the Brazilian Electrical System, that postpone investments in generation, transmission and distribution, as a result of the reduction of electricity demand, especially in peak time; iii) and society, due to lower average tariffs and environmental impacts associated with the electric sector [1].

The application of the International Performance Measurement and Verification Protocol (IPMVP) is mandatory as a reference for Measurement and Verification (M&V) among other steps involved in evaluation of electricity saving and peak demand reduction of an EEP.

The IPMVP establishes rigorous criteria that lead many EEP to economic unviability, mainly due to long periods of measurement. To solve this problem, the Brazilian Association of Electricity Distributors (ABRADEE) developed M&V procedures from IPMVP to apply in EEP by final use, with contributions of consultancies and partnerships. Thus, a new M&V methodology by end use has been defined and approved by ANEEL, and passed on to electricity distribution companies in September 2014.

The annual consumption of electricity avoided, which represents annual electricity saving by the EEP depends on some factors that have random behavior over time such as: the number of residents of the housing project that received the EEA, the bath habit of these people, changes in family income, and acquisition or replacement of electrical appliances in these houses.

In this context, taking in account that gains obtained with energy efficiency are included in long-term energy planning in Brazil [2, 3], this paper presents the stochastic modeling for generation of future scenarios of electricity saving from Energy Efficiency Project involving solar water heating for low-income families. The random variable Annual Electricity Saving resulting from EEP is modeled by using the stochastic process called the Geometric Brownian Motion with Mean Reversion (GBM-MR).

2. Application of M&V procedures adapted from IPMVP

The methodology containing the M&V procedures adapted from IPMVP includes the following steps: i) definitions related to key parameters of M&V, independent variables with potential to influence electricity consumption by electric showers, measurement periods, among others; ii) definition of the number of samples for M&V in EEP, respecting the level of precision previously defined; iii) verification of the existence of correlation between key parameters and independent variables, and in affirmative case, calculation adjustments in order to eliminate the influence on the measured values of different measurement periods; iv) calculation of the annual electricity saving and the reduction of the peak demand with EEP; v) obtainment of the uncertainty associated with the results.

One of the results to be obtained by the application of these procedures is the annual electricity consumption avoided by replacing electric showers for solar water heating systems in homes of low-income families, which represents the electricity saving by PEE, once there is no additional electric water heating. Thus, the electricity saving (ES) is given by:

$$ES = C_{bl} - C_{rp} \tag{1}$$

Where C_{bl} corresponds to the electricity consumption in baseline period (period prior to the installation of solar heating system in the house) and C_{rp} represents the electricity consumption after the system installation, which corresponds to the reporting period.

The electricity consumption (C) in both periods is obtained by using equation (2), if there is correlation between the key parameter Electric Power during the use of the shower and the independent variable Outdoor Temperature. This variable was obtained from measurements of the nearest station of Brazilian National Institute of Meteorology - INMET. This consideration is necessary to eliminate the influence of outdoor temperature on the power measurements in order to relate the conditions of consumption during baseline and reporting periods. If this correlation does not occur, the electricity consumption (C) can be calculated by using equation (3).

$$C = \left(LinC_{pt} + Dec_{pt}.T_{standard}\right).Time_{bath} \tag{2}$$

where $LinC_{pt}$ corresponds to the linear coefficient and Dec_{pt} is the declivity, both resulting of the linear regression analysis between the power of the shower and the outdoor temperature, $T_{standard}$ refers to the standard temperature adopted for alignment of consumption conditions and $Time_{bath}$ represents the average bath time, which depends on the behavior of the local consumers.

$$C = Power_{shower}.Time_{bath} \tag{3}$$

Where $Power_{shower}$ is the average electric power resulting of the M&V.

The electricity saving is obtained by the deterministic expressions presented and it is dependent on some variables, such as the electric power of the shower, the bath time (key parameters) and the outdoor temperature (independent variable). These variables have random behavior over time, since they are influenced by factors such as number of residents, bathing habits, family income and electrical appliance present in the residence.

