

## Control Design of a Two Degree of Freedom Combined with Repetitive Controller Applied to a Single Phase Inverter Power Generation in the Context of Microgrids.

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**Abstract-** This paper present the design, analysis and implementation of a control scheme Two Degrees Of Freedom 2DOF combined with a Repetitive Control. This reduces Total Harmonics Distortion in voltage ( $THDv$ ), when inverters that operates as voltage sources in microgrids. The controller is designed to be applied to inverters operating in island mode. The goal is to keep the waveform, frequency and amplitude of the grid voltage, when it has linear or nonlinear loads. The analysis and design of the control system is presented in detail.

### Keywords

Two Degree of Freedom, Repetitive Control, Microgrids

### 1. Introduction

Currently, as a result of increased electricity demand, they are promoting new forms of generation. Based on renewable energy such as wind, solar and fuel cell. These systems can work injected energy to the grid or to a separate load, it depends on the needs to be taken. In this context, this requires that you have systems that allow flexible work both ways. These new generation schemes, called Distributed Generation (DG) [1], [2].

In this scheme of generation, it is necessary to implement interfaces for the connection of these small units. These can be connected to the grid (operation in connection to grid). As well as, providing power to loads in the absence of the grid (island operation mode). These interfaces are known microgrids [3].

Considering that one of the ways to operate the inverter is in island mode, driving local loads without a grid connection. Must, ensure the quality of supply ensuring the waveform, amplitude and frequency of the signal to the load. To achieve this, it is proposed to implement a Two Degrees of Freedom

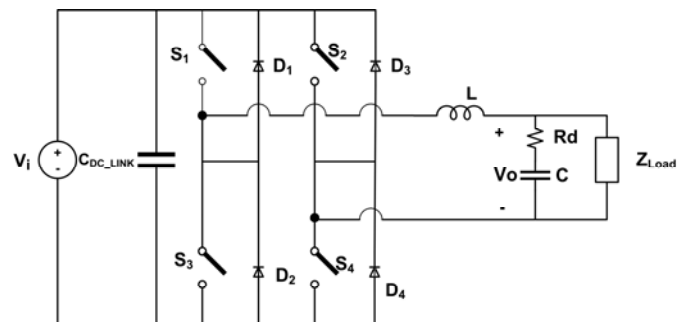


Fig. 1. Circuit scheme of the inverter for operation in island mode.

(2DOF) control combined with a repetitive controller, which in the presence of nonlinear loads reduce the Total Harmonics Distortion in voltage ( $THDv$ ) to be supplied the charge. Its application is justified, because the harmonic currents increase the loss, damage the quality of the voltage waveform, produce extra power for neutral, and can also cause resonance and interference [4].

### 2. System Description

The topology of the system under study is using a diagram of a single-phase full-bridge inverter as shown in Fig. 1. The input is an array of panels, with a power of 2kW and a voltage of 400V. The output voltage waveform is sine, with the amplitude  $230 V_{rms}$ .

For analysis, the inverter can approach a Buck circuit [5], taking into account that the point of operation, periodic signals are handled time-varying.

The expression that relates the output voltage to input voltage in terms of duty cycle D is presented in equation (1).

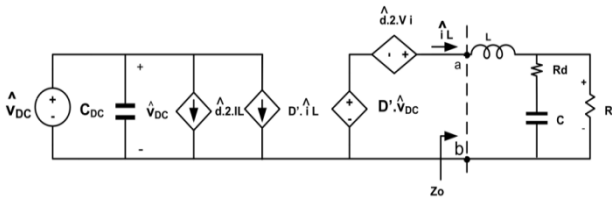


Fig 2. Loop control of voltage and current ACC investor island mode

$$V_o = V_{DC} \cdot (2 \cdot D - 1) \quad (1)$$

Assuming that, each of the averaged variables can be decomposed into a constant term for the operating point (Capital letters). And in terms of small signal variations (“^”) represents the perturbation around the operating point the equation (1) takes the following aspect:

$$V_o + \hat{v}_o = (V_{DC} + \hat{v}_{DC}) \cdot (2 \cdot (D + \hat{d}) - 1) \quad (2)$$

