

## Analysis of probabilistic properties of harmonic currents of loads connected to the high voltage networks

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**Abstract.** The paper presents the results of a statistical analysis of the probabilistic properties of harmonic currents of the loads connected to the high voltage networks. The analysis was based on the measurements of current of three facilities: an aluminum smelter shop, a paper mill and a railway traction substation. These loads have a large capacity and receive electric power from the 220 kV network. The main technological equipment of the facilities is a source of harmonics. The facilities have different load curves. The load of the aluminum smelter shop during a day time remains almost constant. The load of the paper mill varies during a day time due to the shift operation. The load of the traction substation is abruptly variable. The analysis shows that the arrays of measured harmonic currents are non-stationary and contain anomalous elements. In the currents of each load there are prevalent harmonic components. It is established that in the process of measurement the direction of harmonic current flows changed. The harmonic currents sometimes flow from the network to the load and sometimes vice versa. The probability density functions for the values of harmonic currents in most of the cases are a composition of several known distributions.

### Key words

Power quality, harmonics, measurement, statistical analysis, high voltage network.

### 1. Introduction

Experimental studies on the harmonic currents and voltages in the electrical network have been conducted since long ago. The results of measurements in the low- and medium- voltage networks are presented in the substantial and well known publications [1-5]. Their authors present the results of research into the harmonic conditions in the network with different types of loads. In the last decade intensive measurements of parameters of the harmonic components have been continued, which is explained by the emergence and wide use of new nonlinear electric equipment and increase in the values of harmonic currents and voltages in the networks. For example, the

authors of [6] present the results of measurements at the network node feeding the nonlinear loads of the retransmission TV stations. In [7] the authors demonstrate the results of measured parameters of the harmonics generated by energy saving lamps. The paper [8] presents the harmonics measured in the medical center and their impact on the work of the medical equipment. In [9] the authors present the results of large-scale measurements of power quality indices in the 110 kV networks of Siberia, which cover 23% of Russia's territory. The measurements confirmed the presence of harmonics in the electrical networks. In many cases the harmonic voltage levels exceed the standards established in [10].

This paper presents the results of an analysis of harmonic currents of nonlinear loads connected to the nodes of the 220 kV network. The analysis is made on the basis of measured state parameters for 10 harmonics at the nodes connecting the supply network to three facilities: an aluminum smelter shop, a paper mill and a railway traction substation. Measurements were performed with the aid of the device "OMSK", which measures not only the indices of power quality but also currents, powers and other parameters. The measurements were carried out for 24 hours with a time interval of 1 minute. The considered facilities have a large capacity and receive electric power from the 220 kV network. The main process equipment is a source of harmonics. The facilities have different load curves. The load of the aluminum smelter shop remains virtually the same throughout a day. The load of the paper mill changes throughout the day because of the shift operation. The load of the traction substation is abruptly variable. The facilities are located in one area at a distance of several hundreds of kilometers from one another, but are powered by the 220 kV network of the interconnected power system. Besides the named nonlinear loads, there are other large-capacity nonlinear

loads in the 220 kV network. Their harmonic currents also spread across the 220 kV network and contribute to the formation of harmonic conditions.

The goal of the research is to study the probabilistic properties of harmonic currents for different nonlinear loads connected to the nodes of the 220 kV network. The obtained results will be used to develop the models of nonlinear loads connected to the high voltage networks, determine the contribution of the nonlinear loads to the voltage distortion, and select the technologies for the reduction in the harmonic levels and calculation of non-sinusoidal conditions in the high voltage electrical networks.

## 2. Analysis of measured parameters

The arrays of measured state variables represent time series of data, which can be analyzed using the methods of statistical analysis [11]. The results of the measurements are used to determine the following maximum and average load power for one phase: 50.2 MW and 46.6 MW - the aluminum smelter shop, 46.3 MW and 23.9 MW - the paper mill, 55.5 MW and 17.2 MW - the traction substation. The daily diagrams of the root mean square currents measured at the nodes connecting the studied loads to the 220 kV supply network are presented in Fig. 1.