Thus, since the electricity saving is function of key parameters and independent variable mentioned, the Annual Electricity Saving achieved by a PEE is considered as a random variable.

3. Stochastic Modelling of Electricity Saving by Solar Water Heating Systems

Several stochastic processes have been used in the Brazilian Electricity Market to model the uncertainties present in this, such as the spot price, affluence, electrical demand and consumption of electricity. These random variables can be modeled as time series by using the Monte Carlo simulation technique, associated with the stochastic process called Random Walk [4].

Once the annual electricity saving obtained by EEP is dependent of variables with random behavior, it is necessary to generate future scenarios of this random variable by using an adequate stochastic process.

The Geometric Brownian Motion (GBM) is a particular case of Ito's process, which in turn corresponds to the generalization of Brownian motion with drift [5]. It is assessed that the GBM follows a normal distribution function within a T interval, with mean $\left(\mu - \frac{1}{2}.\sigma^2\right).T$ and variance $\sigma^2.T$, and it is represented by equation (4).

$$\Delta a = \left(\mu - \frac{1}{2}.\sigma^2\right).\Delta t + \sigma.\varphi.\sqrt{\Delta t} \tag{4}$$

In equation (4) a corresponds to the random variable that follows the GBM, μ is the constant that represents the percentage drift of the random variable, t represents the time, σ is the constant that represents the percentage volatility random variable and φ corresponds to a random variable with standard normal distribution – *N(0,1)*.

According to [6], while it is considered that the behavior of a random variable follows a Markov process and has independent increments, it should not be assumed that the variations of this random variable follow a normal distribution, if this variable cannot have value less than zero. In this case, it can be assumed that the random variable follows a *log-normal* distribution function, i.e., the changes in the logarithm of the random variable follows the normal distribution. Thus, the GBM can be represented by equations (5) and (6).

$$\ln a_{t+1} - \ln a_t = \left(\mu - \frac{1}{2}.\sigma^2\right).\Delta t + \sigma.\varphi.\sqrt{\Delta t} \tag{5}$$

$$a_{t+1} = a_t.e^{\left[\left(\mu - \frac{1}{2}.\sigma^2\right).\Delta t + \sigma.\varphi.\sqrt{\Delta t}\right]} \tag{6}$$

According to [7], when a random variable follows a GBM, their values tend to diverge from the original starting point, since the variance grows linearly with time. In this context, the process of BGM with Mean Reversion, also called Ornstein-Uhlenbeck process, forces the values obtained to the random variable over time to revert in direction of the equilibrium position, i.e., the starting value (mean value, for example). According to [6], there is a force of reversion acting on the random variable pulling it to a long-term equilibrium level. This occurs at certain reversion speed, represented by η parameter.

The process of Ornstein-Uhlenbeck for stochastic variable has as deterministic term (trend): $\eta.(\bar{a} - a_t).\mathrm{dt}$. The equation (7) results from the application of Ito's lemma [4] to the variable $ln\,a$, where η is the mean reversion speed and $\bar{a}$ represents the mean value of the random variable.

$$\ln a_{t+1} - \ln a_t = \left[\eta.(\bar{a} - a_t) - \frac{1}{2}.\sigma^2\right].\Delta t + \sigma.\varphi.\sqrt{\Delta t} \tag{7}$$

Equation (8) represents the stochastic modeling of mean reversion, whose deduction is presented in [4] and [6].

$$a_{t+1} = a_t.e^{\left\{\left[\eta.(\ln \bar{a} - \ln a_t) - \frac{1}{2}.\sigma^2\right].\Delta t + \sigma.\varphi.\sqrt{\Delta t}\right\}} \tag{8}$$

According to [4], the expected variation of the random variable depends on the difference between its value at any given instant of time and the mean. Soon, if in this given instant of time the value of a is greater than$\bar{a}$, it will have a drop in the value of the next instant of time Δt, occurring the opposite if a is less than the starting value.Thus the mean reversion process occurs in a GBM.
The random variable Annual Electricity Saving by EEP (*EE*) can be obtained by equation (9).