Separating the constant and variable components is obtained:

$$V_o = V_{DC} \cdot (2 \cdot D - 1) \quad (3)$$

$$\hat{v}_o = \hat{v}_{DC} \cdot (2 \cdot D - 1) + 2 \cdot V_{DC} \cdot \hat{d} \quad (4)$$

Similarly is can obtain the constant and variable components for the relationship input current to output current.

$$I_i = I_L \cdot (2 \cdot D - 1) \quad (5)$$

$$\hat{i}_i = \hat{i}_L \cdot (2 \cdot D - 1) + 2 \cdot I_L \cdot \hat{d} \quad (6)$$

### 3. Small Signal Model

From equations (5) and (6) gives the small-signal model of the inverter shown in Fig. 2.

From small-signal model and control scheme using average current mode control (ACC), ACC is shown in Fig. 3. It needs to regulate the inductor current inverter output ( $\hat{i}_L$ ) and the output voltage ( $\hat{v}_o$ ) of this to be delivered to the load. To meet this objective, it is determined the following transfer functions for this mode of operation, where:  $Z_{load} = R$ .

Where in figure 3,  $G_s$  represents the current control loop, and  $RD(s)$ , is the digital delay, which is defined as:

$$RD(s) = \frac{1 - \left(\frac{s \cdot T_s}{2}\right) + \left(\frac{(s \cdot T_s)^2}{12}\right)}{1 + \left(\frac{s \cdot T_s}{2}\right) + \left(\frac{(s \cdot T_s)^2}{12}\right)} \quad (7)$$

where,  $T_s$  is the switching period ( $T_s = 62.5 \mu s$ ),  $R_i$  is the gain of the current sensor ( $R_i = 0.2$ ),  $\beta$  is the gain of voltage loop sensor, ( $\beta = 0.006$ ) and  $F_m$  is the gain of the bipolar PWM modulator which is defined as:

$$F_m = \frac{1}{V_{pp-Triangular}} = 1 \quad (8)$$

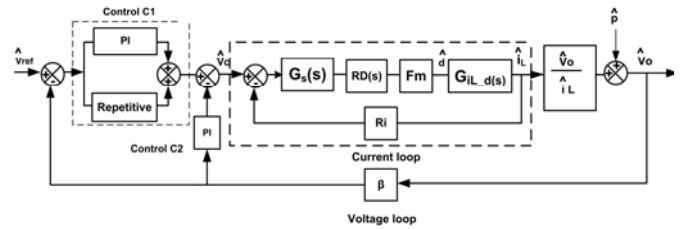


Fig.3. ACC average current control for inverter operation in island mode

The transfer functions for this mode of operation are:

Transfer function relating the output voltage to duty cycle

$$G_{VO-d}(s) = \left. \frac{\hat{v}_o}{\hat{d}} \right|_{i_{DC}=i_o=0} = \frac{2 \cdot V_{DC} \cdot Z_{eq}}{Z_{eq} + s \cdot L} \quad (9)$$

Transfer function relating the output current to duty cycle.

$$G_{iL-d}(s) = \left. \frac{\hat{i}_L}{\hat{d}} \right|_{i_{DC}=i_o=0} = \frac{2 \cdot V_{DC}}{Z_{eq} + s \cdot L} \quad (10)$$

Table 1 shows the design parameters of the inverter.

