



# Optimal Allocation of Multiple Distributed Generations including Uncertainties in Distribution Networks by k-Means Clustering and Particle Swarm Optimization Algorithms

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**Abstract.** Climate change is the one of the most important issues faced globally and reasons of it must be reduced immediately in every area. Installing distributed power generation (DG) is one of the powerful options for reducing carbon emissions in power generation. However, improper allocation of these assets has several drawbacks. Efficient, novel and robust algorithm which is combination of both k-Means clustering and Particle Swarm Optimization is proposed in order to allocate DGs. Proposed algorithm clusters distribution network buses and selects to most proper cluster to allocate DG in this way it reduces possible buses. Furthermore, sizing and generation constraints of DGs are quite important for allocation. Therefore, several cases including different DG sizes and types are implemented to obtain the best results. Moreover, multiple DG cases are included in the study. Finally, DGs have considered as wind turbines for best cases and cases have analysed in 24 hourly bases including uncertainties both demand and production side. 33 Bus test feeder power losses are reduced up to 69%, 86%, 90% at best cases and 39%, 53%, 55% at including uncertainties by proposed algorithm for cases 1, 2, 3 DG installed, respectively.

**Key words.** Distributed power generation, improving voltage profile, k-Means clustering, particle-swarm optimization (PSO), power loss reduction.

## 1. Introduction

Nowadays, climate change is increasingly becoming a vital issue and fighting against it can be ensured by reducing fossil fuels or carbon emissions in area of power systems. Increasing efficiency of power production, encouraging the usage of renewable sources and decreasing power losses are the fundamental options for fighting against climate change. A review paper states that installation of distributed generation (DG) units are able to decrease carbon emissions 41% in United Kingdom [1]. Therefore, installing DG units, most specifically renewable sources, has received much attention in the past decade due to the developing technology and DGs great benefits. The most specific feature of DGs is the capability of locating near load-side thereby, DGs are significantly capable of reducing the

transmission losses. Furthermore, the principal impacts of DGs penetration to distribution networks includes improving voltage regulation, improving voltage stability, increasing reliability, reducing the distribution network losses and increasing the line capacities [2]–[4]. Depending on DGs impacts, both power quality and system operators' profits will have increase. More importantly, renewable DGs (wind, photovoltaic etc.) are society's best option to stop climate change immediately for cleaner future. However, inappropriate placing and inefficient sizing of these assets have several critical consequences such as increased system operation cost, high short-circuit currents etc. [5], [6]. Therefore, appropriate placing and sizing of DGs under the operational constraints is attracting considerable interest due to both improving network performance and increasing system operators' profits while avoiding its possible drawbacks.

There is a vast amount of literature on different optimization algorithms implemented for allocation of DGs. Several methods have recently been proposed which they are mostly meta-heuristic algorithms including ant lion [7], artificial bee colony [4], crow search algorithm [8], genetic algorithms [9], grey wolf algorithm [10], harmony search algorithm [11], tabu search [3], whale optimization algorithm [12] and several variants of particle swarm optimization [13][14][15]. There are also several deterministic algorithms such as analytical [16], efficient analytical method [17], MILP [18] and MINPL [19]. Some of these studies are differs from the other depending on their multi-objectives including voltage, power indices and network losses [15]. Studies generally differs from each other according to their algorithm used, required computation time and effectiveness or success of algorithm to achieve objectives. Moreover, optimal allocation of only 1 DG unit is not quite enough for future distribution networks. Several papers are allocated multiple DGs in to the distribution networks [20] [2]. Also, different types of DG

have great importance for allocating DGs. Different generation limits are implemented to analyze impacts of DGs [21].

Different than other studies in the literature, a novel algorithm is proposed for more robust placing of the DGs. Despite this valuable interest on this subject, problem has not yet been analyzed enough with various scenarios whether distribution networks are capable to accomplish future needs. Therefore, optimal allocation and sizing of DGs is considered up to four units in this study. Different DG types are included with different DG real and reactive generation capacities. Allocated DGs are analyzed regarding their performance on voltage profile, power losses and line currents for both instant and hourly performances. Hourly performances of the networks are examined applying uncertainties of both generation and loads sides. Uncertainty of production side is implemented using wind speed probability density function at each DG. Proposed algorithm is implemented to radial 33 bus test feeder using programming language Python. Results are compared with each other on subjects of improving voltages, reducing the system losses and line currents.