$$EE_{t+1} = EE_t.e^{\left\{\left[\eta.(\ln \overline{\mathrm{EE}} - \ln \mathrm{EE}_t) - \frac{1}{2}.\sigma^2\right].\Delta t + \sigma.\varphi.\sqrt{\Delta t}\right\}} \tag{9}$$

The steps that make up the Monte Carlo simulation are: i) definition of the starting value of the random variable, which is the value at time zero; ii) definition of the analysis period and the time interval between the forecasts, in the same unit of time; iii) generation of random numbers converted to numbers with standard normal distribution by computational tool (these are applied to the φ parameter); iv) application of one step of the random walk process, once other parameters and variable components of stochastic modeling expression are known; v) repetition of steps of random walk for values of the random variable with standard normal distribution obtained for each instant of time during the simulation period [8].

The stochastic behavior of the random variable can be represented by curves containing the values obtained for annual electricity saving on the time horizon defined~~,~~ as a family of time series, using the Monte Carlo simulation. The stochastic process can be also represented by the evolution of the probability density function (PDF) of the random variable over time [4].

4. Results of Simulation

The M&V procedures adapted from IPMVP are applied in three case studies in Goiás state, by EEP from CELG Distribution S/A (CELG-D): municipality of Itumbiara, and the housing projects Real Conquista Residential and Orlando de Morais Residential, located in Goiânia, that is capital of Goiás state. In these three locations the number of residences of low income families are 1080, 478 and 544 houses, respectively, contemplated with the replacement of electric showers by solar heating water system, characterizing this way EEP of the local electricity distributor.

It is defined as the starting value to the random variable Electricity Saving (in the initial year) the annual electricity consumption avoided by the EEP Real Conquista Residential (265.92 MWh). Table I presents the input data for simulation.

The volatility (σ) of the random walk is defined as the estimated standard deviation of Annual Electricity Saving, obtained by multiplying the coefficient of variation calculated with the measured values of electric power in the EEP Real Conquista Residential (0.39) for the electricity saving in the initial year (mean).For the mean reversion

Table I. - Input Data for Simulation

Average value (MWh)	265.92
Volatility of the random walk (MWh)	103.71
Mean reversion speed	0.50
Time horizon of simulation (years)	10
Interval between simulation periods (years)	1
Number of scenarios	2000

speed (η) it is assigned the value 0.50. Both parameters of the random walk should have assigned values based on historical data. However, considering the absence of these data, it was not possible to perform appropriate statistic analysis to obtain these values, which led to the aforementioned assignments, considering the pioneering nature of this study.

Figure 2 presents the simulation results for one scenario of the random variable Electricity Saving, which shows how the random walk can occur along the horizon. Figure 3 presents 2000 scenarios generated for this random variable, which shows a range of results obtained from probabilistic laws along time.

Figure 4 shows the PDF for each year of the study horizon, as another way of representation of future scenarios for annual electricity saving obtained by the EEP Real Conquista Residential. The red line shown in this figure represents the mean of the values obtained from time series, in each year, which remains around 265 MWh.

Table II shows the results for the expected value and the standard deviation of annual electricity saving in each year of the simulation period.

In the simulation results presented in Figures 2, 3 and 4 the values of the mean reversion speed (η) and the volatility (σ) of the annual electricity saving were assigned once there are no historical data of them until this moment.

