22nd International Conference on Renewable Energies and Power Quality (ICREPQ'24)
Bilbao (Spain), 26th to 28th June 2024
Renewable Energy and Power Quality Journal (RE&PQJ)
ISSN 2172-038 X, Volume No.22, September 2024

Volume Optimization for a Single-Phase Air-Core Reactor – 2D Finite Element Method and Particle Swarm Optimization Approach

Jeylto Alves de Souza Cruvinel & José Roberto Camacho

Alternative Energies and Electric Systems Protection Laboratory – LEAPSE
Faculty of Electrical Engineering, Universidade Federal de Uberlândia (UFU)
Santa Monica Campus – Uberlândia, Minas Gerais (Brazil)
Phone number: +55.16.99130.2879, +55.34.99971.2710

Abstract. This paper describes the optimization of an air core reactor using Particle Swarm Optimization (*PSO*) combined with the finite element analysis. The aim is to reduce material volume costs in manufacturing and maintenance. *PSO* is a nature-inspired optimization algorithm that simulates swarm behavior in search of the best solution in a multidimensional search space.
This article employs the PSO optimization method as an initial contribution to the design of single-phase air-core reactors, suggesting the possibility of further studies comparing its efficacy with other heuristic algorithms on the subject in the future.

Key words. Air Core Reactor, Particle Swarm Optimization, Finite Element Method, Optimization, Volume Reduction.

1. Introduction

This study proposes a methodology to optimize the volume of single-phase air-core reactors, aiming to minimize the use of material during their construction. The optimization is performed through the integration of the Particle Swarm Optimization (*PSO*) algorithm with electromagnetic calculations through the Finite Element Method (*FEM*). *PSO*, based on the collective behavior of particles in search of global optima, stands out for efficiently exploring solution spaces in optimization problems. By adapting to problem conditions, *PSO* seeks optimal solutions, considering the complexity of the variables involved in the design. This article combines *PSO* with *FEM*, formulating it as an optimization problem subject to a set of constraints, presenting a robust strategy for optimizing the volume of single-phase air-core reactors.

2. Single-Phase Air-Core Reactor

Single-phase air-core reactors are devices employed in medium-voltage distribution systems and high-voltage transmission for various purposes, such as fault current limitation, load flow control, reactive power compensation (shunt reactors), and as an inductive component in tuned harmonic filters [1].
Essentially composed, the air-core reactor consists of a core surrounded by air and a winding, which can be composed of one or more concentric conductors. Typically, these conductors are made of aluminum due to its lower cost compared to copper and its low specific weight. This results in lower investment in winding for a comparable level of losses [2].

3. Finite Element Method (*FEM*)

In this work, the finite element method was employed to calculate the electromagnetic parameters of the air-core reactor. Finite elements provide more accurate calculations and allow addressing phenomena whose effects are challenging to visualize in conventional analytical methods, as they are calculated based on the mapping of magnetic potentials along space in two or three dimensions.

The asymmetric cylindrical coordinate model is an approach to represent three-dimensional designs through a symmetrical plane concerning a vertical axis [3]. This plane covers all symmetrical rotations around the central axis, resulting in computational resource savings without compromising accuracy.

4. Problem Formulation

In this model, four independent variables will be used as input for the reactor optimization process. These variables include the height (*h*), the base (*b*) of the conductor's cross-section, the average radius of each coil (*R*), and the quantity of conductors (*N*). Meanwhile, the volume (*V*), current density (J_{femm}), quality factor (Q_{femm}), and inductance (L_{femm}) will be the output variables from the Finite Element Method (*FEM*). Among these design variables, the base, height, and average radius are considered continuous, while the quantity of conductors is treated as a discrete value. The representation of this model is shown in Figure 1.

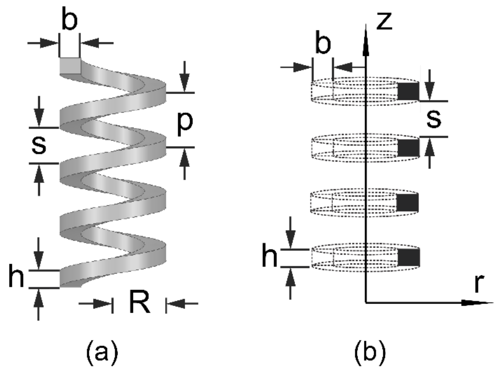

Fig. 1. (a) Schematic of the Air-Core Reactor Dimensions and (b) Representation of a Solid in an Axisymmetric Coordinate System.