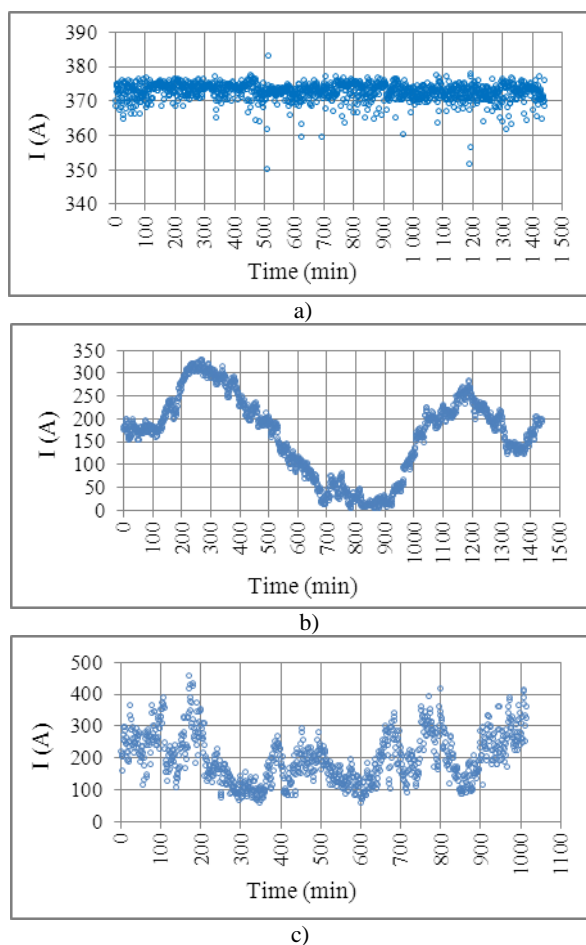


Fig.1. Scatter diagrams of load currents of the aluminum smelter shop (a), paper mill (b), traction substation (c).

The load current values for the aluminum smelter shop are quite stable due to continuous process of aluminum production (Fig. 1a) The load current of the paper mill varies throughout a day according to the shift operation of the facilities (Fig. 1b). The load current of the traction substation is abruptly variable which is due to a certain train schedule during a day (Fig. 1c).

### A. Check of the measured data for the anomalous elements

The analysis of the measured data shows that the arrays of data contain the elements, which differ greatly from the rest of the elements. Their values can be either much larger or much smaller. Similar measured values in special literature are called “anomalous” or “outliers” [12-13]. They distort the picture of conditions and do not allow us to construct a real histogram and correctly determine the probability density function. Also, they make the time series of measured parameters non-stationary. There are a lot of methods developed to detect the anomalous measurements, including the Irwin method [12] and Von Neumann test [13]. These methods were applied in this research.

The anomalous elements in the measured data of the paper mill and traction substation were detected using the Irwin method. For example, in the 25-th harmonic current of phase A of the traction substation we detected two elements with the values considerably exceeding the values of the other elements. These are the elements number 333 and 507. They are well seen from the scatter diagram of the time series of the 25-th harmonic current in Fig. 2. The anomalous elements are also present in the currents of the 19-th and 23-rd harmonics of the traction substation and in the currents of 10 harmonics of the paper mill.

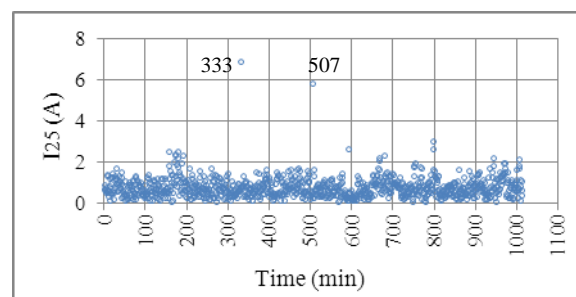


Fig. 2. Anomalous elements.