This paper is structured so that the introduction is followed by proposed methodology, which is given in Section II, after that Section III gives details on the modelling of the wind turbines, allocation of DGs depending on several cases is given in Section IV and best cases are analyzed 24 hourly basis including uncertainties given in Section V. Finally, conclusions are summarized in Section VI.

## 2. Methodology

Proposed algorithm is combination of k-means clustering and particle swarm optimization algorithms.

### A. k- Means clustering algorithm

k- Means clustering is the one of the most used clustering algorithm and it is described as an unsupervised machine learning method which aims for dividing dataset into k clusters [22]. This method is chosen on account of the fact that it's easy implementation and effective results. Algorithm objective is maximizing the similarities in each cluster and minimizing the similarities between clusters. Algorithm achieves its objective by minimizing the distances between data in each cluster. Equations are given in followings.

$$S_i^{(t)} = \{x_p: \|x_p - \mu_i^{(t)}\|^2 \leq \|x_p - \mu_j^{(t)}\|^2 \forall j, 1 \leq j \leq k\} \quad (1)$$

$$\mu_i^{(t)} = \frac{1}{|S_i^{(t)}|} \sum_{x_j \in S_i^{(t)}} x_j \quad (2)$$

Basic workflow of algorithm is given below.

1. k<sup>th</sup> number of clusters are determined randomly.
2. Mean distance of data are evaluated for each clusters and data set are re-clustered depending on mean values.
3. New cluster means are evaluated.
4. Process is repeated until clusters are stable.

### B. Particle Swarm Optimization (PSO) algorithm

Other method that used in this study is PSO which is population-based meta-heuristic optimization technique [23]. In PSO, initial conditions of population are randomly determined, and optimal solution is updated by each generation. Particle positions are updated according to their velocities at each iteration according to given equations.

$$v_i^{(t+1)} = wv_i^{(t)} + c_1 * rand * (p_{best} - x_i^{(t)}) + c_2 * rand * (g_{best} - x_i^{(t)}) \quad (3)$$

$$x_i^{(t+1)} = x_i^{(t)} + v_i^{(t+1)} \quad (4)$$

### C. Proposed Algorithm

Proposed algorithm divides network into several pieces by k-Mean clustering thereafter starts to detect the most proper placement with PSO for DGs depending on objective of the problem. Voltage magnitude and angle data obtained by initial power flow results are used for clustering. Regardless from the distribution network size, buses can be clustered into 3 different clusters. Voltage magnitudes and angles in each cluster are relatively close to each other at buses. Therefore, connectivity between buses will not be interrupted inside of clusters. However, it should be noted that connectivity between buses at same cluster might fail if number of clusters are increased too much. k-Means Clustering algorithm is implemented on the 33- Bus feeder base case to in order to give an example. Clusters are visualized depending on the results of k-Means clustering algorithm. Clusters created by algorithm are shown within power flow results in the Fig. 1.

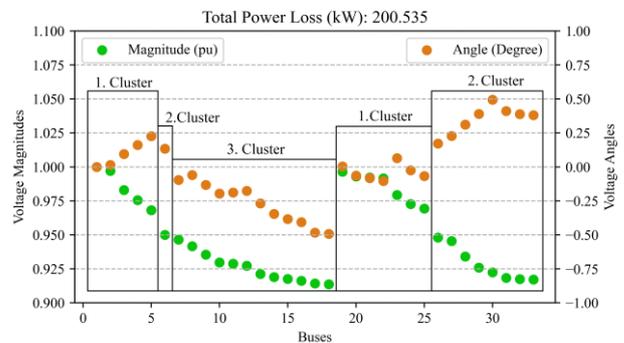


Fig. 1. Clustering results of base case

As it is given in the figure, buses are clustered. Depending on these results, DG will be allocated on the one of these clusters. By this way, search space of buses will be reduced significantly while allocating.