The essential components of design optimization will be discussed next, covering parameters, constraints, limits, and the objective function.

A. Limits of Design Variables

The design variables in Figure 1 are constrained by the following inequalities:
:

$$h_{min} \leq h \leq h_{max} \quad (1)$$

$$b_{min} \leq b \leq b_{max} \quad (2)$$

$$R_{min} \leq R \leq R_{max} \quad (3)$$

$$N_{min} \leq N \leq N_{max} \quad (4)$$

The labels "*max*" and "*min*" indicate the upper and lower bounds, respectively.

B. Equality and Inequality Constraints

This model aims to equate the inductance calculated by the *FEMM software* to the desired inductance of the reactor ($L_{femm} = L_0$), using a penalization technique. The inductance penalty P_L is represented by:

$$\mathbf{P_L} = \left((\mathbf{L_{femm}} - \mathbf{L_0}) \cdot \mathbf{w_L}\right)^2 \quad (5)$$

Where L_{femm} is the obtained inductance, L_0 is the desired inductance, and w_L is an inductance weight constant. The variable w_L ensures an appropriate balance in weighting the inductance in the penalty equation.

Inequality constraints are applied for the current density and the quality factor. The current density J_{femm} must be kept below J_{max} ($J_{femm} \leq J_{max}$), with a penalty.

$$\mathbf{P_J} = ((\mathbf{max}(\mathbf{0}, \mathbf{J_{femm}} - \mathbf{J_{max}})^2) \quad (6)$$

Where J_{femm} is calculated through Equation (7).

$$\mathbf{J_{femm}} = \frac{\mathbf{I_0}}{\mathbf{h} \cdot \mathbf{b}} \quad (7)$$

Where h and b represent, respectively, the height and base of the cross-sectional area of the reactor conductors, and I_0 is the nominal current of the reactor.

The quality factor Q_{femm} must be greater than or equal to Q_{min} ($Q_{femm} \geq Q_{min}$), with a penalty.

$$\mathbf{P_Q} = \left(\mathbf{min}(\mathbf{0}, \mathbf{Q_{femm}} - \mathbf{Q_{min}})\right)^2 \quad (7)$$

C. Objective Function

In this optimization project of the dry-type air-core reactor, only the volume minimization objective function will be considered because the losses are a constraint that are related to the resistances, and these will be minimized according the reactor's quality factor.
However, to meet this goal, it is necessary to consider a set of constraints defined earlier. To deal with these constraints, the penalty technique will be adopted [6]. In this context, the penalty function is defined as:

$$\boldsymbol{\phi} = \mathbf{V_i} - \mathbf{r_p}(\mathbf{P_L} + \mathbf{P_Q} + \mathbf{P_J}) \quad (8)$$

The penalty coefficient r_p controls the impact of the penalty function on the objective function, allowing for a balance between seeking an optimal solution and satisfying the constraints [4].
To find the volume V_i in Equation (9), it is necessary to determine the conductor length l_i [5], which is given by Equation (9):

$$\mathbf{l_i} = \mathbf{N}\sqrt{\left(\boldsymbol{\pi}\left(\frac{\mathbf{R_i}}{\mathbf{2}}\right)\right)^2 + (\mathbf{p})^2} \quad (9)$$

Multiplying the length l_i by the cross-sectional area of the conductor ($h_i \times b_i$), and knowing that the pitch p is the sum of h_i and s, we find the volume V_i in Equation (10):

$$\mathbf{V_i} = \mathbf{N h_i b_i}\sqrt{(\mathbf{2\pi R})^2 + (\mathbf{h_i} + \mathbf{s})^2} \quad (10)$$

Where, V_i represents the volume of aluminum of the conductors, *b* and *h* represent the base and height of the cross-sectional area of the reactor conductors, respectively. R represents the radius of the reactor, and *N* represents the number of turns of the reactor.