All the time series of harmonic currents measured at three nodes proved to be non-stationary. The series were checked for stationarity by the Von Neumann test [3]. The test consisted in the calculations of the Von Neumann coefficient. If its value equals two the series is stationary if below two the series is non-stationary. Table I presents the Von Neumann coefficients calculated for harmonic currents of three nodes of one phase. The values of the coefficients are below 2, which confirms the non-stationary state of the time series of the harmonic currents. Notations of the table are: A – aluminum smelter shop, P – paper mill, T – traction substation.

Table I. – Coefficients of Von Neumann

No-de	I3	I5	I7	I9	I11	I13	I17	I19	I23	I25
A	1.1	0.6	0.7	1.1	0.9	0.8	0.9	1.0	0.9	1.0
P	1.5	1.2	1.5	1.0	1.4	1.2	1.4	1.6	0.9	1.5
T	0.3	0.2	0.2	0.4	0.4	0.5	0.9	1.0	1.2	1.4

### B. Harmonics of load currents

The analysis included consideration of the current components for 10 harmonics. The diagrams presented in Fig. 3 demonstrate the order harmonics of measured currents and predominant harmonic current root mean square values. The values of the same harmonic currents at the studied facilities are different. The orders and values of harmonic currents of the aluminum smelter shop are determined by the rectifier circuit. For current rectification we use a three-phase 12-pulse rectifier circuit. The harmonic currents of the traction substation are determined by the traction load. Electric locomotives are driven by DC engines. The engines are powered through single-phase 2-pulse rectifier circuits. At the paper mill the sources of harmonic currents are represented by adjustable speed drive. Table II presents the maximum values (max), minimum values (min), average values (a) and standard deviations (s) for harmonic currents for one phase.

The analysis of the harmonic currents of the aluminum smelter shop shows that throughout the whole period of measurement the harmonic content remains constant. The values of the harmonic currents change but insignificantly. The currents of the 11-th and 13-th harmonics are the largest. The 12-pulse rectifier circuit explains this. The harmonics of the load currents of the paper mill remain the same but the values of the harmonic currents vary during the process of measurements. At different time instants the largest values are observed in the currents of the 3-rd, 7-th, 11-th and 13-th harmonics, which is explained by the change in the mix of operating equipment. The current of the traction substation is dominated by the 3-rd, 5-th and 7-th harmonic currents, which is characteristics of the single-phase 2-pulse rectifier circuit.

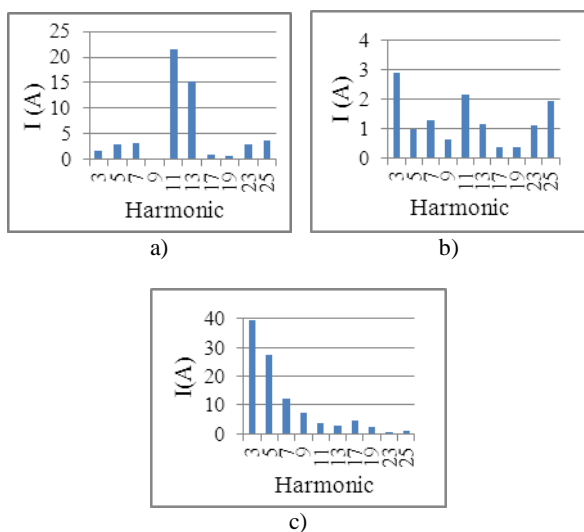


Fig.3. Diagrams of harmonic currents of the aluminum smelter shop (a), paper mill (b), traction substation (c).