After the clustering of network is completed, DGs are allocated to selected cluster depending on DG types with PSO. Selecting one of the clusters lowers the possible placing of the DGs thereby reducing required number of iterations. For this reason, preprocessing of data decreases computational time significantly, yet does not decrease efficiency of the algorithm. Finally, proposed algorithm is repeated as much as number of the DGs while considering DG type and generation constraints.

### 3. Problem Formulation

Several cases including multiple DGs are implemented with different production capabilities and sizes in order to investigate the impacts of DGs to distribution networks. All cases are summarized in Table I.

Table I. Abstract of implemented cases

Number of DG	Maximum Sizes		
1	1	2	3
2	0.833	1.667	2.5
3	0.667	1.333	2
4	0.5	1	1.5
Min. cos $\phi$	0.8	0.9	1

Simulation is repeated for the number of DGs given with minimum power factor and maximum sizes. 36 Cases are presented according to DG number implemented yet only the best cases are presented in detailed. Also configuration of 33 Bus Test Feeder network is shown in Fig. 2.

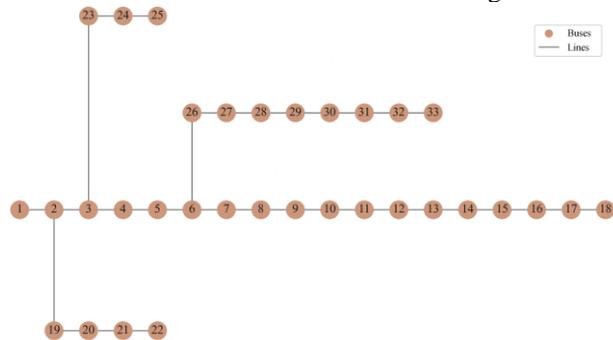


Fig. 2. Configuration of 33 bus test feeder network

DGs are taken as wind turbines according to results of best cases in order to include uncertainties at power generation side. Probability density function of wind speed is given in following equations.

$$f_w(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k\right) \quad (5)$$

$$k = 2 \quad f_r(v) = \frac{2v}{c^2} * \exp\left(-\left(\frac{v}{c}\right)^2\right) \quad (6)$$

$$c \approx 1,128 * v_m \quad (7)$$

Probability density function of wind speed is depended on the average wind speed which is taken 9 m/s. Moreover, 35 m/s is the maximum wind speed generated by algorithm. Power output of wind turbines are expressed as below [24].

$$P_W = \begin{cases} 0 & 0 \leq V < V_i \\ P_r(A + B * V + C * V^2) & V_i \leq V < V_r \\ P_r & V_r \leq V < V_o \\ 0 & V \geq V_o \end{cases} \quad (7)$$

$$A = \frac{1}{(V_i - V_r)^2} \left[ V_i(V_i + V_r) - 4V_i V_r \left(\frac{V_i + V_r}{2V_r}\right)^3 \right] \quad (8)$$

$$B = \frac{1}{(V_i - V_r)^2} \left[ 4(V_i + V_r) \left(\frac{V_i + V_r}{2V_r}\right)^3 - (3V_i + V_r) \right] \quad (9)$$

$$C = \frac{1}{(V_i - V_r)^2} \left[ 2 - 4 \left(\frac{V_i + V_r}{2V_r}\right)^3 \right] \quad (10)$$

$V_i$  is the cut-in speed,  $V_r$  is the rated speed and  $V_o$  is the cut-out speed. It should be noted that wind turbines are taken as producing only active power.

### 4. Allocation of DGs

#### A. 1 DG Installed Cases

1 DG installed case results are given in the Table II.

Table II. Installed 1 DG scenarios results

No	Bus	MW	MVar	Loss (kW)
1	30	0.77	0.64	94.98
2	30	0.85	0.53	96.26
3	30	1.00	0.00	126.60
4	28	1.56	1.25	64.14
5	28	1.70	1.05	65.92
6	6	2.00	0.00	108.23
7	6	2.47	1.70	61.39
8	6	2.55	1.58	61.67
9	6	2.56	0.00	103.85

When the table is analysed, it can be seen that limitations of the sizing and power factor has great importance. Bus 6 is optimal place for 1 DG installed cases similar to several studies. However, it not always the best bus when limitation of DG is tighter which it should be considered in real life applications. Apart from this, cases supplying only active power (3, 6 and 9) is not adequate for reducing the system total losses as much as other cases. Therefore, it is obvious that network needs reactive power. Even though case objectives are close, best result is the case 7. Voltage profile of case 7 is shown in the Fig. 3.