5. Particle Swarm Optimization

Particle Swarm Optimization (*PSO*) is an evolutionary computing technique widely applied to various optimization problems across different domains. Proposed in 1995 by Kennedy and Eberhart [6], the inspiration for the *PSO* algorithm based in the social behavior of animals, such as birds and fish. The algorithm employs a set of particles to guide its search. Each particle has a velocity and is influenced by locally and globally found solutions. In each iteration, particles update their positions relative to both individual and the entire population, seeking promising solutions. Given its simple concept and effectiveness, the PSO has become a popular optimizer and has widely been applied in practical problem solving[7].

There are diverse ways to use *PSO*; in this work, the updates of particle velocities and positions follow the standard *PSO* equations. The velocity update is determined by Equation (12).

$$v_i = wv_i + c_1 r_1 \left(P_{best_i} - x_t\right) + c_2 r_2 \left(\mathrm{G}_{\mathrm{best_i}} - x_i\right) \qquad (12)$$

Where v_i is the velocity of particle *i* in the next iteration, *w* is the inertia coefficient,c_1 and c_2 are acceleration constants, r_1 and r_2 are uniformly distributed random values between 0 and 1, P_{best_i} is the best local position achieved by the particle, G_{best_i} is the best global position achieved by any particle in the population, and x_i is the current position of the particle.

The value of the corresponding position is then updated by:

$$x_i = x_i + v_\mathrm{i} \qquad (13)$$

Where x_i and v_i are, respectively, the position and velocity of particle *i* in the next iteration.

These updates allow particles to explore the search space in an ordered and adaptive manner, seeking increasingly better solutions throughout the iterations of the PSO algorithm.

6. Proposal of Optimization Model

The development of flowchart presented in Figure 2 uses the *PSO* method applied to the Finite Element Method.

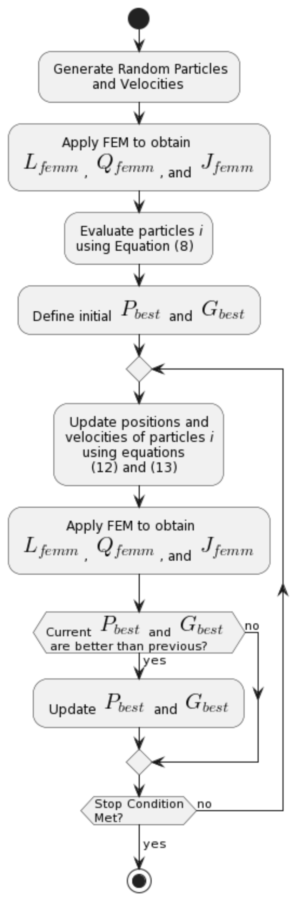

Fig. 1. Flowchart of the proposed optimization model.

In the flowchart presented in Figure 2, it can be observed that in the first stage, random positions and velocities are generated. These values serve as input to the finite element method, which produces the parameters V, L_{femm}, Q_{femm} and J_{femm}. These parameters are then evaluated through Equation (8), which represents the objective function. Subsequently, the best particles, P_{best} and G_{best}, are identified, and the velocities and positions are updated using Equations (12) and (13). This process is repeated until the stop condition is reached, with the criterion of the number of generations being chosen in this work as the indicator to terminate the algorithm.

7. Parameters and Optimization Criteria

To define the constraints boundaries and achieve a solution closer to the desired one in the optimization method, the limit values are provided in Table 1 below.

Table 1. Constraints of construction parameters.

Parameters	Minimum Value	Maximum Value
h	1 cm	4 cm
b	1 cm	4 cm
R	5 cm	50 cm
N	1 turn	100 turns
Spacing	5 mm	5mm

For this simulation, the following parameter values were adopted as specified in Table 2 for the *PSO* algorithm.

Table 2. Parameters used in PSO.

Parameters	Value
Number of Particles	250
Generations	100
Inertia Weight (w)	0.4
Cognitive Weight (c_1)	1.5
Social Weight (c_2)	1.5
Inductance Weight (w_L)	10^6
Penalty Coefficient r_p	100

With the use of these parameters, we aim to obtain the corresponding values of inductance, quality factor, and current density as indicated in Table 3. The material used for this simulation was air and aluminum found in the FEMM software library.

Table 3. Parameters used by the model.