Table II. – Statistical estimates of the harmonic currents (A)

Harmonic	3	5	7	9	11	13	23	25
A	max	3.2	5.7	4.6	1.6	24.5	19.5	6.4
	min	1.3	1.4	1.4	0.1	14.2	7.7	0.1
	a	2.2	3.2	2.9	0.6	21.6	15.6	2.8
	s	0.3	0.8	9.5	0.3	1.7	2.2	1.0
P	max	15.1	3.1	4.7	1.9	6.0	4.4	2.8
	min	0.1	0.0	0.1	0.0	0.0	0.0	0.0
	a	2.5	1.0	1.1	0.7	2.1	1.2	0.7
	s	1.3	0.6	0.5	0.4	0.7	0.5	0.4
T	max	71.4	45.6	23.2	11.3	9.3	5.9	9.6
	min	0.2	5.0	0.3	0.1	0.2	0.0	0.0
	a	16.1	20.6	9.3	3.5	3.7	2.2	1.2
	s	11.8	8.5	4.6	2.2	1.2	2.3	0.7

### C. Phase shift angles between harmonic voltages and currents

The analysis of phase shift angles between the root mean square values of harmonic voltages and currents, i.e.  $\varphi_n = \varphi_{U_n} - \varphi_{I_n}$ , shows that during the measurement period, the angle  $\varphi_n$  took the values in the range from 0 to  $2\pi$  for most of the harmonics of three loads. The exceptions could be seen for the 7-th and 11-th harmonics at the connection node of the aluminum smelter shop. The diagrams of angle  $\varphi_n$  for these harmonics are presented in Fig. 4.

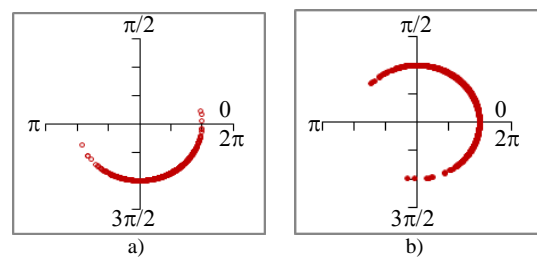


Fig. 4. Diagrams of  $\varphi_n$  for the 7-th (a) and 13-th (b) harmonics of the aluminum smelter shop.

At the connection node of the traction substation angles  $\varphi_n$  of the 5-th and 7-th harmonics (Fig. 5) have special diagrams.

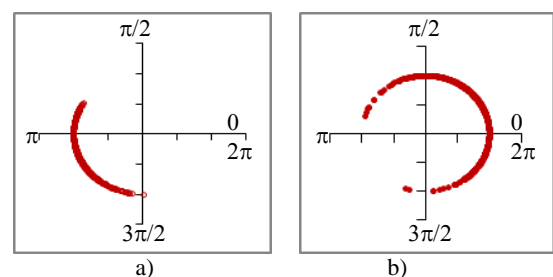


Fig. 5. Diagrams of  $\varphi_n$  for the 5-th (a) and 7-th (b) harmonics of the traction substation.

Since the aluminum smelter shop, traction substation and paper mill are consumers that have nonlinear electrical equipment, we assume that the harmonic currents will flow from the electrical equipment of these facilities to the supply network. The analysis of angles  $\varphi_n$  shows that

the harmonic currents also flow from the supply network to these facilities. Fig. 6 presents a scatter diagram of active and reactive components of the 13-th harmonic current of the aluminum smelter shop. The diagrams show that the directions in which the currents components are drawn change randomly in the course of measurement. We can assume that at a positive value of current it flows from the connection node to the load and at a negative value – from the network to the connection node. In both cases, the reactive component of the current is of a capacitive or inductive kind. Similar character of variation in the directions of harmonic currents is seen in the harmonic currents at the connection nodes of the paper mill and traction substation.

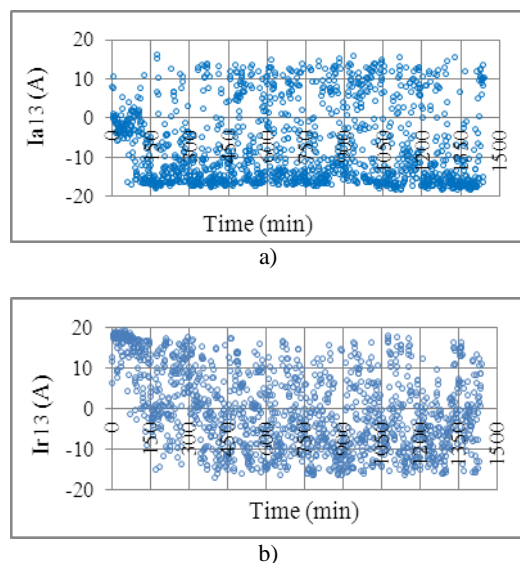


Fig.6 Scatter diagrams of active (a) and reactive (b) currents of the 13-th harmonic of the aluminum smelter shop.