Table I. Inverter Design Parameters

Features	Valor
Inverter rated power	3 kW
Switching Frequency	16 kHz
Frequency reference signal	50 Hz
Inverter output voltage $V_{rms}$	230 V
DC_LINK Voltage	400 V
DC_LINK Capacitance	2 mF
Inductance L	5.46 mH
Filter capacitance C	4.7 uF
Damping resistance $R_d$ .	5 $\Omega$
Load Resistance R	17.16 $\Omega$

### 4. Current Controller Design

The current loop controller is designed with a  $P + Resonant$  control with the following characteristics [6]:

$$G_S(s) = K_P + \frac{K_h \cdot B_h \cdot s}{s^2 + B_h \cdot s + (\omega_h^2)} \quad (11)$$

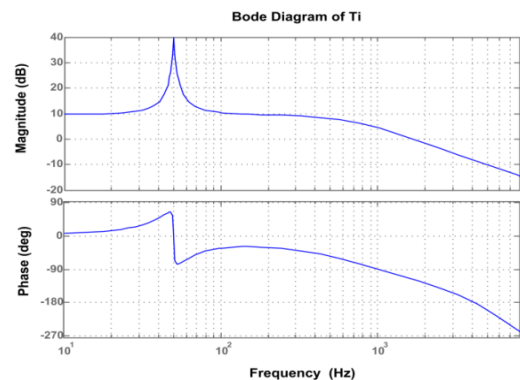


Fig. 4. Bode diagram of the current loop

Where  $K_p$  is proportional controller gain, the  $K_p$  value can be determined by the following expression:

$$K_p = \frac{L * \omega_c \text{ desired}}{R_i * F_m * 2 * V_{DC}} = 3.2 \quad (12)$$

$\omega_h = h * \omega_0$ , is defined as the pulsation multiple of the fundamental resonance.  $K_h$ , is the gain of the resonance peak at the frequency  $\omega_h$ .  $B_h$  is the bandwidth in rad/sec, which has resonance gain.

For this application  $h = 1, k_h = 100, B_h = 2 * \pi, \omega_0 = 2 * \pi * 50$ .

The implementation of this controller produces a current loop with the following characteristics: phase margin of  $62.1^\circ$ , gain margin of 8.24dB and bandwidth of 1.9 kHz. As shown in Fig. 4.

### 5. Voltage Regulator Design

#### A. Control of two degree of freedom 2DOF

The design of this controller is based in 2DOF control settings in combination with a repetitive control.

The control technique of two degrees of freedom can correct both disturbances in the system [7], [8], such as changes in the set point signal [9], [10]. That is, under this technique seeks to independently process the signals of reference and output [11] [12] [13] [114], for which there are different methods of tuning [15]. For this reason, the implementation of this technique is to have greater robust compared with that One Degree of Freedom.

#### B. Repetitive control

The addition of a repetitive controller, [10], [11] to the control scheme 2DOF, will goal to contribute to improving the disturbance rejection, reducing  $THD_v$  the inverter output voltage. This control, operation based on the Internal Model Principle (IMP) [16], for the design of controllers capable of tracking periodic references and reject periodic disturbances [12]. With the addition of this regulator is seeking to maintain the waveform, amplitude and frequency of the signal. [17], [18] [19].

The design of this controller is obtained from the scheme shows in Fig. 5: from which one obtains the following transfer functions:

$$G_{vo_{vref}} = \frac{\hat{v}_o}{\hat{v}_{ref}} = \frac{G_{vo\_vc} * C1 + \text{Repetitive Control}}{1 + (C1 + C2) * G * \beta} \quad (13)$$

$$G_{vo_p} = \frac{\hat{v}_o(s)}{\hat{p}(s)} = \frac{1}{1 + (C1 + \text{Repetitive Control} + C2) * G_{vo\_vc} * \beta} \quad (14)$$

From which it seeks to meet the following conditions:

The characteristics of the regulators who met this objective are:

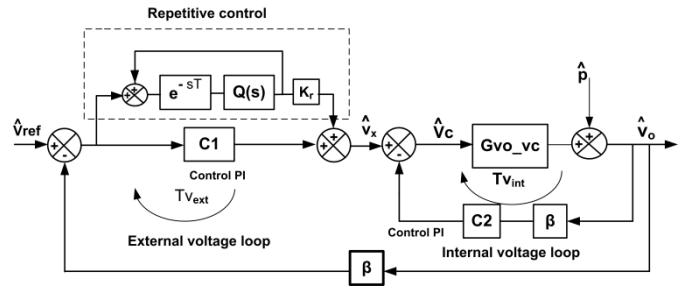


Fig. 5. Block diagram of control scheme Two Degrees of Freedom with Repetitive Control.

$$C2 = 0.0006344 * \frac{(s + 1.14e4)}{s} \quad (17)$$

$$C1 = 0.03988 * \frac{(s + 2.89e4)}{s} \quad (18)$$

$$\text{Repetitive control} = \frac{Q(s) * e^{-sT}}{1 - Q(s) * e^{-sT}} * Kr \quad (19)$$

$$G_{vo\_vref} = \frac{\hat{v}_o}{\hat{v}_{ref}} \approx \frac{1}{\beta} \quad (15)$$

$$G_{vo\_d} = \frac{\hat{v}_o(s)}{\hat{d}(s)} \rightarrow 0 \quad (16)$$

where  $Q(s)$  is a low pass filter of second order infinite response (IIR).

$$Q(s) = \frac{1}{\frac{s^2 + 2\epsilon s}{\omega_q^2} + 1} \quad (20)$$

$e^{-sT}$  is a delay.

where  $T$  is the period of the fundamental signal ( $T=0.02$ ),  $\epsilon$  is the damping coefficient of the filter ( $\epsilon=0.707$ ).

$$\omega_q = 2 * \pi * f \quad (21)$$

where  $f$  is the filter cutoff frequency ( $f = 400\text{Hz}$ ),  $Kr$  is the gain of the Repetitive Control ( $Kr = 0.3$ ).

With the design of these regulators results in a voltage loop gain ( $Tv$ ) given by:

$$Tv_{2DOF+Rep} = (C1 + Rep + C2) * G_{vo\_vc} * \beta \quad (22)$$

$$Tv_{PI} = Gv * G_{vo\_vc} * \beta \quad (23)$$

Fig. 6 shows the Bode diagram for the voltage loop gain expressed in (22). As the voltage loop gain of a PI controller [13] expressed by (23).

PI control has the following feature:

$$Gv = 0.06713 * \frac{s + 1.3e4}{s} \quad (24)$$

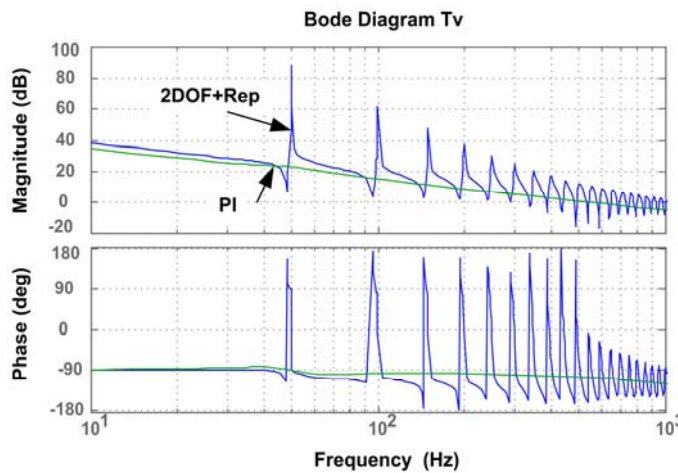


Fig. 6. Bode diagram for the voltage loop  $T_v$ .