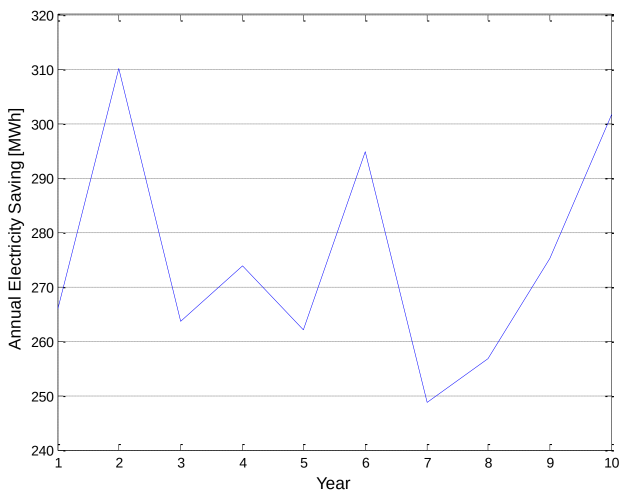


Fig. 2. Behavior of the annual electricity saving over time – one scenario

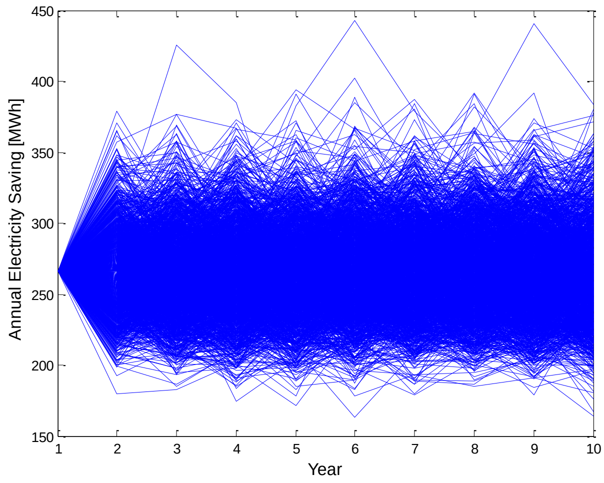


Fig. 3. Behavior of the annual electricity saving over time – 2000 scenarios

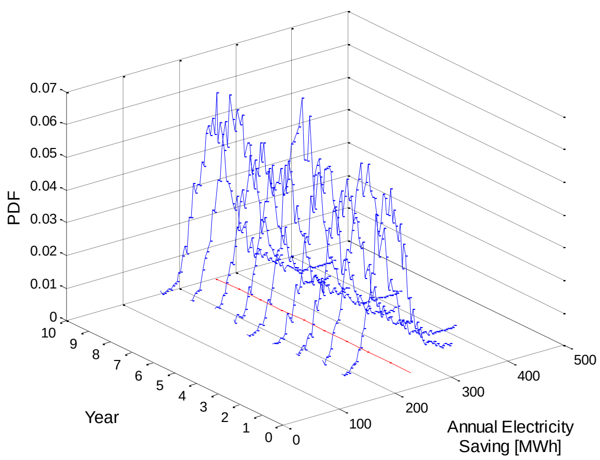


Fig. 4. PDF of the annual electricity saving over time

Table II. - Expected Value and Standard Deviation of Simulated Values

Year	ExpectedValue (MWh)	Standard Deviation (MWh)
1	265.9	0.0
2	266.1	27.4
3	265.0	30.6
4	265.4	31.8
5	265.0	30.9
6	266.5	32.0
7	266.0	31.7
8	266.2	31.6
9	265.3	32.5
10	263.8	32.3

To verify the influence of these parameters on the results, sensitivity analysis was performed. For this objective, it was adopted a variation range of 0.00 to 265.92 MWh with step of 13.30 MWh for the volatility(σ), and range of 0.10 to 10.00 with step of 0.10 for the mean reversion speed(η)[1], both for the year 3 of the initial horizon of the simulation.

Figure 5 shows the behavior of the maximum and minimum values of the annual energy for the 2000 scenarios obtained for the year 3, in function of the variation of the standard deviation and the mean reversion speed. This figure represents the stochastic behavior of the electricity saving for a large range of situations. As expected, there is an increasing of the peak-to-peak amplitude of annual electricity saving with the increasing of both the standard deviation and the mean reversion speed.

The behavior of the expected value and the standard deviation of the annual electricity saving for the 2000 scenarios in the year 3, in function of the variation of the volatility and the mean reversion speed is shown in Figure 6.