#### D. The analysis of correlations

In order to determine the influence of harmonic currents of nonlinear loads connected to the considered nodes on the values of corresponding harmonic voltages at these nodes, we calculated the correlation coefficients ( $r_{U_n, I_n}$ ) between harmonic voltages and currents of phase A. They are presented in Table III. The values of coefficients are estimated in accordance with the Chaddock scale.

At the connection node of the aluminum smelter shop the correlations between all harmonic voltages and currents, except the 11-th harmonic are either missing or weak. The 11-th harmonic has a moderate correlation. The correlations at the connection node of the traction substation are also either missing or weak.

At the connection node of the paper mill the correlations vary. High correlation for the 11-th harmonic is highlighted in bold type, noticeable correlation for the 3-rd, 9-th, 13-th, 23-rd and 25-th harmonics is shown in bold italics. Thus, the values of correlation coefficients at the connection nodes of the aluminum smelter shop and traction substation indicate that the influence of harmonic

currents measured at these nodes on the corresponding harmonic voltages is insignificant.

At the connection node of the paper mill the influence of harmonic currents on the values of harmonic voltages is greater.

Table III. - Correlation coefficients

$n$	$r_{U_n, I_n}$			$r_{I_{an}, I_{rn}}$		
	A	P	T	A	P	T
3	-0.05	<b><i>0.56</i></b>	-0.12	-0.08	-0.24	<b>-0.83</b>
5	0.02	<b><i>0.47</i></b>	0.04	-0.19	0.10	<b>-0.52</b>
7	-0.05	-0.07	-0.01	-0.37	0.25	-0.29
9	-0.05	<b><i>0.62</i></b>	0.04	-0.10	0.05	-0.33
11	-0.43	<b>0.73</b>	-0.06	<b>0.59</b>	-0.17	-0.10
13	0.15	<b><i>0.56</i></b>	0.02	0.13	0.03	-0.29
17	-0.18	0.18	0.23	0.15	-0.08	-0.05
19	0.24	<b><i>0.40</i></b>	0.09	<b>0.40</b>	-0.10	-0.07
23	-0.21	<b><i>0.59</i></b>	0.09	0.13	-0.27	0.04
25	-0.24	<b><i>0.51</i></b>	0.22	-0.03	-0.26	0.11

The analysis of correlation coefficients ( $r_{I_{an}, I_{rn}}$ ) between active and reactive components of harmonic currents shows that the harmonic currents of the aluminum smelter shop have a moderate correlation for the 7-th and 19-th harmonics and a noticeable correlation for the 19-th harmonic. The active and reactive harmonic currents of the paper mill have a weak correlation. At the traction substation, there is a moderate correlation for the 9-th harmonic, noticeable correlation for the 5-th harmonic and high correlation for the 3-rd harmonic. Based on the analysis, we can conclude that there are no significant correlations between active and reactive components of harmonic currents at the studied nodes.