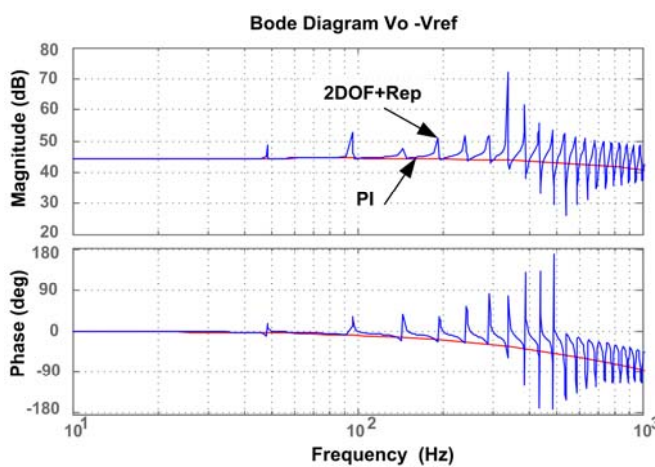


Fig. 7. Bode diagram for setpoint tracking with PI controller and 2DOF + Repetitive Control

Fig.6 shows that with the implementation of the controller of Two Degrees of Freedom in combination with repetitive control have high gain at all frequencies that are multiples of  $1/T$ . Where  $T$ , represents the fundamental period of the reference signal. This behavior ensures, the disturbance rejection and zero error in the reference tracking with spectral content of these frequencies, as shown in fig.7.

However, with the implementation of the PI controller gain in all the multiples of the fundamental frequency is lower. Therefore, the rejection of disturbances is lower compared with the implementation of previous regulatory. The results of the implementation of these regulations will be presented later with simulations.

The graphs in fig.7 are located at 44dB, because the loop gain of the voltage sensor is 0.006. Thus concluded that with the implementation of these controllers, have a good set point tracking of the plant.

With the PI controller, gives a voltage loop gain with the following stability characteristics: phase margin of  $73.5^\circ$ , gain margin of 12 dB and bandwidth of 569 Hz.

On the other side, the control of two degrees of freedom more Repetitive yields the following stability characteristics:  $GM=14.3dB$ ,  $P.M=75.85$  Deg.  $Freq=624Hz$ .

To reinforce the above analysis, it is convenient to calculate the closed-loop impedance of the inverter output. This, in order to verify the impedance characteristics of this inverter with regulators implemented. That is, it is intended that the inverter output impedance at low and high frequencies present a resistive behavior. In order to have good disturbance rejection.

## 6. Calculation of the Impedance Inverter

To calculate the output impedance of the inverter shown in Figure 1. Initially calculate the output impedance open-loop  $Z_{OLA}(s)$ . This calculation is done by analyzing the small-signal model of fig. (2) and the control scheme of fig.3.

The transfer function  $Z_{OLA}(s)$  is calculated making  $\hat{v}_{DC} = \hat{v}_C = 0$ . That is, assuming no changes in voltage in the DC LINK. With this consideration, the output impedance of open loop can be defined as:

$$Z_{O-LA}(s) = \left. \frac{\hat{v}_o}{\hat{i}_o} \right|_{\hat{v}_{DC}=\hat{v}_C=0} = \frac{Z_a * M}{s * Z_a * C_i + M} \quad (25)$$

Where:

$Z_a(s)$ , is the impedance presented to the right of the inductor in the small-signal model of Fig. 2.

$$Z_a(s) = \frac{R * (s * C * R_d + 1)}{s * C * (R_d + R) + 1} \quad (26)$$

Applying KVL and KCL to the small-signal model, defines the following equations:

$$s * C_i * \hat{v}_{DC} * I_L * \hat{d} + D * \hat{i}_L = 0 \quad (27)$$

$$D * \hat{v}_{DC} + V * \hat{d} = \hat{i}_L \quad (28)$$

$$\hat{i}_L = \frac{\hat{v}_o - \hat{i}_o * Z_a}{Z_a} \quad (29)$$

On the other hand, the current loop is:

$$\frac{\hat{d}}{\hat{i}_L} = -Fm * Ri * Gs * RD \quad (30)$$

Solving  $\hat{v}_{DC}$  (28) is obtained:

$$\hat{v}_{DC} = \frac{\hat{i}_L * V * \hat{d}}{D} \quad (31)$$

Substituting (31) and (29) in (27) and development terms, is obtain:

$$s * C_i * Z_L - V * K * s * C_i + D * I_L * K + D^2 \left( \frac{\hat{v}_o - \hat{i}_o * Z_a}{Z_a} \right) = -s * C_i * \hat{v}_o \quad (32)$$