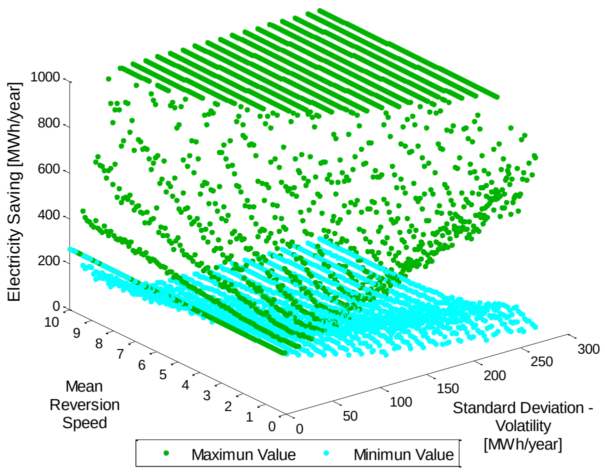
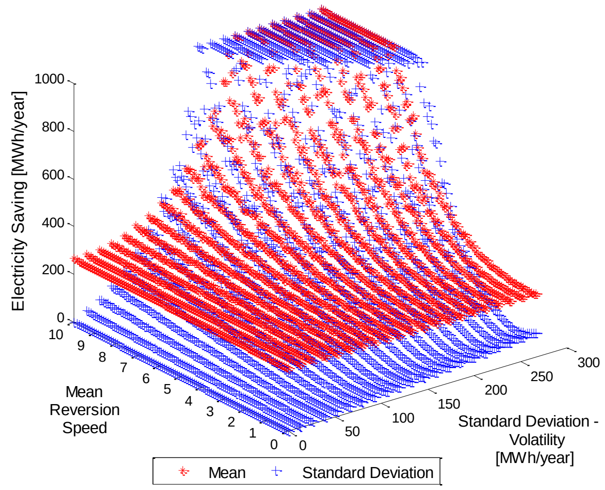


Fig. 5. Amplitude of electricity saving by variation of σ and η



[1]Once obtaining the mean reversion speed from historical data occurs by linear regression, i.e.,it corresponds to the slope (component "a" of the trandeline equation "y=a.x+b"), it is considered a large range of possible values[4].

Fig. 6. Mean and standard deviation of electricity saving by variation of σ and η

As the variation of the parameters mentioned in this sensitivity analysis results in some extremely high values of annual electricity saving, Figure 5 and Figure 6 have the z axis graphically limited to 1000 MWh/year, in order to highlight the behavior throughout the variations.

It can be seen by these figures that increasing the standard deviation leads to greater spread of results, however, it is with the increase of the mean reversion speed that is observed a sharp increase of the mean and standard deviation of the 2000 series.

Such behaviors are best viewed in Figure 7 and Figure 8. In Figure 7, the mean reversion was set at 0.5 and ranged up the standard deviation. In Figure 8 the standard deviation was set at 103.71 MWh and ranged up the reversion speed. It is also possible to verify in Figure 8 that the standard deviation of the 2000 series reaches the minimum value when the speed is equal to 1, i.e, when the slope of the trend line formed by the historical data is equal to 45^{o}or 135^{o}.