Parameters	Value
Inductance (L_0)	$100\mu H$
Minimum Quality Factor (Q_{min})	20
Maximum Current Density (J_{max})	2 A/mm²
Frequency	60 Hz

8. Results and Discussion

Using the parameters described in Tables (1), (2), and (3), simulations of the model were conducted, resulting in the graphs presented in Figures 3 to 8.

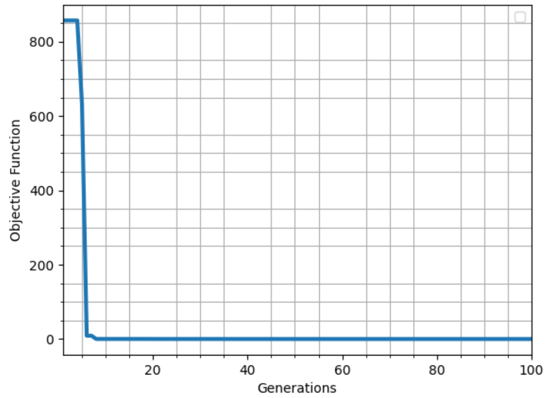

Fig. 3. Graph of the behavior of the objective function over generations.

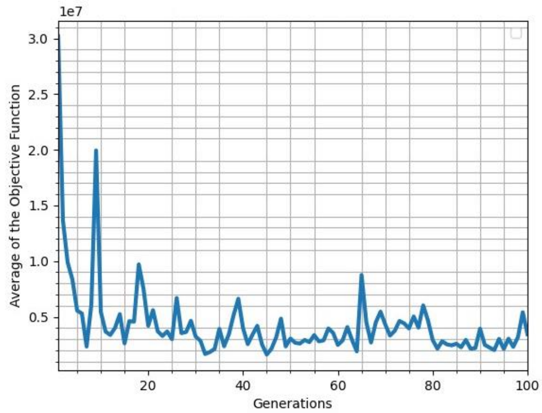

Fig. 4. Graph of the average of the objective function over generations.

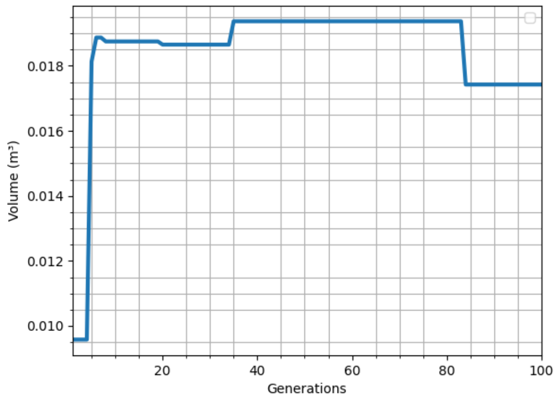

Fig. 5. Graph of the behavior of volume over generations.

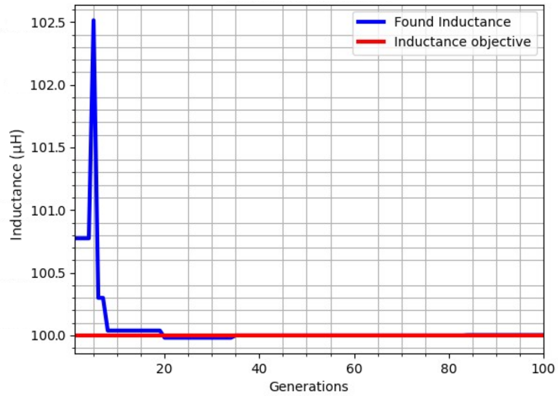

Fig. 6. Graph of the behavior of inductance over generations.

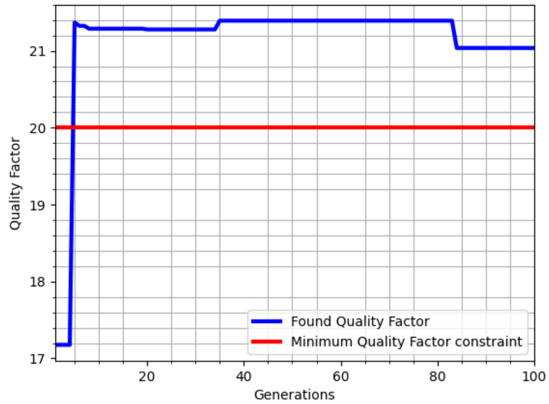

Fig. 7. Graph of the quality factor over generations.