Assuming that:

$$M = s * Ci * Z_L - V * K * s * Ci + D * I_L * K + D^2 \quad (33)$$

Obtained:

$$\frac{M * \hat{v}_O}{Z_a} - M * \hat{i}_O = -s * Ci * \hat{v}_O = Z_{O-LA}(s) \quad (34)$$

Finally, the output impedance of closed loop is:

$$Z_{O-LC}(s) = \left. \frac{\hat{v}_O}{\hat{i}_O} \right|_{\hat{v}_C = \hat{v}_{Ref} = 0} = \frac{Z_{O-LA}(s)}{1 + T_V(s)} \quad (35)$$

This expression was used to determine the output impedance of closed-loop inverter, with the implementation of the PI controllers and 2DOF + Repetitive Controller. Which is shown in fig.8.

Where in the fig.8 is shows that due to the action of the controller implemented in the current loop (*P + Resonant*) and the voltage loop (*2DOF + Repetitive Control*). The behavior of the inverter output impedance is resistive at low and high frequency, since phase is kept at  $0^\circ$ . However, with the implementation of the PI controller in the loop voltage, the characteristics of the inverter output impedance is inductive, presenting a phase of  $90^\circ$  at low and high frequency.

The above justifies the implementation of the control of two degrees of freedom in combination with repetitive controller, has a better performance in disturbance rejection. I.e, with this type of output impedance reduces is the THD of the inverter output voltage. That is, in the frequencies of the harmonics, the output impedance is small so the voltage drop caused by this will result in low distortion. As seen in the simulations performed below.

## 7. Results

These simulations were conducted in PSIM<sup>TM</sup> software [21].

Fig.9 shows the output voltage signal of the investor in operating island mode. This is with a PI controller and control of Two Degrees of Freedom with Repetitive Control feeding linear load. *THD<sub>v</sub>*. Which can be seen, that both controllers have good performance, as it is obtained a 0.7% *THD<sub>v</sub>*.

Fig.10 shows the output voltage signal of the inverter in operating island mode. With the implementation of a PI controller and feeding nonlinear load, with the features shown in table II. Which shows that do not have a good performance because it has a 8.8% *THD<sub>v</sub>*. The value is above the 5% allowed by IEEE Standard 519-1992.

Fig.11 shows the output voltage signal of the investor in the island mode operation. With the implementation of a Control of Two Degrees of Freedom combined with Repetitive Controller, and feeding nonlinear load, with the features shown in Table II. Where it is observed that with this controller gives a very good result, thus obtaining a *THD<sub>v</sub>* of 2.5%.

Table II. Characteristics of the Nonlinear Load

Nonlinear load, with full-wave rectifier	
Features	Value
Load Capacitance C	1.5 mF
Load Resistance R	85 Ω
Crest Factor	4.6