Fig. 3. Voltage profile of case 7

Result shows that voltage profile is much better when it is compared with the base case which is given in Fig. 1. Bus magnitudes are between 0.965 and 1 pu and it directly effects line losses. Comparison of power loss and line current between base case and case 7 is shown in Fig. 4.

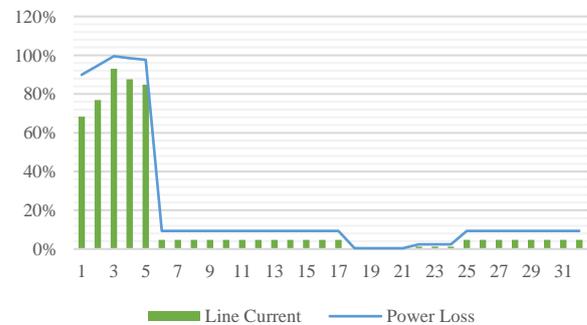


Fig. 4. Percentage of line currents and loss reduction in case 7

Figure shows that beginning of the feeder (lines 1-5) has reduced more than 90% for power loss and 68% for the

line currents. To sum up, average line currents and power losses have been reduced 16% and 69%, respectively.

### B. 2 DG Installed Cases

2 DG installed case results are given in the Table III.

Table III. 2 DG installed case results

No	Bus	MW	MVAr	Bus	MW	MVAr	Loss (kW)
10	31	0.65	0.53	26	0.70	0.46	63.33
11	31	0.71	0.44	6	0.71	0.44	64.43
12	31	0.83	0.00	28	0.83	0.00	109.59
13	30	1.29	1.06	13	0.80	0.40	29.05
14	30	1.42	0.88	28	0.53	0.33	60.21
15	26	1.67	0.00	14	0.67	0.00	89.72
16	28	1.63	1.31	13	0.70	0.33	37.71
17	6	2.13	1.32	31	0.68	0.42	40.14
18	6	2.50	0.00	9	0.42	0.00	99.10

When table is compared with 1DG installed cases, it is obvious that performance has been increased by reducing the losses for each case. DGs with supplying only active power cases still give the poorest results among cases. Thus, supplying reactive power is required for greater improvements in the network. When power generations are compared with case 7, it can be seen that even though DG are supplying lesser power to network, case 13 achieves better results. Therefore, it can be said that division of installing power has providing greater beneficial. Case 13 is shown a great performance and it is far better than among the other cases implemented for now. Voltage profile of case 13 is shown in the Fig. 5.

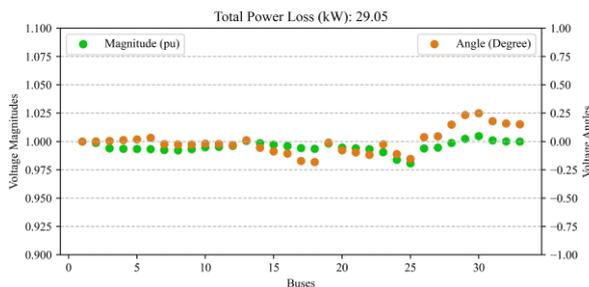


Fig. 5. Voltage profile of case 13

Result shows that bus magnitudes are between 0.981 and 1.005 pu. Maximum voltage is slightly higher than 1 pu. Comparison of power loss and line currents between base case and case 13 is shown in the Fig. 6.



Fig. 6. Percentage of line currents and loss reduction in case 13

Power loss and line currents have reduced effectively not only beginning of the feeder as previous case but also at the

end of the feeder. Power loss of lines 1-10 is reduced more than %83 further, line currents of these lines are reduced %59. Average power loss has been reduced %86 and line currents have been reduced %38.