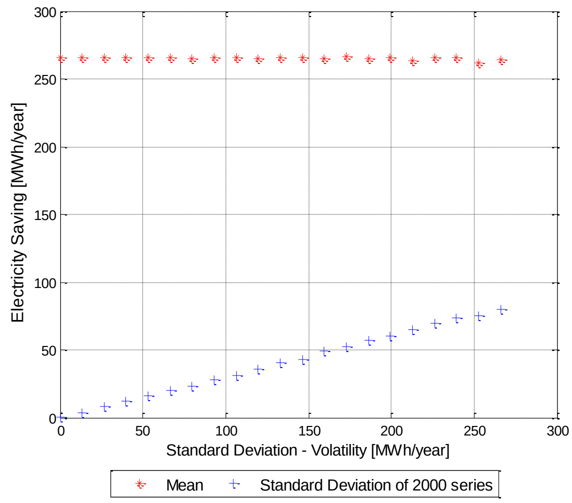


Fig. 7. Mean and standard deviation of electricity saving by variation of σ

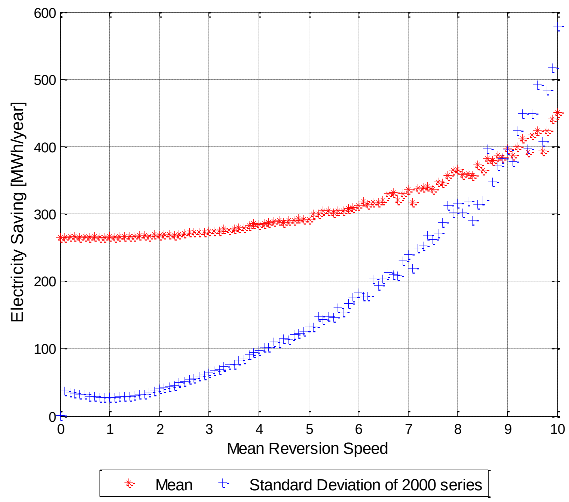


Fig. 8. Mean and standard deviation of electricity saving by variation of η

5. Conclusion

From the input data shown in Table I, it is obtained the behavior of the Annual Electricity Saving, which is represented by the family of time series, as shown in Figure 2 and represented by the evolution of the PDF of the random variable over time, as shown in Figure 4, thus obtaining the projection of this random variable in a time horizon of 10 years.

The sensitivity analysis performed allowed to obtain the behavior of the Annual Electricity Saving from the possible values adopted for the volatility and the mean reversion speed, since there is no historical data for obtaining these. It can be seen that there is great differentiation in the results of the amplitude (Figure 5) and in the mean and standard deviation calculated based on the values of 2000 series (Figures 6, 7 and 8).

The application of the methodology of stochastic modeling to forecast future electricity saving by EEP leads to the conclusion that the reliability of its use is conditioned to obtaining historical data for volatility and mean reversion speed, given the abrupt variation of results. As new results of electricity saving by M&V in EEP will be obtained, it will be constituted a sufficient set of historical data for the assignment of these constants used in stochastic modeling.

Considering these contributions and new information to be used in the stochastic model, as the useful life of equipment, this methodology can be used to obtain prediction of electricity saving by using solar water heating systems in homes of low-income families.

Acknowledgment

Thanks are made to the technical team of Energy Use and Quality Sector of CELG-D by support granted to carry out evaluations.

References

[1] Centrais Elétricas Brasileiras S/A, Energia solar para aquecimento de água do Brasil: contribuições da Eletrobrás Procel e parceiros, Rio de Janeiro: Eletrobrás, 2012.

[2] Ministério de Minas e Energia, Plano Nacional de Eficiência Energética,Brasília: MME, 2011.

[3] Ministério de Minas e Energia, Plano Nacional de Energia 2030**,** Brasília: MME, 2008.

[4] E. Domingues, Análise de risco para otimizar carteiras de ativos físicos em geração de energia elétrica, Itajubá: Tese de Doutorado, Curso de Pós-Graduação em Engenharia Elétrica, Universidade Federal de Itajubá, 2003.

[5] J. Hull, Options, Futures and Other Derivatives, Prentice-Hall, second edition, 1993.

[6] E. Fonseca, Comparação entre simulações pelo Movimento Geométrico Browniano e Movimento de Reversão à Média no cálculo do Fluxo de Caixa*at Risk* do departamento de *downstream* de uma empresa de petróleo**,** Rio de Janeiro : Dissertação de Mestrado, Instituto COPPEAD de Administração, Universidade Federal do Rio de Janeiro, 2006.

[7] A. K.Dixit and R. S.Pindyck, Investment Under Uncertainty, Princeton University Press, Princeton, New Jersey, 1993.

[8] G. S.Fishman, Monte Carlo – Concepts, Algorithms and Applications, IE-Springer-Verlag, New York, 1996.