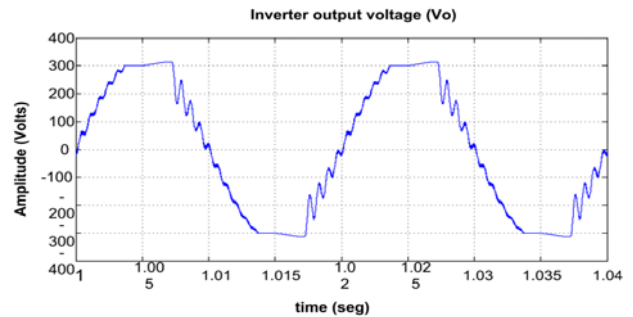


Fig. 10. (a) Waveform of output voltage with nonlinear load,

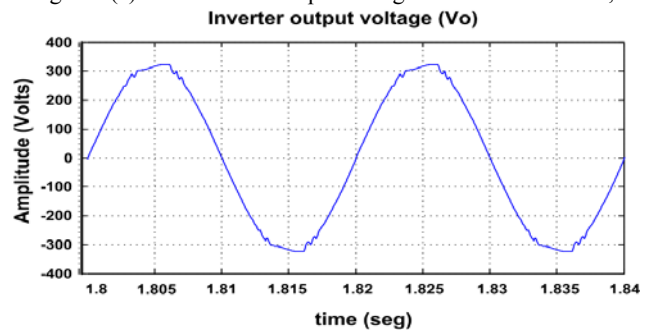


Fig. 11. (a) Waveform with nonlinear load output voltage

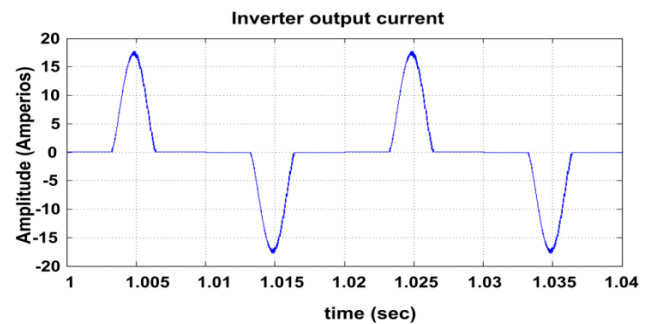


Fig.12. Waveform of the inverter output current, feeding nonlinear load.

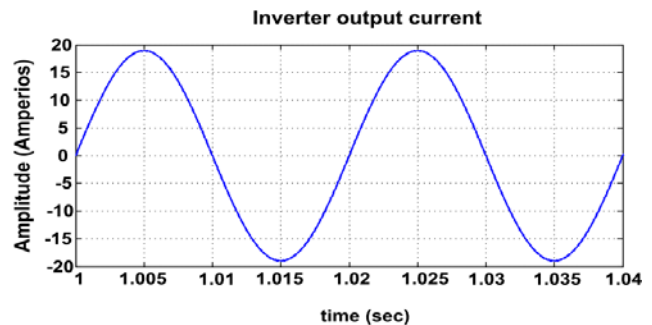


Fig. 13. Waveform of the inverter output current, feeding linear load.

Table III. Summary of Results

Controller	$THD_v$ (%)
PI Controller	8.8
Two Degree of Freedom + Repetitive control	2.5

Fig. 12, shows the waveform of the inverter output current and feeding nonlinear load. Which can be seen that the problem with nonlinear loads, is the nonsinusoidal current. This current is present for short periods of time during each half cycle. In contrast to a linear load where the sinusoidal current is present throughout the half cycle and is similar to the waveform of the applied voltage. Signal shown in fig.13.

Table III, shows the  $THD_v$ , which presents the output voltage of the inverter, feeding nonlinear load. Using a PI controller, and the controller of Two Degree of Freedom in combination with a Repetitive Control.

## 8. Conclusion

In this paper, present the design, development and results of a control scheme based on the configuration of two degrees of freedom in combination with Repetitive Control for an application of control of inverter operating in island mode in the context of microgrids. With this controller is able to regulate the inverter output voltage, keeping the waveform, amplitude and frequency of the grid voltage to linear and nonlinear loads. That is, when comparing the results obtained with the application of PI control and control of two degrees of freedom with Repetitive Control was observed that the latter was obtained  $THD_v$  lower than the first. That is to say, 2.5 and 8.8% respectively.

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