### C. Installed 3 DG

3 DG installed case results are given in the Table IV.

Table IV. Installed 3 DG scenarios results

No	↓	*	o	↓	*	o	↓	*	o	L
19	31	0.52	0.42	28	0.52	0.42	29	0.52	0.41	56.61
20	31	0.57	0.35	28	0.57	0.35	29	0.57	0.35	58.23
21	31	0.67	0.00	28	0.67	0.00	28	0.49	0.00	108.07
22	30	1.03	0.85	4	1.16	0.66	13	0.72	0.37	19.98
23	30	1.13	0.70	28	0.81	0.50	28	0.00	0.00	57.76
24	28	1.33	0.00	6	1.22	0.00	16	0.41	0.00	87.97
25	28	1.56	1.25	13	0.72	0.34	31	0.15	0.15	35.12
26	28	1.70	1.05	31	0.41	0.26	29	0.00	0.00	59.19
27	6	2.00	0.00	13	0.58	0.00	32	0.42	0.00	82.34

(↓): DG bus (\*): MW (o): MVAr (L): Loss (kW)

Case 22 has the best result among all other cases. Even though results are better than case 13, it should be noted that total installed power is also greater too. Additionally, when Table IV and Table IV are compared, 3<sup>rd</sup> DG is not quite beneficial as much as 2<sup>nd</sup> DG for instant power cases. However, decisions should always take depending on costs. Voltage profile of Case 22 is shown in the Fig. 5.

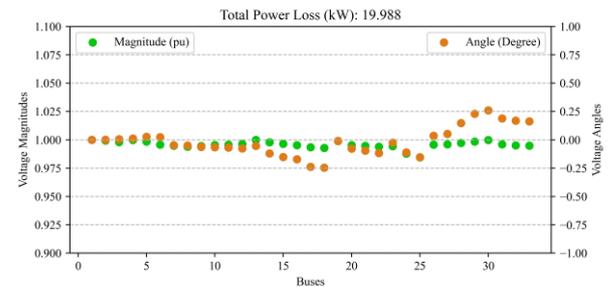


Fig. 7. Voltage profile of case 22

Result shows that bus magnitudes are between 0.985 and 1.005 pu. This outstanding performance resulted with reducing the losses as same. Comparison of power loss and line currents between base case and case 22 is shown in Fig. 8.



Fig. 8. Percentage of line currents and loss reduction in case 22

Power loss of lines 1-10 is reduced more than %91 further, line currents of these lines are reduced %69. Total performance of case is remarkable, average power loss has been reduced %90 and line currents have been reduced %43.

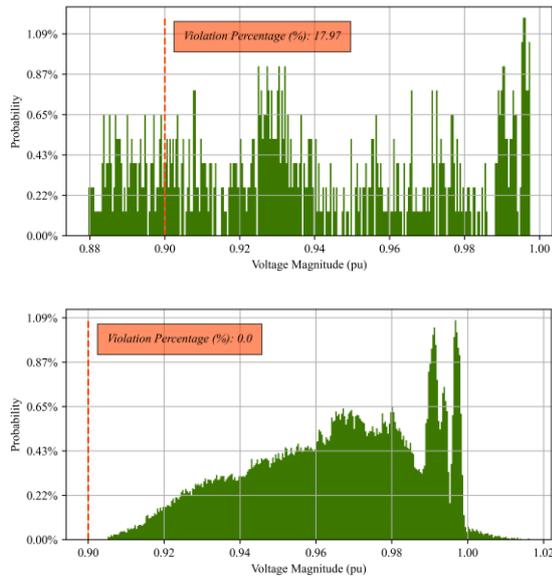


Fig. 10. Voltage probability functions of buses except 1st bus (Clockwise from top-left: base case, 1 DG, 3DG, 2DG)

#### D. Installed 4 DG

4 DG case was not better than the case 22, therefore it is not given in this paper because of page limit obligation. Additionally, it can be said that installing more DGs is not beneficial after installation of 3rd DG for 33 Bus Feeder.

### 5. Network Analysis including Uncertainties

Hourly bus loads generated randomly depending on base load profile are given in the Fig. 9.