#### E. Probability density functions of harmonic currents

To determine the probability density functions of the measured harmonic currents, we constructed the histograms and checked them using Pearson's chi-squared test [11]. As a rule, the histograms are asymmetrical. Only a few of them satisfy a Gaussian distribution, which is confirmed by Pearson's chi-squared test. The constructed histograms are complex figures that represent combinations of the known probability density functions such as the Gaussian distribution, the exponential distribution and the Rayleigh distribution. Figure 7 presents scatter diagrams and histograms of currents for the 3-rd and 11-th harmonics for the aluminum smelter shop. The current for the 3-rd harmonic is distributed according to the Gaussian distribution, which is proved by Pearson's chi-squared test (Fig.7a). The figure shows a histogram constructed on the basis of the measurement results and a theoretical curve of the Gaussian distribution, corresponding to the mean value and standard deviation of the measured current for the 3-rd harmonic. The rest of the harmonic currents of the aluminum smelter shop have complex individual distributions. Figure 7b presents a histogram of current for the 11-th harmonic, which is canonical for a 12-pulse rectifier circuit. The histogram of current for the 11-th harmonic has a complex distribution.



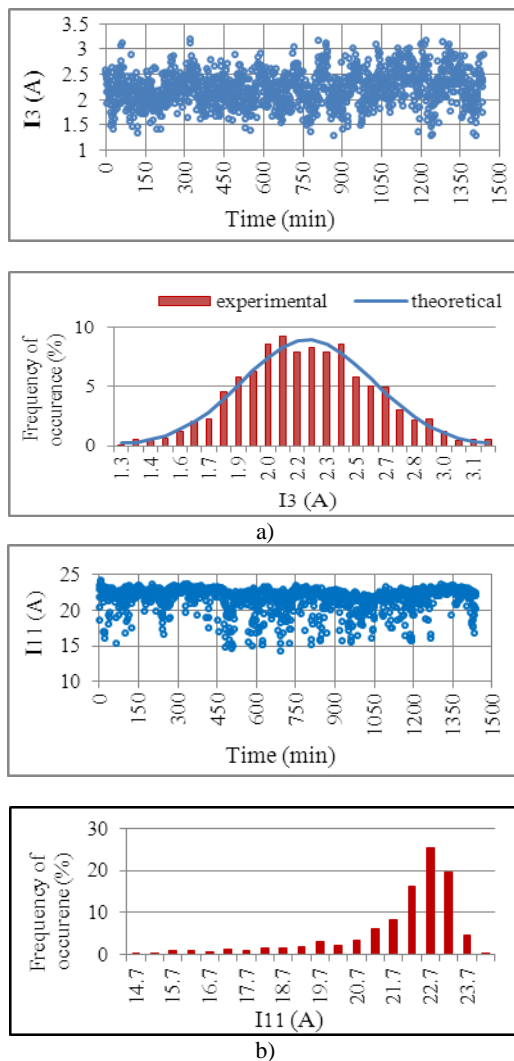


Fig.7. Scatter diagrams and histograms of currents for the 3-rd (a) and 11-th (b) harmonics of the aluminum smelter shop.

Figure 8 shows scatter diagrams and histograms of currents for the 3-rd and 11-th harmonics of the traction substation. The potential probability density functions of these harmonic currents are not confirmed by Pearson's chi-squared test. The demonstrated histograms and the histograms of the other harmonics also represent the combinations of the exponential distribution, the Rayleigh distribution, and possibly other distributions.

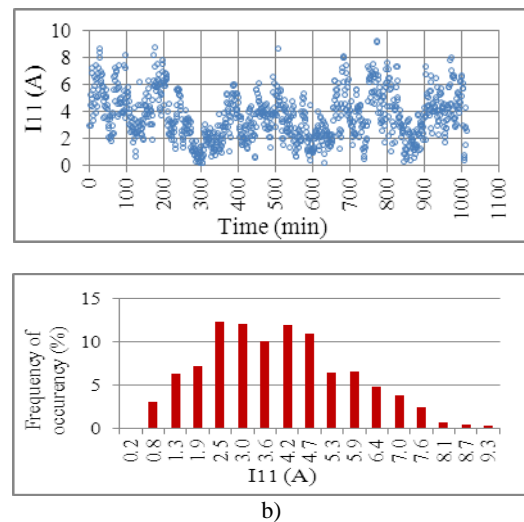
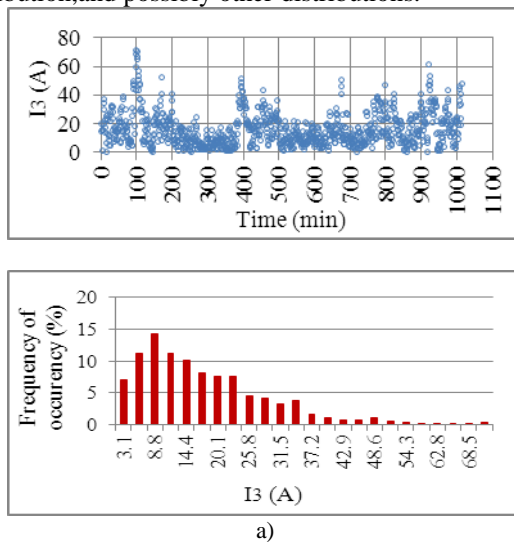