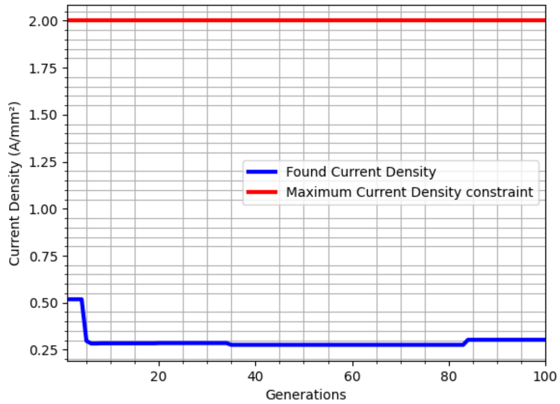

Fig. 8. Graph of the current density over generations.

Upon analyzing the graphs, we observe that the algorithm converged to the desired parameters by the 12th generation. In Figures 6, 7, and 8, we observe that the inductance approached 100μH significantly, the quality factor exceeded 20, and the current density remained below 1, as specified earlier. The detailed values of the parameters found are presented in Table (4).

Table 4. Table with the parameters found for the smallest volume reactor.

Parameters	**Reactor with minimum volume**
ϕ	0.017879
V	0.017424 m³
L_{femm}	100.002131 μH
Q_{femm}	21.036159
J_{femm}	0.302813 A/mm²
h	2.347385 cm
b	2.813654 cm
R	38.167172 cm
N	11 turns

Thus, based on the results in Table (4), it was possible to create the visual representation of the air-core reactor simulation in Figure 9.

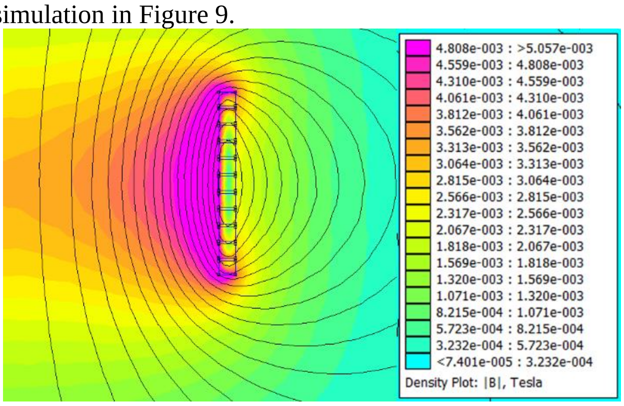

Fig. 9. Simulation of the magnetic flux density of the reactor from Table (4) using the FEMM software.

The simulations were conducted using an AMD EPYC 7F72 24-Core Processor (48CPUs) ~ 3.2GHz processor together with an NVIDIA RTX A6000 graphics card. 25,000 reactors were simulated using the finite element method, and the execution took 23 hours. The model computer code is in Python, and the *FEMM* software used for parallel processing with 48 cores. In this work, the research used the public domain PyFEMM, Multiprocessing, NumPy, and Matplotlib libraries.

9. Conclusion

This paper shows clearly that Particle Swarm Optimization, when associated with the finite element method, is an effective tool in the development of single-phase air-core reactors. This approach allows for the determination of desired parameters and the reduction of material volume used, resulting in savings in expenses and operation and transportation costs. Furthermore, this methodology proves to be highly practical and accurate in modeling physical problems, especially when integrated with finite elements. The use of axial symmetry of the reactor in the 2D finite element model simplifies the process, reducing computational complexity. The conclusion is that particle swarm optimization applied to finite elements methods can significantly enhance the efficiency in the design and production of air-core reactors and many other electrical devices.

Acknowledgments

Our thanks to the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES), Financiadora de Estudos e Projetos (FINEP), and the Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG) for the financial support in the elaboration of this research; to the colleagues of the research group at the Laboratory of Alternative Energies and Protection of Electrical Systems (LEAPSE), and to the Postgraduate Program at the School of Electrical Engineering (FEELT) of the Universidade Federal de Uberlândia (UFU) for the physical facilities used and the resources offered on their premises.

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