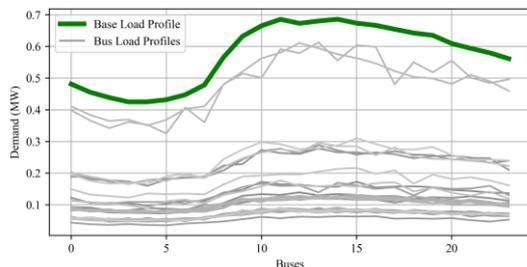
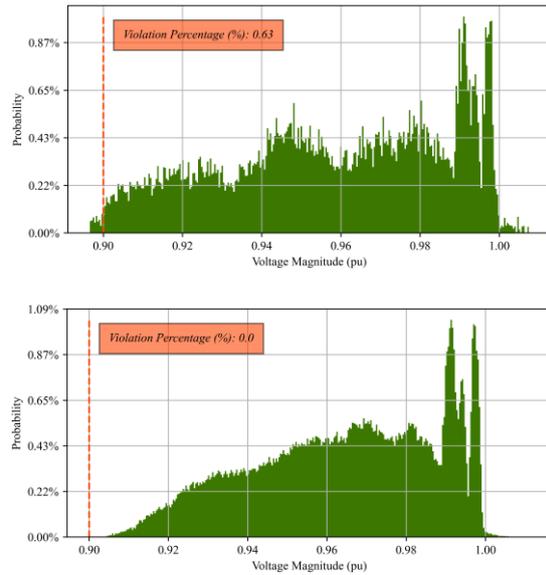


Fig. 9. Hourly base load profile and bus load profiles

Network has analysed according to hourly loads generated randomly. At the same time, wind speed is randomly generated depending on the previous equations. Then, output power of wind turbines is evaluated according to given equations. Based on the findings of this paper, network needs reactive power to improve voltage profile. Therefore, installed reactive power is considered continuously supplying power to network. Wind turbines were taken as supplying only the active power to network. Implementation has repeated 500 times for each hour. Voltage probability density function except 1st bus, which is 1pu for all hours and iterations, of base and best cases are shown in the Fig. 10. Figure shows that bus voltages violates limits, which is 0.9 pu, without any DG installed. Violation probability has been reduced to 0.63% from 17.97% at 1DG installed case. Furthermore, there are no violation at buses in 2 and 3 DG installed cases. At last, the



findings of this study are summarized in Table V and Table VI, respectively.

Table V. Results of best cases

No	Loss (kW)	Line Curr.	Loss	V. Min.	V. Max.	V. Avg.
7	61.39	16%	69%	0.965	1	0.985
13	29.05	38%	86%	0.981	1.005	0.996
22	19.98	43%	90%	0.985	1	0.996

Table VI. Results of best cases including uncertainties

Cases	Avg.	Std.	Viol.	Min.	Max.
Base	Loss	279.44	94.38	-	139.852
	Voltage	0.94	0.04	18%	0.880
1 DG	Loss	169.30	77.55	-	43.793
	Voltage	0.96	0.03	1%	0.897
2 DG	Loss	129.85	65.83	-	21.900
	Voltage	0.97	0.02	0%	0.905
3 DG	Loss	124.59	63.41	-	15.065
	Voltage	0.96	0.02	0%	0.904

### 6. Conclusion

In this paper, impacts of installing DGs to distribution networks has been analysed. Comprehensive results have demonstrated that DGs has been improved significantly voltage profile, reduced power loss and line currents. Power losses have been reduced 69%, 86%, 90% and line current have been reduced 16%, 38%, 43% for 1, 2, 3 DG installed cases, respectively. Moreover, impacts of DGs are confirmed including uncertainties with both demand and generation sides. Average power loss have been reduced 39%, 53%, 55% for 1, 2, 3 DG installed cases, respectively. Further, probability of under voltages have been solved for 2 and 3 DG installed cases.

Findings would seem to show that dividing the installed power up to a certain point has greater advantages. Also, installing one more DG unit might not be needed and if it is needed size would be relatively smaller. However, installing 3 DG is adequate according to results. At last, reactive power must be supplied to system whether by compensation systems or synchronous generators. It is

clear that supplying only active power is not adequate for improvements in the network.

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