Fig.8. Scatter diagrams and histograms of currents for the 3-rd (a) and 11-th (b) harmonics of the traction substation.

The 3-rd harmonic current of the paper mill harmonic is shown in Fig. 9a. The current for the 11-th harmonic is distributed according to the Gaussian distribution, which is proved by Pearson's chi-squared test. The other harmonic currents represent combinations of different distributions.

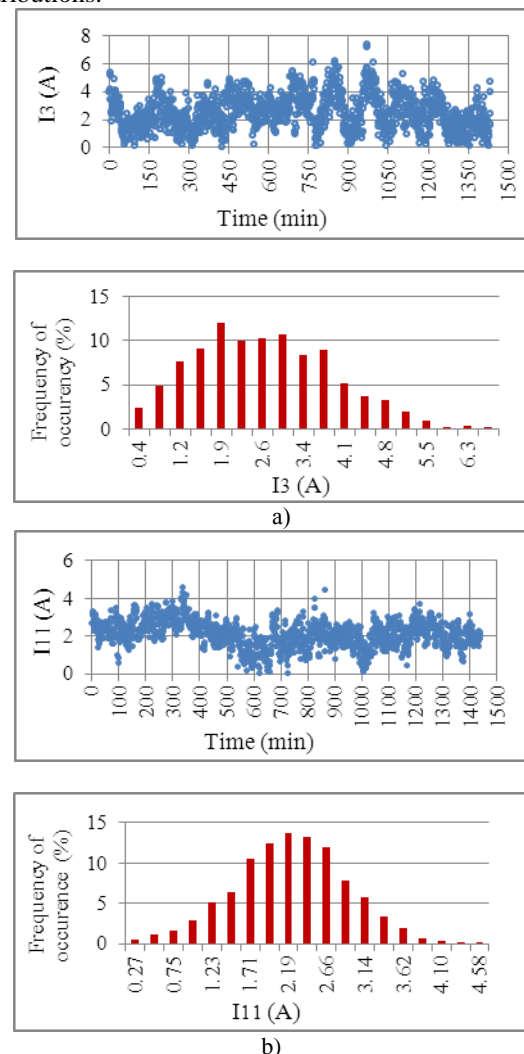


Fig.9. Scatter diagrams and histograms of currents for the 3-rd (a) and 11-th (b) harmonics of the paper mill.

### 3. Conclusion

1) The analysis of the measured harmonic currents at the nodes of connection of three different loads makes it possible to conclude that the harmonic conditions in the 220 kV network are of probabilistic nature.

2) The root mean square values of harmonic currents of the studied loads connected to the nodes of the 220 kV network is different. It is determined not only by the technological equipment at the facilities, but also by the harmonic currents of many other nonlinear loads connected to the 220 kV network and by the harmonic currents coming from the networks with other voltages.

3) The directions of harmonic current flows are changed. The harmonic currents sometimes flow from the network to the load and sometimes vice versa.

4) The measured sets of harmonic currents are non-stationary and include anomalous elements.

5) Harmonic currents have weak correlations between active and reactive components of currents and the corresponding harmonic voltages.

6) The probability density functions of harmonic currents represent combinations of several known of distributions as a rule

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